Response to anonymous referee #1

*We thank referee #1 for reading the manuscript carefully and providing thoughtful and constructive comments. The comments are noted with **RC,** our responses with **AC**, and the intended additions or changes in the manuscript are underlined.*

RC1.1 Based on some exploratory analysis in Rupp et al. (2009), I do have a lingering doubt related to the apparent asymmetry and the need to explicitly account for it fully.

Asymmetry has been considered before although differently. For example, Olsson (1998) and Güntner et al. (2001) developed the distributions of the breakdown coefficients separately for time intervals that start or end a rainfall sequence or are within a rainfall sequence. They showed that starting/ending intervals had distinctly asymmetric distributions.

> **AC1.1** *We thank Reviewer #1 for this important comment. Yes, the dependency of the cascade generator on the external pattern of precipitation (i.e. the dependency of the cumulative distribution function (CDF) of the breakdown coefficients (BDCs) on the external pattern of precipitation), has been highlighted/commented in a number of publications. We were also aware of the work of Rupp et al. (2009) and of their analyses/doubts on the need/way to account for it in a MRC model.*
>
> *To our knowledge, accounting for this dependency in a MRC has mainly (almost only) been carried out with empirical MRC where the CDFs of the BDCs is estimated empirically and described with empirical CDFs. This allows to describe CDFs with different shapes. Analytical scaling MRC presented up to now, such as those described in Rupp et al. (2009), can conversely not account for this dependency. The reason is that the CDF of the BDCs is modelled with a symmetric distribution where one assumes equal wet/dry or dry/wet probabilities (p01 and p10) and a symmetric distribution of non-zero weights. For instance, the analytical model uses a symmetric beta distribution in Rupp et al. (2009), a mixed beta – normal distribution in Licznar et al. (2011a), a 3N-B distribution in Licznar et al. (2011b) where a symmetric beta distribution is combined to 3 two-side truncated normal distributions.*
>
> *One noticeable exception is the analytical MRC of McIntyre et al. (2016), who used a 2-parameters beta distribution. This distribution, which can be asymmetric, was used to model the asymmetric CDFs found for precipitation amounts in asymmetric precipitation sequences, namely the starting/ending intervals (for the respectively named "followed" and "preceded" precipitation amounts in their work). McIntyre et al. (2016) did not consider scaling models to estimate the CDF parameters (the CDF of the BDCs was estimated for each temporal scale for different categories of rainfall, defined from the volume of precipitation and the external pattern class (isolated, enclosed, preceded and followed)). The number of parameters for their model is then considerable, and is potentially too large to allow for a robust estimation.*
>
> *In our work, we fill this methodological gap. We consider an analytical model which is by construction asymmetric and whose parameters are related – thanks to simple scaling laws, to the asymmetry of the local precipitation pattern. This allows us to*

keep the number of parameters very low, and to combine 1) scaling relationships with temporal scales and intensity and 2) "scaling dependency" on the external pattern structure. This is a first novelty. Next, we introduce the asymmetry index and we show that the asymmetry level of the cascade generator (i.e. the asymmetry level of the CDF used for the distribution of W's) depends on this index in a continuous way. This allows conditioning the MRC on the external pattern but without the need to consider external pattern classes. This is another strength of our approach.

As shown in Fig. 1, the "intensity" of the asymmetry of the local precipitation sequence can vary a lot from one external pattern to the other. This is not only a question of asymmetry class ('ending/starting", …). As shown in Hingray and Ben Haha (2005) (Fig. 4 of this work given below), the distribution of W is not expected to be the same in configurations b) (high gradient in a steep descending pattern) and c) (low gradient in a steep descending pattern). The asymmetry index introduced in our work is shown to allow the distinction between such configurations.

The introduction of our paper will be strengthened to better describe previous issues.

