

## Response to anonymous referee #2

*We thank referee #2 for reading the manuscript carefully and providing thoughtful and constructive comments. The comments are noted with **RC**, our responses with **AC**, and the intended additions or changes in the manuscript are underlined.*

**RC2.1** The authors compare their results with a very narrow field of the latest developments (here: analytic solvable micro-canonical cascade models). However, the results can be interpreted from a larger perspective (at least micro-canonical cascade models, better: cascade models in general). For example the scale-dependency (model A vs. B, bounded vs. unbounded model) and position-dependency. The discussion would benefit from it and the reader would be provided with a broader perspective on the scientific field. It is also important because some findings (which are new for the analytic solvable MRC) are quite common to apply for other cascade models.

***AC2.1** Thank you for this remark. We agree that our results could be interpreted from a larger perspective for other disaggregation approaches, for other cascade model approaches. We believe that the impact of the local asymmetry of precipitation on precipitation variability will be worth further investigations and may improve our understanding and modelling of some precipitation properties. We will add a comment on this in the conclusion.*

**RC2.2** Section 4.1 The authors highlight the improvement by introducing the asymmetry in comparison to model A and model B. From my understanding neither model A nor model B takes into account the position-dependency of the current time step to disaggregate. To my knowledge the latest references on micro-canonical cascade models all take into account position dependency (so starting, enclosed, ending, isolated position classes depending on the wetness state of:  $\{R_{t-1}, R_t, R_{t+1}\}$ ). As mentioned before, for analytic solvable MRC it is maybe not common to take into account the positions/patterns from the coarser scale, but it is common for MRC in general. Here, the asymmetry can be interpreted as an extension of the position-dependency, since it takes into account the intensity of the surrounding time steps rather than the wetness state only (so real vs. boolean). So it is not surprising that A+ and B+ outperform A and B, respectively, but A and B do not represent the state-of-science. I recommend to add a position-dependent cascade model to evaluate the added value of the asymmetry in comparison to the wetness state. Even if both approaches result in similar statistics, the asymmetry would have the benefit of being more parameter parsimonious. It is important here to show the reader the clear benefit of the introduced model approach.

