Comments for proof corrections

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1 Corrections improperly taken into account

- Page 22 line 67: as indicated in my previous corrections, do not replace "for $p_w \geqslant 0$ model in Eq. (4)" by "then $p_w \geqslant 0$ model in Eq. (4)". It should be "for $p_w \geqslant 0$ the model in Eq. (4)".
- Page 11 line 32 "The Green's function underlying" instead of simply"Green's function underlying".
- Page 27 line 19 "denote as K and L" should simply be "denote K and L".
- Page 27 formula on $\overline{\boldsymbol{x}}_K$: still incorrect spacing:

$$\overline{\boldsymbol{x}}_K \in \mathring{K} \quad \forall K \in \mathcal{T} \quad \text{and}$$

$$\frac{\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K}{|\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K|} = \boldsymbol{n}_{K,\sigma} \quad \text{for} \quad \sigma \in \mathcal{F}_{int},\ \sigma = \{K, L\}$$

2 Answers to the queries

CE1 Ok.

CE2 We prefer to remove the final "s" of simulations in this case, to get LES.

CE3 From the Cambridge dictionary, it is said that : "Consist of something = to be made of or formed from something" while "Consist in something = to have something as a main and necessary part or quality". We have used "consist in" in this sense. It is therefore relevant for us to keep "consists in" in this context because we consider the use of a filtrering strategy specifically.

CE4 This is worse than before. Let's go for "space scales and timescales".

CE5 You are right indeed. Thank you for explaining this.

CE6 Yes, use "considered to be" instead.

CE7 Yes, if you cannot leave CA alone we would prefer parenthetical commas instead. However, why do you dislike CA written like this on line 16 but are perfectly happy with $CA_\epsilon(K)$ on the previous line ? Both are used in the same way.

CE8 Same as above.

CE9 Ok.

CE10 Let's go for "of the family of MFD algorithms".

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CE11 Ok.

CE12 The true problem in “the mathematical requirement that is probably too strong” is the “the”. You can replace it by “this mathematical requirement that is probably too strong”.

CE13 We do have a problem here, “there exists a” is really an established mathematical terminology. You cannot alter it without modifying its meaning. Moreover “a constant $C > 0$ exists independently of h” has really not the same meaning as “there exists a constant $C > 0$ independent of h”.The first one implies that a constant C exists whatever the value of h, but the value of C can depend on h, while the second one implies that the value of C not only exists, but that its value does not depend on h. The formulation we have used is so classical that I cannot imagine how to modify it, and in fact why. Apart from the fact that I have read and used it countless times in articles, I have also made some verification. Everyone seems to accept the mathematical formulation as a correct sentence. This has the same grammatical structure as “there is a”. We insist on restoring “there exists a constant $C > 0$ independent of h”.

CE14 Same as above.

CE15 From the Cambridge dictionary, it is said that : ”Consist of something = to be made of or formed from something” while ”Consist in something = to have something as a main and necessary part or quality”. We have used “consist in” in this sense. It is therefore relevant for us to keep “consists in” in this context.

CE16 Simply write LES models.

CE17 From the Cambridge dictionary, it is said that : ”Consist of something = to be made of or formed from something” while ”Consist in something = to have something as a main and necessary part or quality”. We have used “consist in” in this sense. It is therefore relevant for us to keep “consists in” in this context because we consider the use of the differential filter specifically.

CE18 We were probably not clear enough there. The new version is “For quantities such as the water flux for which Neumann is a more natural boundary condition Neumann everywhere”, which we agree makes no sense. The original one was “For quantities such as the water flux for which Neumann everywhere is a more natural boundary condition”. “Neumann everywhere” is an established two-words noun for this boundary condition. If you really cannot bear “Neumann everywhere”, you could use “For quantities such as the water flux for which Neumann is always the natural boundary condition”, without any everywhere.

CE19 Same as above.

CE20 Ok

CE21 No, the meaning is modified. As a workaround, you can use “we can guess, using the convergence curves of Fig. 15, which filter sizes are giving a correct solution”

CE22 This is not correct. “Fixed elevation”, is an established formulation, and it is strange not to use it. However, if you cannot stand it, we can accept the former version “Model boundary conditions are fixed elevations”, but this is much less precise.