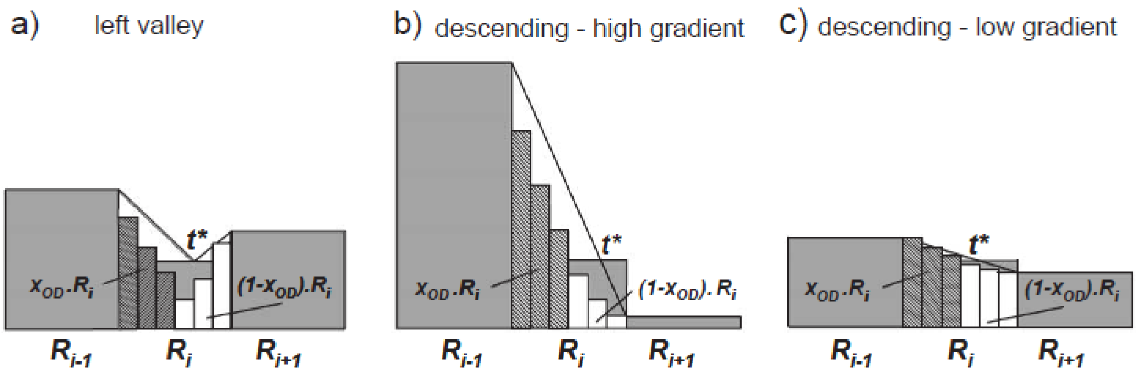

Fig. 4. Graphical illustration of the rainfall fraction x_{OD} used in the pattern-based Microcanonical Random Cascade model (pbRC) for three rainfall sequences $[R_{i-1}, R_i, R_{i+1}]$: (a) left valley pattern; (b) descending pattern-high gradient; (c) descending pattern-low gradient. The grey rainfall volume spanning the three first 10-min bars and calculated according to Eq. (7) is $R_1=G(0.5*T)=x_{OD}\cdot R_i$. The white volume spanning the three last bars is $R_2=(1-x_{OD})R_i$.

Figure 1. Illustration of possible breaking coefficient for a steep and a non-steep decreasing pattern precipitation sequence (from Hingray and Ben Haha (2005)).

RC1.2 Rupp et al. (2009) showed that models that did not explicitly incorporate asymmetry did not generate asymmetry at the time steps at which the rainfall was simulated. However, when they resampled their synthetic rainfall at an interval of the same duration but offset in time by small amount, asymmetry in the breakdown coefficients was introduced. The breakdown coefficients from the resampled series were remarkably like the breakdown coefficients from the observed data (see their Figure 16).

Rupp et al. (2009) concluded that at least some of apparent asymmetry in the breakdown coefficients arises from imposing a discrete, regularly timed sampling interval to an irregularly timed phenomenon. To what degree, then, are the authors simply reproducing an artifact of sampling by incorporating asymmetry explicitly into their models? I think this issue needs discussion.

AC1.2 *We thank Reviewer #1 for this very interesting comment. We had indeed seen this analysis of Rupp et al. (2009) and their offset experiment. Note that they do not strictly conclude that at least some of the apparent asymmetry in the breakdown coefficients arises from imposing a discrete, regularly timed sampling interval to an irregularly timed phenomenon.*
They only say: "We suspect that the asymmetry in the starting and ending distributions is largely an artifact of sampling a semi-continuous and irregularly times process at discrete, regularly spaced intervals" (paragraph [41]).

And after their "offset experiment", which results they presented in Figure 16. Their conclusion was (paragraph [44]):

"While we do not present definitive evidence that the variability in the cascade weights among class intervals is completely an artifact of the sampling method, our preliminary analysis raises interesting issues that warrant further investigation. For one, if the dependency is largely an artifact, is the approach of Olsson [1998] and Gunter et al [2001] to reproduce it explicitly warranted, particularly because it substantially increases the number of model parameters required? Also, if we sample our rainfall events such that the sampling intervals begin and end exactly when the rain actually begins and ends, will weights in the middle of an event still differ from those near its onset or termination?"