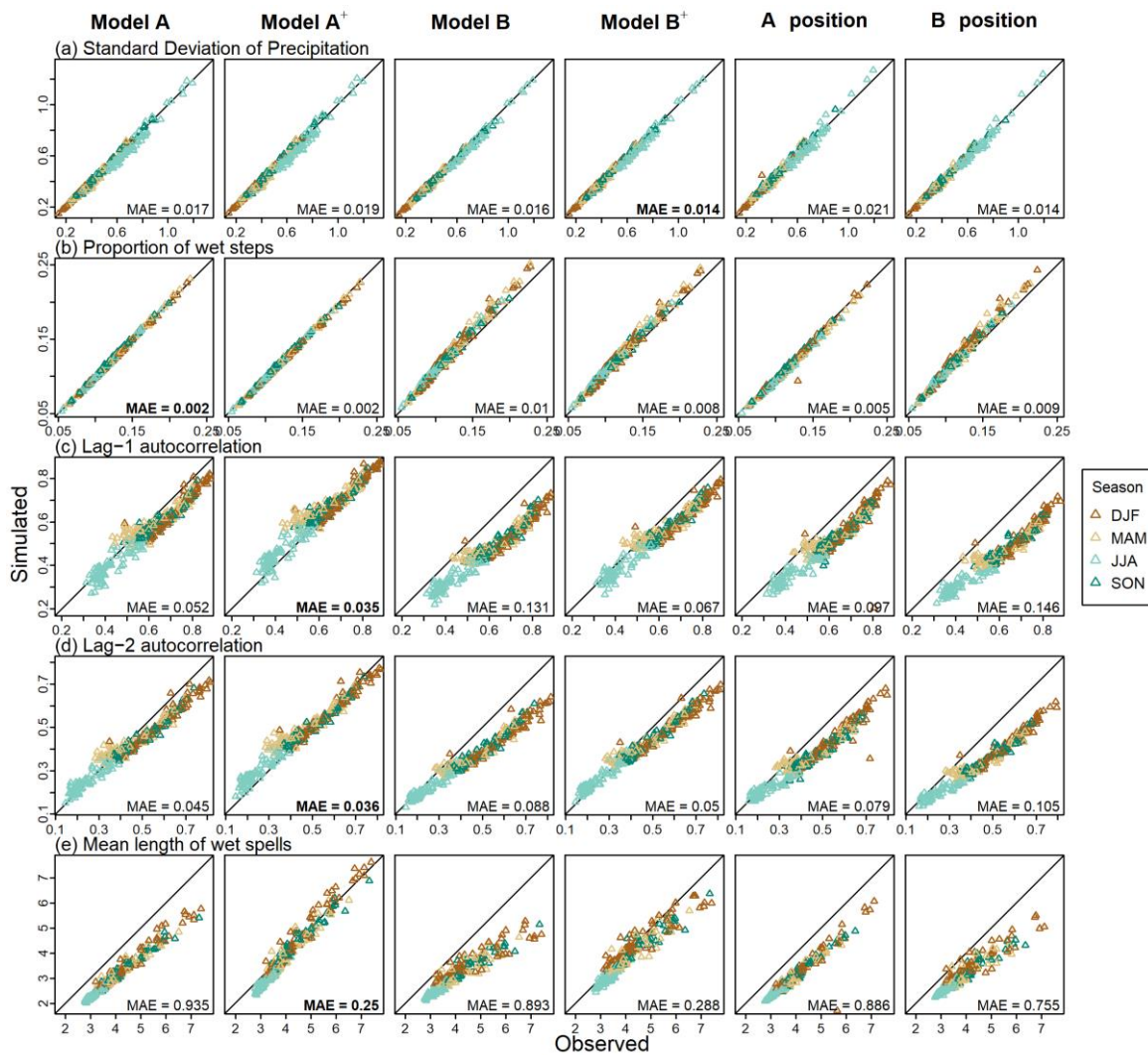
***AC2.2** Yes, you are right, the asymmetry can be interpreted as an extension of the position-dependency already accounted for in a number of previous works. Models A and B, do not account for the position dependency. They are not the state-of-the-art models if empirical cascade models are considered. They are however state to the art models if analytical models (with scaling models included) are considered. To our knowledge, the gap between both approaches was unsolved to date. Our modelling approach proposes a bridge between both approaches: accounting for asymmetry in an analytical way with a continuous asymmetry index which allows to develop analytical scaling laws.*

*The point you raised is however very relevant. Is there some added value of the asymmetry approach in comparison to the position dependency approach? To assess this, we considered two more models “A position” and “B position” (“Ap” and “Bp”). These two models are based on models A and B respectively, but we added a dependency to the position class (starting, enclosed, ending, isolated). Therefore, for each station, we estimated a set of parameters by season and by position class. In the same manner, as explained in the manuscript, the observed quasi-daily amounts were disaggregated to time series of 40-minute resolution following these two models.*

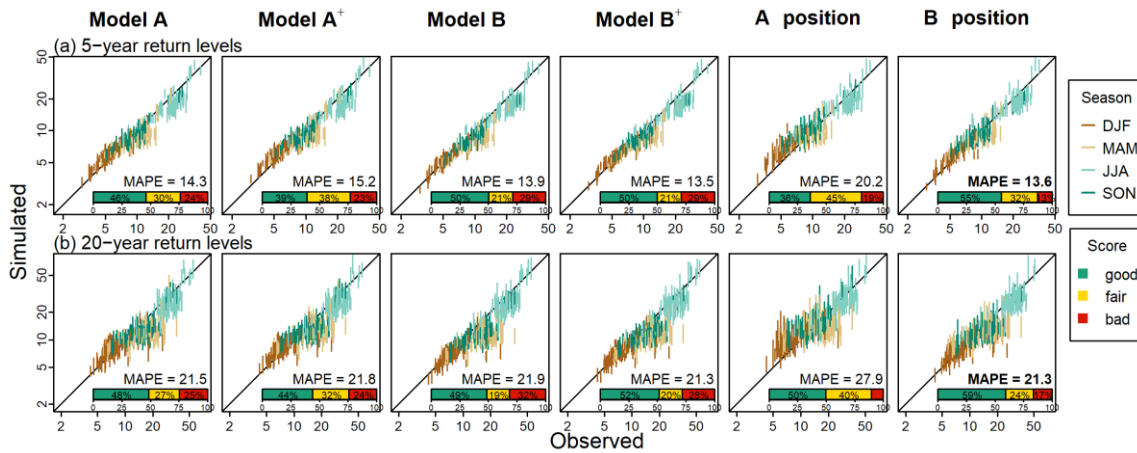
*Results are shown in Figure 6 and can be interpreted as follows:*

