Response to Reviewer Comments 2

We thank the reviewer for the comments. Note that we use the abbreviation **RC** for instance refer to reviewer comments 2, **EC** for editor and **AR** for our authors' response in the following. To highlight the nature of our reply, we use two colours; with green for agreement and yellow for misunderstanding. We explain all our reasoning and changes made to the manuscript. Removed text is shown in red, e.g., ~~this text has been removed.~~. New text is shown in blue, e.g., this text has been added.

RC: Overall, the manuscript is well written. The authors porposaed "mean stress solution" by introducing Biot's consolidation theory to estimate the groundwater fluctuation due to tidal loading.

> **AR:** Thanks for the positive feedback, time spend and all the great comments that intend to improve our work.

RC: Line 110, it is state "Biot's consolidation theory assume $\xi = 0$ when undrained condition. Why? please add reference.

> **AR:** We agree: By definition, ξ represents the amount of fluid content in a system over a period of time (i.e. $\frac{dm}{dt}$, where m is mass of fluid in the system). Thus, if the system is undrained, which occurs when no fluid is entering or leaving a system, $\xi = 0$. We added references to this statement.
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> Suggested revision at line 110: Biot's consolidation theory assumes $\xi = 0$ when undrained conditions apply within the porous medium (Cheng, 2016; Verruijt, 2013; Wang, 2017)

RC: Line 114: The assumption "exp(iwt)" implies only the first-order approximation is adopted. This requires justification.

AR: We agree: If the z-axis is vertical and the only body force is the atmospheric loading, the mechanical equilibrium equation reduces to,

$$\frac{\partial \sigma_{zz}}{\partial z} = -F_z \tag{1}$$

then σ_{zz} is independent of z, but it can be a function of time. Then the Mean Stress Equation in Cartesian coordinates can be written as,

$$\frac{\alpha}{KB}\left[\frac{B}{3}\frac{d\sigma_{kk}}{dt} + \frac{\partial p}{\partial t}\right] - \frac{k}{\mu}\nabla^2 p = Q. \tag{2}$$

The quasi-static assumption of instantaneous mechanical equilibrium requires that the loading period be long relative to the times for elastic wave propagation (which is the case for tides). Under this pseudo steady-state approximation, time and z are independent and the solution is first order,

$$p(z,t) = \tilde{p}(z)exp(i\omega t) \tag{3}$$

where $\tilde{p}(z)$ is the (complex) amplitude, which depends only on z.

Suggested revision at line 114: We solved Eq. 1 for steady state conditions to obtain the periodic water level in an open borehole. This assumes that the instantaneous mechanical equilibrium requires the loading period to be long relative to the times for elastic wave propagation. Furthermore, the initial transient when the surface loading starts is neglected. Under this pseudo steady-state approximation, i.e., time and depth are independent, the solution is first order and has the form $h_w^{AT} = h_{w,o}^{AT} e^{i\omega t}$ due to atmospheric loading, where ω is the angular frequency of the tide signal and superscript AT stands for atmospheric tides, for example S_1 at 1 cycle per day (CPD) or the atmospheric response to S_2 at 2 cpd (Merritt, 2004; McMillan et al., 2019).

RC: The assumption of "Mean stress" needs justification.

AR: We agree: The poroelastic constitutive relationship is

$$\xi = \frac{\alpha}{K}\frac{\sigma_{kk}}{3} + \frac{\alpha}{KB}p, \tag{4}$$

wWhere α represents the Biot conefficeint, K the bulk modulus, B the Skeptom coefficient and σ_{kk} the mean stress. If a change in pore pressure is only cause by a vertical loading and only vertical deformation is allowed, then

$$\frac{\sigma_{kk}}{3}_{\varepsilon_{xx}=\varepsilon_{yy}=\xi=0} = \frac{1}{3}\frac{1+\nu_u}{1-\nu_u}\sigma_{zz}. \tag{5}$$

Now, the fluid continuity equation is

$$\frac{\partial \xi}{\partial t} - \frac{k}{\mu}\nabla^2 p = Q. \tag{6}$$

Substituting for ξ from the constitutive equation Eq. 4 gives

$$\frac{\alpha}{KB}\left[\frac{B}{3}\frac{\partial \sigma_{kk}}{\partial t} + \frac{\partial p}{\partial t}\right] - \frac{k}{\mu}\nabla^2 p = Q, \tag{7}$$

which is presented as Mean Stress Equation.
Suggested revision at line 102: [...] For example, when atmospheric pressure rises and, the formation undergoes compressive stress resulting in an increased in the confined pore pressure (Domenico and Schwartz, 1997). If a change in pore pressure is only caused by a vertical loading and only vertical deformation is allowed, then the mean stress can be computed as

$$\frac{\sigma}{3}_{\varepsilon_{xx}=\varepsilon_{yy}=\xi=0} = \frac{1}{3}\frac{1+\nu_u}{1-\nu_u}\sigma_{zz}. \tag{8}$$

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RC: In this note, only tow borehole data are used. As shown in Figure 5(c), two data point is obviously insufficient. The authors may need to add more data points.

AR: We partially agree, it is indeed acknowledged that relying on just two data points is inadequate, and a more extensive investigation is necessary to thoroughly evaluate the reliability and performance of the analytical solution using field data. The primary aim of this paper is to develop a new analytical solution. In addition, we have applied this to field data and validated its results by comparison with established solutions. Further, we present a comprehensive workflow for signal treatment of field data, with a particular emphasis on the utilisation of the analytical solution, rather than focusing solely on the field data itself.

References

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