To our knowledge, unfortunately, no other work has been carried out to investigate these interesting issues. Our work strongly suggests that their "suspicion" was likely wrong. Our work suggests that there is no one single cascade generator for a given time scale and given intensity class, but a large variety depending on the asymmetry importance of the local precipitation pattern. The cascade generator is asymmetric and as it was demonstrated empirically by the works of Olsson (1998), Gunter et al. (2001), McIntyre et al. (2016) and others, this asymmetry is determined by the asymmetry of the local precipitation sequence around the precipitation amount to disaggregate. In line with the work of Hingray and Ben Haha (2005), our work additionally shows that the asymmetry of the cascade generator can be more or less important, depending the importance of the asymmetry of the local precipitation sequence.

The main argument of Rupp et al. (2009) to their conclusion recalled above is based on their offset experiment. However, another conclusion could (should) likely be given. This is at least what suggests the following offset experiment we carried out to answer this issue raised by Reviewer #1.

Offset experiment. *The offset experiment was carried out on 40min time series data, but similar results are expected for other temporal aggregation levels. Precipitation data available for this experiment have a 10-minute resolution*

- *For a given station, in order to obtain 40-minute time series we aggregate 10-minute time series by using different time offsets: no offset, 10 min, 20 min and 30 min. Four time series have been thus obtained with the same resolution, 40-minute. They are all derived from the same 10-minute initial time series.*
- *For each of these 4 offset 40min time series, we calculate a set of different metrics. Obviously, we would expect the statistics to be independent of the offset experiment. For illustration, some results are shown for different stations in Figure 2 for standard deviation, autocorrelation at lag-1 and for 5 and 20yrs return levels.*
- *The initial 40min time series (without offset) was next disaggregated to 10min producing 30 time series scenarios.*
- *The same offset experiment is performed for each of the 30 disaggregated time series scenarios. For each scenario, 4 offset 40-minute time series were produced with the 4 different offsets. The process was repeated for each station. The results obtained with the 4 models A, A^+, B, B^+ are presented in Figure 3 (the MAPE metric is presented for different statistics).*

The conclusions from our offset experiments are:

- *Whatever the statistics considered, the estimates calculated on observations for different offsets are very similar. This is highlighted in Figure 2 for 12 stations spread across Switzerland.*
- *When calculated on disaggregated data, the estimates calculated for different offsets are no more similar. More precisely, the estimates obtained for the three non-zero offsets (10, 20, 30-minute) are similar to each other but often significantly different from the reference estimate (with the 0-minute offset). This is highlighted in Figure 3. Each box plot represents the estimated value of a given statistics for 81 stations. Estimates are given for standard deviation (first row), lag-1 correlation (2^{nd} row), and 5 and 10-years return levels ($3+4^{th}$ rows) for the four seasons (the 4 columns) and the different models (model A, B, A+ B+ in the x-axis). Whatever the season, whatever the statistics, the red boxplot (offset 0-min) is very often significantly different from the green/blue/magenta boxplots (10, 20, 30-minute).*
- *Models A and B (without asymmetry) are much more sensitive to temporal offset than models A^+ and B^+.*
- *The model the less sensitive to temporal offset is model B^+.*