- For Standard deviation, (a), and the proportion of wet steps, (b): Models Ap / Bp perform similarly as models A/B and models A<sup>+</sup>/B<sup>+</sup>.*
- For Lag-1, (c), and Lag-2, (d), autocorrelation: Models Ap and Bp perform worse than models A and B (and then much worse A<sup>+</sup> and B<sup>+</sup>)*
- For mean length of wet spells, (e): Ap is slightly better than A but Ap is still worse than model A<sup>+</sup>.*
- For return levels Figure 7, results are almost the same for all models.*

*Overall, in our case, conditioning the analytical models to the position class improves the performance of classical analytical models. As shown in Figure 6 it is however less efficient than considering scaling laws with a continuous asymmetry index. This will be worth further investigations to assess if these results could be valid in other climates. We believe that this better performance of the models A<sup>+</sup>/B<sup>+</sup> is due to the fact that the models are additionally able to make a distinction between different “starting” sequences (or different ending sequences) as the asymmetry index is also a measure of the “intensity” of the asymmetry (i.e. steep decreasing intensity over the three consecutive precipitation amounts or only slow decreasing) (see Figure 1 of our response to comment RC1.1). As they are additionally highly parsimonious, Models A<sup>+</sup> and B<sup>+</sup> appear to be really promising alternative to the class-conditioned models.*



**Figure 6. Conditioning the MRC on ending/starting classes.** Observed versus simulated statistics for each considered model at a 40-minute temporal resolution for different metrics. Four first columns correspond to the results and models presented on the manuscript, the fifth and sixth column correspond to the results obtained when in the model A, respectively model B, the position class dependency is added.



**Figure 7. The interest of conditioning the MRC on position classes (ending/starting/enclosed/isolated).** Observed versus simulated return levels at the 40-minute temporal resolution for (a) 5-year and (b) 20-year return periods, for each model and at each site. The four first columns correspond to the results and models presented in the manuscript, the fifth and sixth columns correspond to the results obtained when in model A, respectively model B, the position class dependency is added.

**RC2.3** The seasonal classification is not common for all cascade models and more common for the pulse models (NSRP & BLRP). The authors show the seasonal variations of parameters in Fig. 8, but I'm still curious how this would affect the results. How would the results look if there is one parameter set applied for the disaggregation of the whole time series?

**AC2.3** This is an interesting issue to discuss, thank you for bringing it up. Actually, it not really uncommon for the cascade models to consider a seasonal dependency, see for example Olsson (1998), Günter et al. (2001), Onof et al. (2005), McIntyre et al. (2015). The seasonality of the scaling properties of rainfall – in view of modelling have been also highlighted in other works, especially in Molnar and Burlando (2008) for Switzerland. Conditioning on the season was thus rather natural for us.

To show the added value of this seasonal conditioning, we performed the parameter estimation procedure without accounting for seasonality, so only a set of parameters was obtained for each station. In the same manner, as explained in the manuscript, the observed quasi-daily amounts were disaggregated to time series of 40-minute resolution. In order to do a fair comparison with results obtained when considering a seasonal dependency, the evaluation is done on a seasonal basis. The results are not at all satisfactory, neither for standard metrics, see Figure 8, nor for return levels, Figure 9. A huge gap between the results obtained for summer and for other seasons can be noticed, resulting from the mix between short convective events in summer and other precipitation events in other seasons while estimating the parameters.

This point will be briefly mentioned in the manuscript.