CE23 From the Cambridge dictionary, it is said that : ”Consist of something = to be made of or formed from something” while ”Consist in something = to have something as a main and necessary part or quality”. We have used “consist in” in this sense. It is therefore relevant for us to keep “consists in” in this context as we describe the nature of the test.

CE24-CE25 As indicated in our previous comments, if the problem is the face that the sentence has no verb, you can replace “since”, by “is missing because”. This would give “The first one is missing because numerical noise depends on the software and algorithms used and the number of processors, among other factors. The second one is missing because it is almost impossible to track how the numerical errors are generated.”

CE26 No, the meaning is modified. We suggest "As we can observe in Fig. 23, if the two models of course do not produce exactly the same results, their general behavior remains very similar"

CE27 Same as above.

CE28 Same as CE13 : this really is an established mathematical terminology, that cannot be altered. We insist on restoring "there exists a family of centroids $(\overline{\boldsymbol{x}}_K)_{K\in\mathcal{T}}$"

CE29 Same as CE13 : this really is an established mathematical terminology, that cannot be altered. We insist on restoring "there exist subsets $\mathcal{F}_{ext}^{\mathcal{N}}$ and $\mathcal{F}_{ext}^{\mathcal{D}}$ such that", without any parenthesis.

TS1 We give details for each equation that can be split:

. Page 4 equation 4:

$$\left|\begin{array}{l}\boldsymbol{Q}_w = -\dfrac{k_m h_w \eta_w(h_w)}{s_{ref}^{p_w}}||\nabla(h_s+b)||^{p_w}\nabla(h_s+b),\\[2ex] div\,(\boldsymbol{Q}_w) = S_w \quad \text{in } \Omega,\\[2ex] \boldsymbol{Q}_w \cdot n = B_w \quad \text{on } \partial\Omega_{in},\end{array}\right.$$

instead of:

$$\left|\begin{array}{ll}-div\left(k_m h_w \eta_w(h_w) s_{ref}^{-p_w}||\nabla(h_s+b)||^{p_w}\nabla(h_s+b)\right) = S_w & \text{in } \Omega,\\ -k_m h_w \eta_w(h_w) s_{ref}^{-p_w}||\nabla(h_s+b)||^{p_w}\nabla(h_s+b)\cdot n = B_w & \text{on } \partial\Omega_{in},\end{array}\right.$$

. Page 13 equation 19:

$$\left|\begin{array}{l}\boldsymbol{Q}_w = -\dfrac{k_m h_w \eta_w(h_w)}{s_{ref}^{p_w}}||\nabla(\mathcal{F}_\alpha(h_s+b))||^{p_w}\nabla(\mathcal{F}_\alpha(h_s+b)),\\[2ex] div\,(\boldsymbol{Q}_w) = S_w \quad \text{in } \Omega,\\[2ex] \boldsymbol{Q}_w \cdot n = B_w \quad \text{on } \partial\Omega_{in},\end{array}\right.$$

instead of:

$$\left|\begin{array}{ll}-div\left(k_m h_w \eta_w(h_w) s_{ref}^{-p_w}||\nabla(\mathcal{F}_\alpha(h_s+b))||^{p_w}\nabla(\mathcal{F}_\alpha(h_s+b))\right) = S_w & \text{in } \Omega,\\ -k_m h_w \eta_w(h_w) s_{ref}^{-p_w}||\nabla(\mathcal{F}_\alpha(h_s+b))||^{p_w}\nabla(\mathcal{F}_\alpha(h_s+b))\cdot n = B_w & \text{on } \partial\Omega,\end{array}\right.$$

. Page 24 equation 22:

$$\left|\begin{array}{l}\boldsymbol{Q}_w = -\dfrac{k_m h_w \eta_w\,(h_w)}{s_{ref}^{p_w}}||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b),\\[2ex] div\,(\boldsymbol{Q}_w) = S_w \quad \text{in } \Omega,\\ \boldsymbol{Q}_w \cdot n = B_w \quad \text{on } \partial\Omega_{\mathcal{N}},\\ h_w = 0 \quad \text{on } \partial\Omega_{\mathcal{D}},\end{array}\right.$$

instead of:

$$\left|\begin{array}{ll} -div\left(k_m h_w \eta_w\left(h_w\right) s_{ref}^{-p_w} ||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b)\right) = S_w & \text{in } \Omega, \\ -k_m h_w \eta_w\left(h_w\right) s_{ref}^{-p_w} ||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b)\cdot n = B_w & \text{on } \partial\Omega_{\mathcal{N}}, \\ h_w = 0 & \text{on } \partial\Omega_{\mathcal{D}}, \end{array}\right.$$