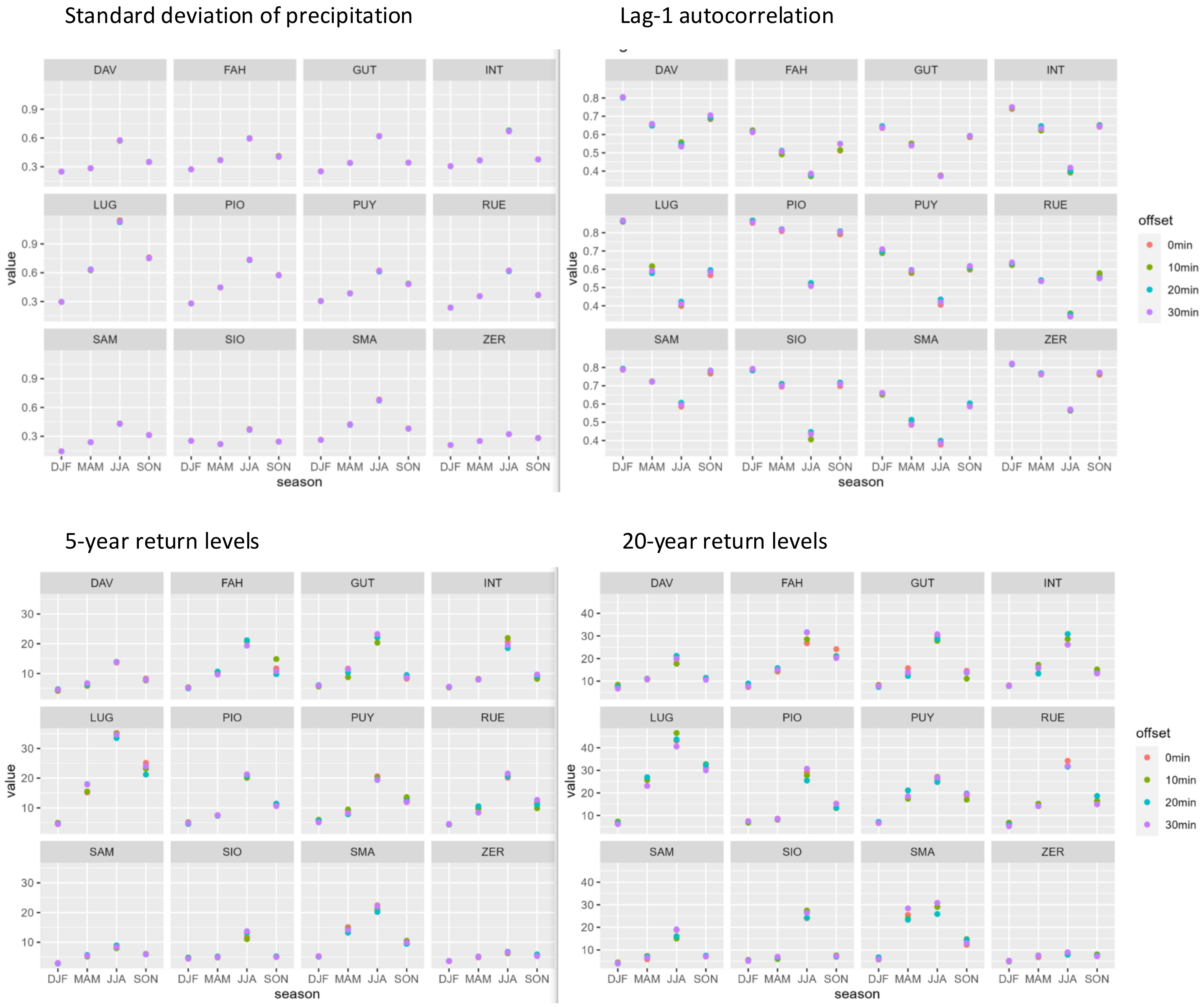

*Figure 2. **Effect of the offset on observed time series statistics.** Observed metrics as estimated on 40-minute time series obtained for different time axis offsets. On the left is shown the standard deviation and on the right autocorrelation at lag-1. Each panel corresponds to a given station (results presented for 12 stations).*