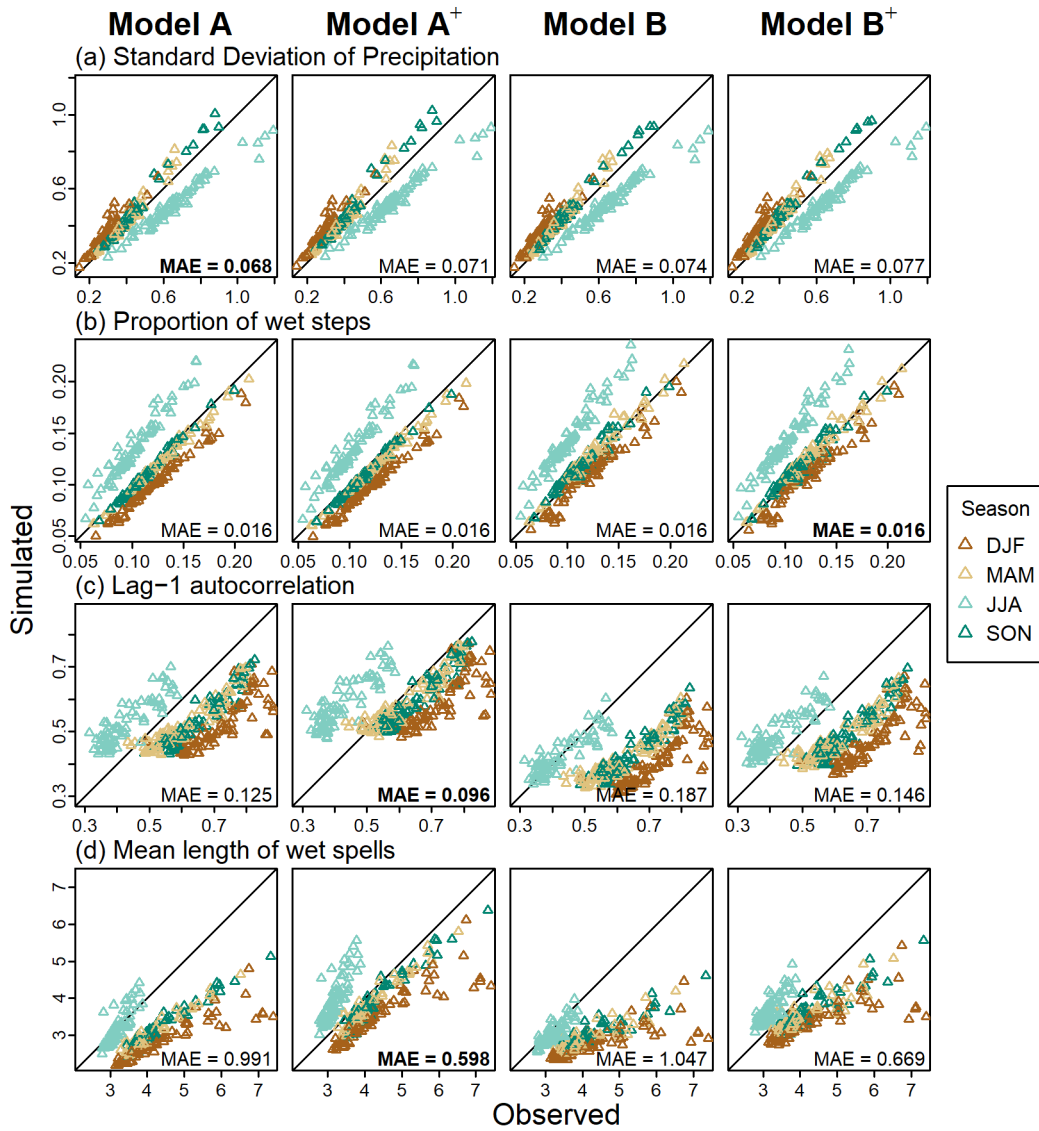


Figure 8. Results without seasonal stratification (results to be compared with results of Figure 5 in the manuscript). Observed versus simulated statistics for each considered model at a 40-minute temporal resolution for different metrics. Each triangle represents a site and a season. Same analytical models as in the manuscript but no seasonal stratification is done on parameter estimation.