. Page 26 equation B1:

$$\left|\begin{array}{l} \dfrac{\partial h_w}{\partial t} + div(h_w \boldsymbol{u}_w) = 0, \\ \dfrac{\partial}{\partial t}(h_w \boldsymbol{u}_w) + div(h_w \boldsymbol{u}_w \otimes \boldsymbol{u}_w) + g h_w \nabla(h_s + b + h_w) + \\ = -\kappa_w\left(h_w, ||\nabla(h_w+h_s+b)||\right)|\boldsymbol{u}_w|^{r_w}\boldsymbol{u}_w, \end{array}\right.$$

. Page 26 second too small equation of first column (please do not forget to use displaystyle on each line to correctly render fractions):

$$\left|\begin{array}{l} \dfrac{\partial \hat{h}_w}{\partial \hat{t}} + \hat{div}(\hat{h}_w \hat{\boldsymbol{u}}_w) = 0, \\[2ex] \dfrac{\partial}{\partial \hat{t}}(\hat{h}_w \hat{\boldsymbol{u}}_w) + \hat{div}(\hat{h}_w \hat{\boldsymbol{u}}_w \otimes \hat{\boldsymbol{u}}_w) \\[2ex] +g\dfrac{H_{s,c}T_c^2}{L_c^2}\hat{h}_w\hat{\nabla}(\hat{h}_s+\hat{b}) + g\dfrac{H_{w,c}T_c^2}{L_c^2}\hat{h}_w\hat{\nabla}(\hat{h}_w), \\[2ex] = -\kappa_w\left(h_w, ||\nabla(h_w+h_s+b)||\right)\dfrac{L_c}{H_{w,c}}\left(\dfrac{L_c}{T_c}\right)^{r_w-1}|\hat{\boldsymbol{u}}_w|^{r_w}\hat{\boldsymbol{u}}_w. \end{array}\right.$$

. Page 26 too small equation in second column:

$$\left|\begin{array}{l} \boldsymbol{Q}_w = -\dfrac{k_m h_w \eta_w\left(h_w\right)}{s_{ref}^{p_w}}||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b), \\[2ex] div\left(\boldsymbol{Q}_w\right) = S_w \quad \text{in } \Omega, \\ \boldsymbol{Q}_w\cdot n = B_w \quad \text{on } \partial\Omega_{\mathcal{N}}, \\ h_w = 0 \quad \text{on } \partial\Omega_{\mathcal{D}}, \end{array}\right.$$

instead of:

$$\left|\begin{array}{ll} -div\left(k_m h_w \eta_w\left(h_w\right) s_{ref}^{-p_w} ||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b)\right) = S_w & \text{in } \Omega, \\ -k_m h_w \eta_w\left(h_w\right) s_{ref}^{-p_w} ||\nabla(h_w+h_s+b)||^{p_w}\nabla(h_w+h_s+b)\cdot n = B_w & \text{on } \partial\Omega_{\mathcal{N}}, \\ h_w = 0 & \text{on } \partial\Omega_{\mathcal{D}}, \end{array}\right.$$

. Page 27 equation C1 (please do not forget to use displaystyle to correctly render fractions):

$$
\left|
\begin{array}{l}
\alpha^2 \displaystyle\sum_{\sigma\in\mathcal{F}_K\cap\mathcal{F}_{int}} \frac{|\sigma|}{\overline{d}_{KL}}(\mathcal{F}_{\alpha,K}(u_{\mathcal{T}}) - \mathcal{F}_{\alpha,L}(u_{\mathcal{T}})) \\
+|K|\mathcal{F}_{\alpha,K}(u_{\mathcal{T}}) = |K|u_K \quad \text{for all } K\in\mathcal{T}, \\
\\
\mathcal{F}_{\alpha,\sigma}(u_{\mathcal{T}}) = \mathcal{F}_{\alpha,K}(u_{\mathcal{T}}) \quad \text{for all } K\in\mathcal{T} \text{ and all } \sigma\in\mathcal{F}_K\cap\mathcal{F}_{ext}^{\mathcal{N}}, \\
\\
\mathcal{F}_{\alpha,\sigma}(u_{\mathcal{T}}) = 0 \quad \text{for all } K\in\mathcal{T} \text{ and all } \sigma\in\mathcal{F}_K\cap\mathcal{F}_{ext}^{\mathcal{D}}.
\end{array}
\right.
$$