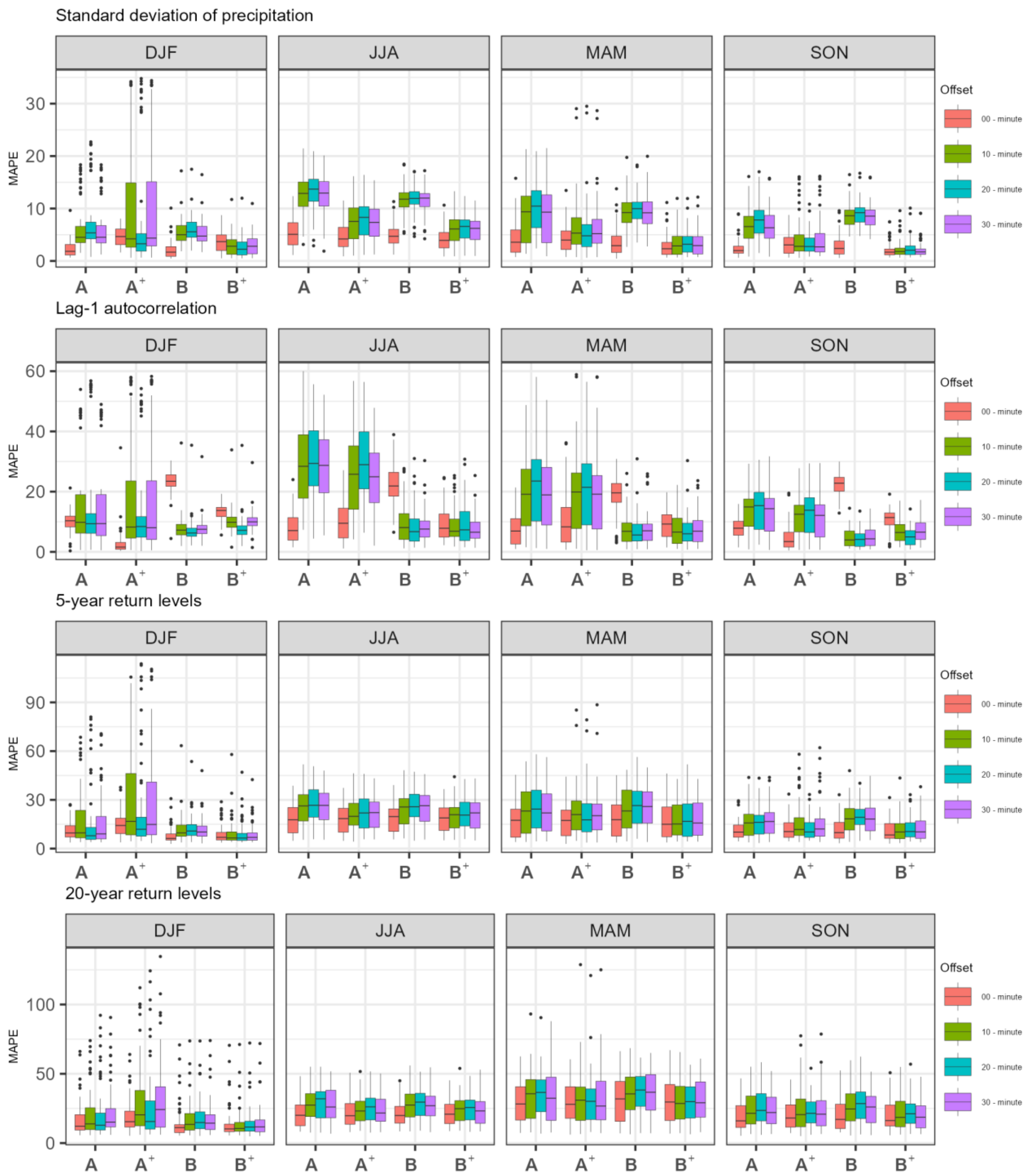

Figure 3. ***Effect of the offset on disaggregated time series statistics.*** *Mean Absolute Percentage Error (MAPE) between the observed and disaggregated values for different statistics (first row: standard deviation, 2^{nd} row: lag-1 autocorrelation, 3 and 4^{th} row: 5 and 10-years return level). MAPE is given as a function of the time offset (boxplots of different colors: red - no offset, green, blue, magenta: 10, 20, 30min time offset, respectively), season (DJF, MAM, JJA, SON columns) and disaggregation model (A, A^+, B, B^+). Each boxplot summarizes the single-site performances obtained for 81 stations spread over Switzerland and for the 40-minute temporal aggregation level.*