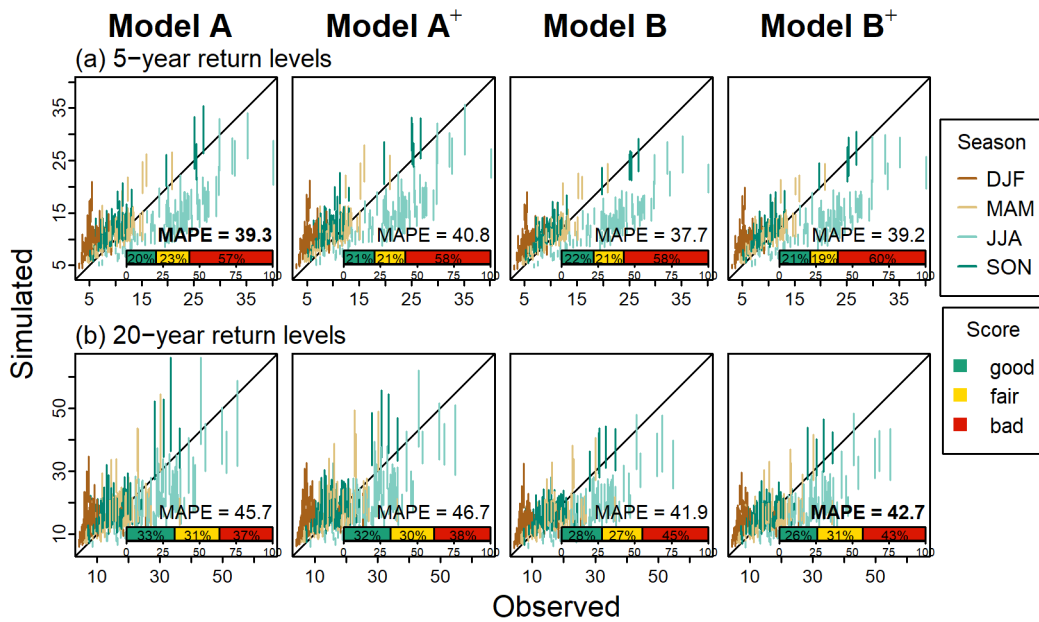


Figure 9. **Results without seasonal stratification** (results to be compared with results of Figure 6 in the manuscript). Observed versus simulated return levels at the 40-minute temporal resolution for (a) 5-year and (b) 20-year return periods, for each model and at each site. Same analytical models as in the manuscript but no seasonal stratification is done for parameter estimation.

**RC2.4** L2 The term ‘simple scaling law’ can be confusing. What does ‘simple’ refer to? Linear? Please clarify.

**AC2.4** Thank you for pointing this possible confusing term. We will modify this sentence as follows: This class of models applies scaling models to represent the dependence of the cascade generator on the temporal scale and the precipitation intensity.

**RC2.5** L5 ‘...is usually disregarded.’ I would add the following extension to this sentence: ‘...or taken into account in a simplified way.’ (or similar), since there are possibilities out there taking into account the wetness state of the surrounding time steps.

**AC2.5** We agree that many empirical cascade models account for the external pattern of precipitation, nevertheless, as far as we are aware this is not the case for analytical scaling models. Our statement was intended for analytical models. For clarification, we will modify the sentence as follows: Although determinant, the dependence on the external precipitation pattern is usually disregarded in the analytical scaling models.

**RC2.6** L176 The term ‘shadow breakdown coefficient’ sounds spectacular, but it is not clear what ‘shadow’ exactly refers to? From my understanding it takes into account the position-dependency as well as the rainfall amounts, because higher rainfall amounts would cause more shadow. However, this name should be introduced/defined so that other authors know when to use it.

**AC2.6** Thank you for your comment. The term “shadow breakdown coefficient” will be replaced by “hidden breakdown coefficient”.



**RC2.7** Section 2.2 When introducing  $Z_t$  the authors could state the intended application briefly and refer to Sec. 2.4 with the detailed description: It only affects  $p_{01}$  and  $p_{10}$ ,  $p_x$  remains unaffected.

*AC2.7 Thank you for the suggestion. We will do so.*

**RC2.8** Fig. 3c For very low and very high values of  $z$  tipping points can be identified. How can it be explained? By the measuring resolution of the measuring instrument, leading to minimum values of e.g. 0.1mm?

