Reply to the supplementary comment from Mark Weber:

There was one point I missed in my review. For trends from monthly mean ozone time series a correction is applied in the regression to account for autoregression (AR1). This correction does not change trends so much but increases the uncertainties due to the reduction of degree-of-freedom associated with AR. It can be applied to both OLS and Ridge regression and should be done. If not, at least a good reason should be given why it is not needed here.

Reply: We thank the reviewer for his comments and suggestions about applying a correction in the regression to account for the autoregression (AR1) for the trends from monthly mean ozone time series.

We have updated our results by including a lag-1 autocorrelation correction process in the OLS regression model with the Cochrane-Orcutt method (1949). The Cochrane-Orcutt method is a popular approach used to correct for first-order autocorrelation (AR1) in the residuals of a regression model with ordinary least squares (OLS) method (e.g. Dhomse et al., 2006; Ball et al., 2019; Petropavlovskikh et al., 2019; Bognar et al., 2022; Godin-Beekmann et al., 2022). The procedure is performed iteratively with the covariance matrix updated for each iteration until the autocorrelation coefficient has converged sufficiently (Cochrane-Orcutt, 1949; Prais and Winsten, 1954).

As mentioned by the reviewer, the trend coefficients do not change much but the uncertainties increase to some extent with this correction. It should be noted that the residuals in some region of the tropical mid-lower stratosphere are still large and auto-correlated after the AR1 correction with the Cochrane-Orcutt method. Hence, some limitations and assumptions of the Cochrane-Orcutt method should be noted, e.g.:

(1) Limited to AR1 Autocorrelation: The Cochrane-Orcutt method is specifically designed to handle first-order autocorrelation (AR1). If the autocorrelation in the residuals follows a higher-order AR process or a different pattern, this method may not be appropriate or effective.

(2) Relying on AR1 Parameter Estimation: Estimating the AR1 parameter involves making assumptions about the structure of autocorrelation and may not be reliable, especially with small sample sizes or noisy data.

(3) Parameter Interpretation: After applying the Cochrane-Orcutt correction, the estimated regression coefficients and their interpretation can be affected. The coefficients of the corrected model may not have a direct interpretation in the same way as those from the original model.

(4) Efficiency Loss: Correcting for autocorrelation may lead to a loss of statistical efficiency in parameter estimates, potentially resulting in wider confidence intervals and reduced power to detect significant effects.

(5) Diagnostics: Assessing the adequacy of the correction and the presence of any remaining autocorrelation may be challenging. Model diagnostics become essential to ensure the correction's appropriateness and to identify any model misspecification issues.

(6) Data Transformation: The method involves transforming the data and iteratively estimating parameters, which may lead to additional complexities and computational burden,

especially for large datasets.

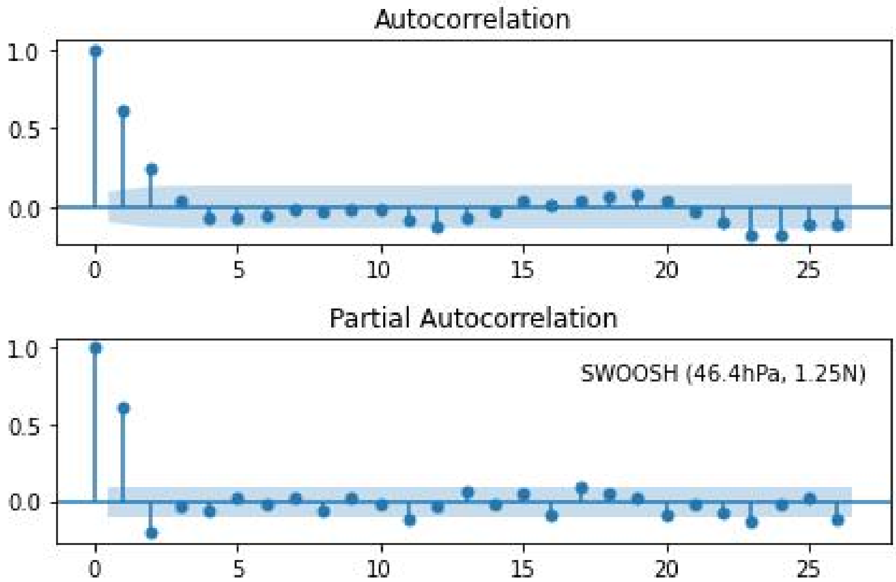

Figure RC1: Estimates of a higher-order AR structure (AR2) of the residuals using autocorrelation and partial autocorrelation based on SWOOSH dataset.

Figure RC1 shows a case of the AR2 structure estimated by the autocorrelation and partial autocorrelation function of the residuals. Despite the limitations of the Cochrane-Orcutt method, the method of the usual least squares can still yield the best linear unbiased estimates of the regression coefficients provided the autocorrelated error terms are taken into account (Cochrane-Orcutt, 1949).

In the Ridge regression, an additional constraint (an L2 penalty) in the cost function is introduced to constrain the magnitudes and fluctuations of the coefficient estimates. This constraint helps to reduce the variance of the model at the expense of no longer being unbiased. For our current MLR setup, we choose not to apply the AR1correction to Ridge regression. If we still apply the AR1 correction to Ridge regression as for the OLS regression, the estimated regression coefficients can be affected as the correlation between the regression model and underlying data becomes very poor after "correction", and the regression in this case is in an "under-fitting" state with a very large tuning parameter. Besides, when applying the AR1 correction to Ridge regression, the autocorrelation coefficient does not always converge during iteration which makes it impossible to obtain the covariance matrix as in OLS regression. Hence, care is needed when applying the AR1 correction to Ridge regression and more detailed work can be carried out in future studies.