Conclusions: *The results of these offset experiments, with observations first and with disaggregated series next, strongly suggest that cascade models that disregard the asymmetry of the cascade generator and its dependency to the asymmetry of the local precipitation definitively break some important precipitation variability features. A comment on this interesting point will be also added in the discussion.*

Model parsimony argument. *One last argument of Rupp et al. (2009) to disregard the asymmetry dependency was the large amount of model parameters required to describe this. This was indeed a critical point of the empirical models of Olsson (1998) and Gunter et al. (2001). Their models are based on empirical ECDFs, which are different from one "asymmetry" class to the other. Their models could thus not easily account for the dependency on intensity (at least not with the scaling relationships of analytical MRC developed by Rupp et al. (2009)). This was indeed an important limitation. This model parsimony argument was also an issue in the analytical model of McIntyre et al. 2016. As mentioned above, they did not consider scaling laws, which are needed to reduce the number of parameters.*

Our approach fills this gap. We account for the local asymmetry of precipitation in a very parsimonious way, with an asymmetry index which is continuous. Our model can be then analytical, both for the statistical distribution (which is a non-symmetric beta distribution) and for the scaling relationships linking the parameters of the model to different features of the rainfall amount to disaggregate (intensity, asymmetry, temporal scale).

With our continuous asymmetry index, we do not have to define classes, allowing a much more robust estimation of model parameters. We are then able to present maps over Switzerland for the 5 parameters of the model. The very large spatial homogeneity of the parameters clearly shows the robustness of the estimates and strongly suggests the relevance of the model with respect to the different features that are of importance for the cascade generator.

Is asymmetry an artifact of sampling? *Notwithstanding previous elements, from what can be understand from observations, asymmetry in precipitation-related data is not an artifact of sampling.*

- *The asymmetry index we introduce is defined per see. It just characterizes the asymmetry of any given (observed, simulated) temporal rainfall sequence* $\{R_{t-1}, R_t, R_{t+1}\}$. *As mentioned in the manuscript, for a given time t, the farthest the value of Z is from 0.5, the more asymmetric the sequence is. A Z value close to 0.5 means that* R_{t-1} *and* R_{t+1} *are very close to each other, or that they are very small when compared to the amount to disaggregate. This has no link with the sampling artifacts.*
- *As shown in the manuscript, this rainfall sequence asymmetry translates directly to the asymmetry of the ECDF of the breakdown coefficients. This is clear from Figure 3 of the manuscript where statistical characteristics of the breakdown coefficients W are presented as a function of the*

asymmetry index Z. Some comments will be given in the discussion on these issues.

RC1.3 33-35: Yes, many types and variations of disaggregation models exist. Although it would be excessive to describe them all here, I suggest referencing one or two review papers.

> **AC1.3** *Thank you for this suggestion. There is however to our knowledge no real review paper on disaggregation for the generation of high-resolution data. The review paper of Srikanthan and McMahon (2001) reviews some disaggregation techniques but it is somehow dated and does not consider the disaggregation to sub-daily resolutions. The paper of Koutsoyiannis (2003) gives an interesting and rather large view of different disaggregation techniques but it is not really a review and some approaches are missing (e.g. Method of Fragments). We will nevertheless mention it in the manuscript.*

RC1.4 157: Winter should be defined.

> **AC1.4** *As pointed out by you below in RC1.9, it should actually be Autumn which will also be defined as suggested.*

RC1.5 169: The text claims to be referring to Figure 1d but I think it should be Figure 1b.

> **AC1.5** *Thank you for pointing out this error. We will fix it.*

RC1.6 172: Model B has 5 parameters, not three. I_0 and I_1 should be included in the count of parameters.