*AC2.8 As mentioned in Section 2.2 of the manuscript,  $Z_t$  values close to 0 indicate sequences with very little rain on the first two time steps when compared to the last one (very steep "ascending" sequences), whereas  $Z_t$  values close to 1 indicate sequences with very little rain on the two last time steps when compared to the first one (very steep "descending" sequences). This means that the value of the precipitation amount  $R_t$  for the central step has to be rather low for those configurations. As a consequence; the observed weights for those rainfall configurations are typically 0 or 1 (this can be observed in the histograms given for the response AC1.11 for the two configurations  $Z=0.05$  and  $Z=0.95$ ). This leaves only a small sample to estimate the mean of the weights  $0 < W < 1$ . On the other hand, in order to reduce the effects of measurement artifacts, weights considered for the analysis are only calculated from precipitation amounts  $R_t$  larger than 0.8 mm. This is expected to drastically reduce the sample size which we believe is the main reason for the tipping points. This issue will be worth further investigation but as this is not a central issue in our work, we will not discuss/mention it in the manuscript.*

**RC2.9** Table 1-caption. Please add the information that the number of parameters is not taking into account any seasonal variation. So four seasons would lead to  $4 \times$  parameter number mentioned in the table.

*AC2.9 Thank you for your valid suggestion. Such information will be added to the table's caption.*

**RC2.10** L271-274 The description is valid and does not be changed. Nevertheless I'm curious why the authors stop the disaggregation procedure at 40mins and don't go all the way to 10min? Did the scaling behaviour change for finer resolution (often scale invariance hold for  $\sim 1d \rightarrow \sim 1h$ )?

*AC2.10 Of course, the disaggregation can go on to finer resolutions. The choice of stopping at 40-minutes was mainly based on the application needs. By contract with the Swiss Confederation, we have to produce weather scenarios for small catchments (from 10 to 1000  $km^2$ ). The time resolution retained for hydrological simulation in the project was thus 30min. On the other hand, the artifact induced by the measurement precision would make difficult the evaluation of disaggregated scenarios at lower temporal resolutions (e.g. Paschalis et al. (2013)).*

**RC2.11** Section 2.6 Maybe I've just not seen it: Which distribution function is used to estimate the return periods analysed in Fig. 6?

*AC2.11 Actually, no distribution is fitted to the data. We use the Gringorten plotting position to plot annual maximums, and then the quantile for a given non-exceedance probability is determined by linear interpolation of annual maxima. This will be precised in the revised manuscript.*

**RC2.12** L379-384 The scale-dependency often plays a minor role if the scaling behaviour is linear, which is often the case for the analysed range of resolution in this study. I suggest

to add a figure on scaling behaviour (the typical  $M_q$  (Moments of order  $q=1,2,3,..$ )-temporal resolution-plot) to verify the finding that A not necessarily outperforms B. Other cascade models apply scale-invariance already for this range of temporal resolutions (e.g. Günther et al, 2001)

**AC2.12** Please find below in Figure 10 the moments of order 1 to 4 estimated on observation data (points) and on the generated scenarios (the lines show the median among the metrics estimated on 30 scenarios). The results here concern the station of Zurich. Each column corresponds to a model and each row to a season. We find the differences between models are only minor for moments of order 1 to 3, while for 4-order moments more differences can be noticed and depend on the season. To our opinion, this interesting scaling behaviour of precipitation is rather out of scope of our study. We will therefore not mention it in the revised version.