We have added a paragraph in the revised manuscript to clarify the differences using OLS and Ridge regression models (Lines 231-245). In Figures RC2-3, the updated ozone trend profiles with AR1 correction applied to the OLS regression are shown and compared with Ridge regression results (with no AR1 correction). Please also see Figures 2-3 in the revised manuscript.

We also updated the other figures with corrected OLS regression and more detailed modifications of the updated results are marked in red in the revised manuscript. The related code and data files are uploaded on github (https://github.com/AmyLee07/Data-and-code-for-OLS-and-Ridge-regression.git).

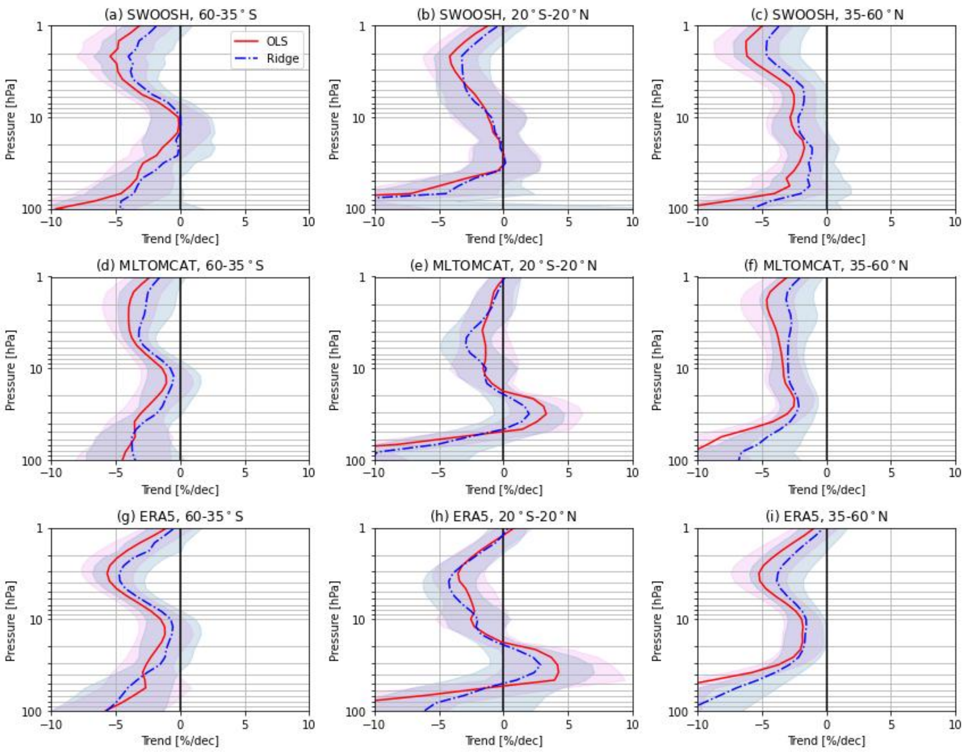

Figure RC2: Profiles of annual mean stratospheric ozone trends (% per decade) compared between OLS and Ridge regression methods for three latitude bands (60-35°S, 20°S-20°N and 35-60°N) from (a-c) SWOOSH, (d-f) ML-TOMCAT, and (g-i) ERA5 model simulation over the period 1984-1997. Shaded regions are 2-σ uncertainties. (Data during 1991-1994 are removed).

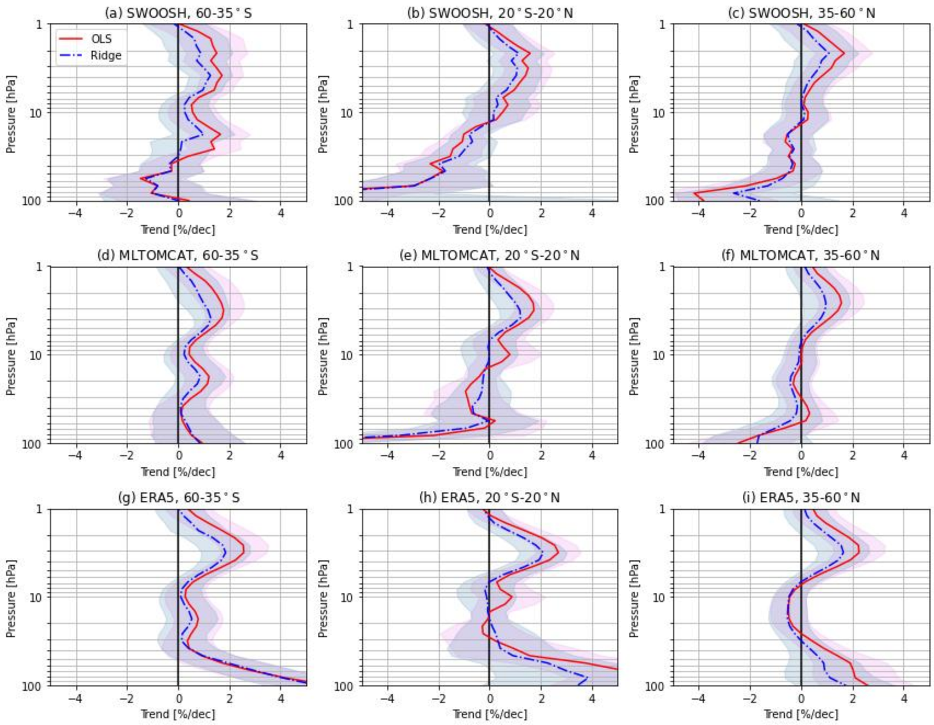

Figure RC3: Same as Figure RC2 but for the post-1998 time periods (1998-2020) for SWOOSH, ML-TOMCAT and ERA5 model simulation.

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