. Page 27 equation C2 (please do not forget to use displaystyle to correctly render fractions):

$$
\left|
\begin{array}{l}
\displaystyle\frac{|K|}{\Delta t^n}(h_{s,K}^{n+1} - h_{s,K}^n) + \sum_{\sigma\in\mathcal{F}_K\cap\mathcal{F}_{int}} \frac{|\sigma|}{\overline{d}_{KL}s_{ref}^{p_w}}\eta_{s,\sigma}^{n+1}\Delta\Psi_{KL}^{n,n+1} \\
\displaystyle+ \sum_{\sigma\in\mathcal{F}_K\cap\mathcal{F}_{ext}^{\mathcal{D}}} \frac{|\sigma|}{\overline{d}_{K\sigma}s_{ref}^{p_w}}\eta_{s,\sigma}^{n+1}\Delta\Psi_{K\sigma}^{n,n+1} \\
\displaystyle- \sum_{\sigma\in\mathcal{F}_K\cap\mathcal{F}_{ext}^{\mathcal{N}}} |\sigma|B_{s,\sigma}^{n+1} = |K|S_{s,K}^n \quad \text{for all } K\in\mathcal{T}, \\
h_{s,\sigma}^{n+1} + b_\sigma^{n+1} = h_{s,K}^{n+1} + b_K^{n+1} + \boldsymbol{G}_{s,K}^{n+1}\cdot(\overline{\boldsymbol{x}}_\sigma - \overline{\boldsymbol{x}}_K) \\
\\
\qquad \text{for all } K\in\mathcal{T} \text{ and all } \sigma\in\mathcal{F}_K\cap\mathcal{F}_{ext}^{\mathcal{N}}, \\
\\
h_{s,\sigma}^{n+1} = 0 \quad \text{for all } \sigma\in\mathcal{F}_{ext}^{\mathcal{D}},
\end{array}
\right.
$$

. Page 28 equation C7 (you can forget numbering this equation): use an itemize environment

. if $\sigma\in\mathcal{F}_{ext}^{\mathcal{D}}$:

$$q_{w,\sigma}^{n+1} = \mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})$$

. if $\sigma\in\mathcal{F}_{int}$ and $\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) > 0$ and $\mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) > 0$

$$q_{w,\sigma}^{n+1} = \frac{\overline{d}_{KL}\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})}{\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\overline{d}_{L\sigma} + \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\overline{d}_{K\sigma}}$$

. if $\sigma\in\mathcal{F}_{int}$ and $\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) = 0$ or $\mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) = 0$

$$q_{w,\sigma}^{n+1} = \frac{1}{2}(\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) + \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}))$$

instead of the array:

$$q_{w,\sigma}^{n+1} = \left| \begin{array}{ll} \mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) & \text{if } \sigma \in \mathcal{F}_{ext}^{\mathcal{D}} \\ \dfrac{\overline{d}_{KL}\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})}{\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\overline{d}_{L\sigma} + \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})\overline{d}_{K\sigma}} & \text{if } \sigma \in \mathcal{F}_{int} \text{ and } \mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) > 0 \text{ and } \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) > 0, \\ \dfrac{1}{2}(\mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) + \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1})) & \text{if } \sigma \in \mathcal{F}_{int} \text{ and } \mathcal{F}_{\alpha,K}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) = 0 \text{ or } \mathcal{F}_{\alpha,L}^{\mathcal{N}}(q_{w,\mathcal{T}}^{n+1}) = 0. \end{array} \right.$$

. Page 28 equation C9 (you can forget numbering this equation): use an itemize environment

  . If $\mathcal{T}_\sigma = \{K, L\}$

$$\boldsymbol{R}_{s,\sigma}^n =$$

$$\frac{1}{\overline{d}_{KL}^2}\left(h_{s,L}^n + b_L^n - h_{s,K}^n - b_K^n - \boldsymbol{G}_{s,\sigma}^n \cdot (\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K)\right)(\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K)$$