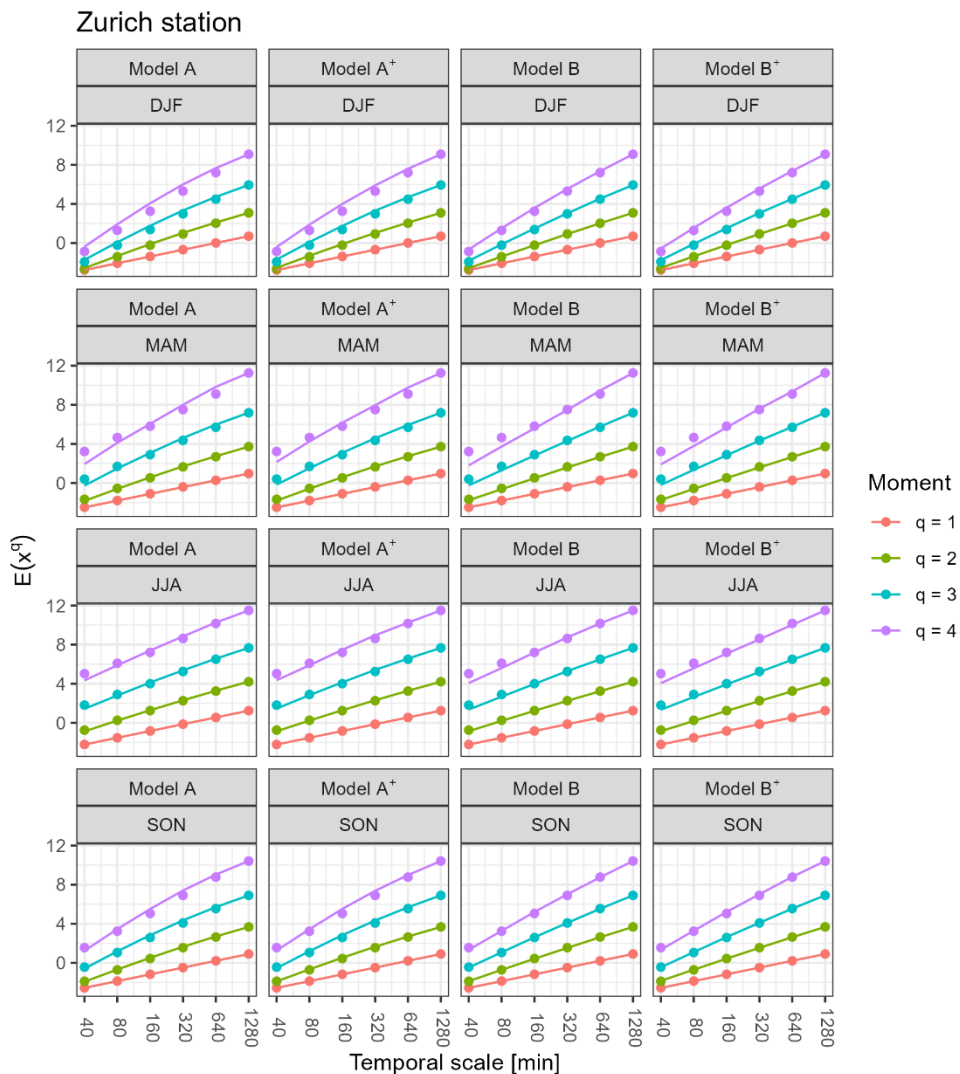


Figure 10. Log-log plots of the empirical  $q$  moments versus temporal scale. Each color corresponds to one moment, dots to moments estimated on observations, and full lines correspond the median of estimated moments on the 30 scenarios. Each column to one model and each row to one season. The analysis is performed on the data for Zurich station.

**RC2.13** Fig. 5a) ‘Standard deviation’ – of what?



**AC2.13** Thank you for noting that the description needs to be detailed. We will add “Standard deviation of precipitation”.

**RC2.14** L376-378 This is maybe true for analytical developments, but for non-analytical approaches position-dependencies are most often taken into account. This information should be added here for the sake of completeness.

**AC2.14** Our focus here was on the analytical scaling models. Anyway, this is a valid point and we will complete this information as suggested.

**RC2.15** L408-420 The persistence/intermittency is a weakness of micro-canonical cascade models. Müller-Thomy (2020) has introduced an extended position-dependency that improves the autocorrelation for all lags. Here, lag-1 and lag-2 are studied, results for other lags are not shown. Are results similar for all lags? Would the involvement of the extended position-dependency (would be an extended asymmetry approach) also an (additional) improvement for the analytical MRC?

**AC2.15** Yes, results are similar for all lags. We had mentioned it in the manuscript. Figure 11 below presents results obtained for other lags (lag-3 to lag-6 estimated on 40-minute data). Accounting for the asymmetry index significantly improves the reproduction of the lag 1 and 2. The added value decreases with higher degree lags but is still important for lag 3 and 4 (see Figure 11 below).

Thank you also for the interesting question relative to the “Extended position dependency”. We have shown in our paper that the asymmetry of the CDF depends on the precipitation structure of the precipitation sequence  $\{R_{t-1}, R_t, R_{t+1}\}$  and that the hidden breakdown coefficient  $Z$  is a skilful predictor for this dependence. We could also expect that the CDF asymmetry additionally depends on the precipitation structure at different coarser temporal resolutions and that  $n^{\text{th}}$  order hidden breakdown coefficient  $Z_n$  defined with the extended precipitation sequence  $\{R_{t-n}, \dots, R_{t-1}, R_t, R_{t+1}, \dots, R_{t+n}\}$  could be of interest there. This will however introduce some additional complexity to the model. This will be worth further investigation.

**RC2.16** Fig. 9 Which temporal resolution is shown here?

**AC2.16** Thank you for pointing out that the specification of the temporal resolution is missing for the Figure 9a. We will specify that the temporal resolution is 40-minutes.

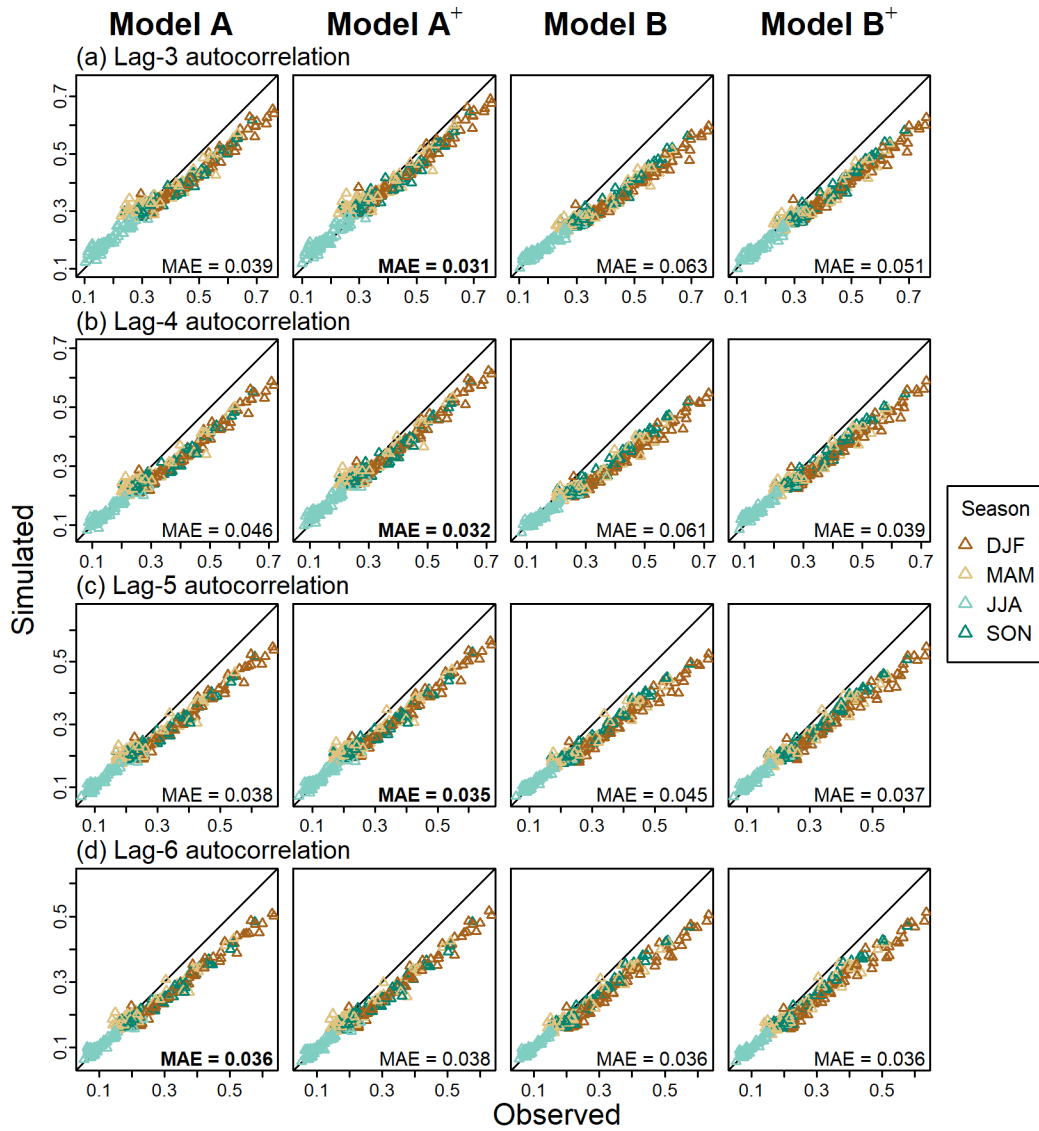


Figure 11. Observed versus simulated statistics for each considered model at a 40-minute temporal resolution for lags of higher order.

## References

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