  . If $\mathcal{T}_\sigma = \{K\}$

$$\boldsymbol{R}_{s,\sigma}^n =$$

$$\frac{1}{\overline{d}_{K\sigma}^2}\left(h_{s,\sigma}^n + b_\sigma^n - h_{s,K}^n - b_K^n - \boldsymbol{G}_{s,\sigma}^n \cdot (\overline{\boldsymbol{x}}_\sigma - \overline{\boldsymbol{x}}_K)\right)(\overline{\boldsymbol{x}}_\sigma - \overline{\boldsymbol{x}}_K)$$

instead of the array:

$$\boldsymbol{R}_{s,\sigma}^n = \left| \begin{array}{ll} \dfrac{1}{\overline{d}_{KL}^2}\left(h_{s,L}^n + b_L^n - h_{s,K}^n - b_K^n - \boldsymbol{G}_{s,\sigma}^n \cdot (\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K)\right)(\overline{\boldsymbol{x}}_L - \overline{\boldsymbol{x}}_K) & \text{if } \mathcal{T}_\sigma = \{K, L\}, \\ \dfrac{1}{\overline{d}_{K\sigma}^2}\left(h_{s,\sigma}^n + b_\sigma^n - h_{s,K}^n - b_K^n - \boldsymbol{G}_{s,\sigma}^n \cdot (\overline{\boldsymbol{x}}_\sigma - \overline{\boldsymbol{x}}_K)\right)(\overline{\boldsymbol{x}}_\sigma - \overline{\boldsymbol{x}}_K) & \text{if } \mathcal{T}_\sigma = \{K\}. \end{array} \right.$$

. Equation C11 is not only too small, but not rendered correctly. The subscripts are larger than the main terms. Use displaystyle to correctly render the sums:

$$\left| \begin{array}{l} \tilde{q}_K^{n+1} - \sum\limits_{\sigma \in \mathcal{F}_K \cap \mathcal{F}_{int}, \mathcal{F}_{\alpha,K}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n) < \mathcal{F}_{\alpha,L}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n)} \\ \tau_{KL}^{n,n+1} \dfrac{\tilde{q}_L^{n+1}}{s_L^{n,n+1}} \left(\mathcal{F}_{\alpha,L}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n) - \mathcal{F}_{\alpha,K}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n)\right) \\ - \sum\limits_{\sigma \in \mathcal{F}_K \cap \mathcal{F}_{ext}} |\sigma| B_{w,\sigma}^{n+1} = |K| S_{w,K}^n \quad \text{for all } K \in \mathcal{T}, \\ s_K^{n,n+1} = \sum\limits_{\sigma \in \mathcal{F}_K \cap \mathcal{F}_{int}, \mathcal{F}_{\alpha,K}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n) \geqslant \mathcal{F}_{\alpha,L}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n)} \\ \tau_{KL}^{n,n+1} \left(\mathcal{F}_{\alpha,K}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n) - \mathcal{F}_{\alpha,L}(h_{s,\mathcal{T}}^n + b_{\mathcal{T}}^n)\right) \\ \\ \tau_{KL}^{n,n+1} = \dfrac{|\sigma| k_{m,\sigma}^{n+1}}{\overline{d}_{KL} s_{ref}^{p_w}} ||\boldsymbol{G}_{\mathcal{F},s,\sigma}^n||^{p_w}, \end{array} \right.$$

TS2 This is just a typo. If you want the full story, the very first version of the paper we wrote used two distinct values $p_{s,1}$ and $p_{s,2}$ for the exponent of the two flux terms, and later we decided that a single one, p_s, was enough and simpler since we never consider in practice any case with two distinct values. Thus, we should have in principle replaced $p_{s,1}$ and $p_{s,2}$ by p_s everywhere, and we have apparently simply missed this one.

TS3 We use PETSc in an indirect way, through another proprietary software, which we are not authorized to cite. We do not have access to the date when the people who made it last accessed the PETSc repository. We could invent one, but we would rather not to. Notice that the URL is not the URL of the PETSc repository, but the URL the authors of PETSc indicate on their webpage "how to cite PETSc", since there is no DOI. Notice that we have added this quite useless URL because you insisted on it. We could invent an access date, but we do not see the point. If you insist, use the submission date.

TS4 Notice that we have added this URL at your request, which is the only one we have found for the thesis (old thesis have no DOI). We use the paper version of the thesis (the thesis occured in our institute long ago) so we have never used the URL, which we added only because you insisted on it. However If you really need a access date, please use the submission date.