> **AC1.6** *We agree that I_0 and I_1 can also be counted as parameters of the model. However, these parameters are kept fixed for all seasons and stations and do not depend on the specific precipitation data at the stations. Here, we will precise that only the free parameters of the model are counted, i.e. the ones that need to be estimated and vary through seasons and stations.*

RC1.7 204: Why use lower-case z for the asymmetry index here and below when it was previously upper-case?

> **AC1.7** *We used lower-case in order to point out that z is a realization of the random variable Z. Anyway, we recognise that this can create confusion to the reader so we will uniformize the notation in the whole manuscript.*

RC1.8 275-279 & 385-398: Licznar et al. (2011) explore in some detail the artifacts arising from measurement resolution. They present a method of adding small amounts of random noise to discretized observations in an attempt to extract the underlying distribution of W at low intensities. It is at least worth a mention even if the authors don't want to take that approach.

> **AC1.8** *The artifacts arising from measurement resolution lead to critical estimation issues, indeed. Thank you for pointing out this issue here. Actually, during our preliminary analysis, we employed a similar approach of "jittering" observation records. The objective was 1) to understand the influence of rain gauge tipping resolution on the distribution of cascade weights (especially on the non-zero probability amount p_x and on the shape of the distribution for the W^+ breakdown coefficient) 2) to determine which data have to be disregarded to allow a relevant estimation of the model parameters, which would be not too much contaminated by the measurement resolution artifact.*

__The jittering process considered in the present study__ consisted as follows. In Switzerland, sub-daily resolution precipitation is measured with tipping bucket rain gauges, with a measurement resolution of 0.1mm. In the measurement process, one rainfall pulse is recorded once the 0.1mm rainfall bucket is filled. The duration needed to fill the bucket is obviously not known. It may be a few seconds in the case of very intense rainfall rates or several hours or days in the case of very light rainfall rates. Our jittering process is based on the previous consideration. When X mm have been recorded within a given time step (and then displayed in the time series of precipitation data), we considered that:

- *(X-0.1) mm really belong to this time step and*
- *Only part of the remaining 0.1mm belongs to this time step (this 0.1mm is obviously the first tipping bucket pulse recorded during this time step). The fraction Y of the 0.1 mm belonging to the current time step was sorted randomly. The complementary fraction (1-Y) was attributed to the previous time step.*

__Generation of jittered time series.__ Our jittering process thus only applies to the first 0.1mm recorded amount (i.e. first recorded tipping bucket pulse) of each time step. It was applied to the original high-resolution time series, available at 10min resolution. We generated 20 jittered time series from the original one and re-estimated the characteristics of the statistical distributions required to describe the breakdown coefficients.

Comments from our jittering analyses.

As mentioned in many previous papers, in the initial time series, rainfall amounts can only take discrete values. With the 0.1mm resolution, only 0.1, 0.2, 0.3…. 1, 1.1, … values are possible. The probability occurrence of breakdown coefficients with a value of 0 or 0.5, 1/3 or 2/3, 1/4 and 3/4 is overestimated (these values correspond to rainfall amounts of 0.1, or 0.2, or 0.3m, 0.4mm respectively, which are rather frequent). With this jittering process, rainfall values can take all possible positive values and the overestimation mentioned above is largely reduced. We thus largely reduce the artifact due to the tipping bucket resolution. As mentioned in the paper of Licznar et al. (2011b), the statistical distributions of W for the jittered time series are "very smooth and elegant, especially for small time scales ranging from 10 to 80min".

These jittering analyses highlighted also the following:

__Impact of the jittering on precipitation amounts and statistics.__ By construction, the largest is the initial rainfall amount for a given time step, the lowest is the difference between this initial value and the jittered value. For instance, for a 1mm initial amount, the jittered values can take values between 0.9mm and 1mm. The largest possible difference is 10%. For a 10mm initial amount, the jittered values can take values between 9.9mm and 10mm. The largest possible difference is 1%. The jittering process has thus almost no influence on rainfall properties for moderate to large precipitation amounts. This is for instance the case for precipitation maxima (especially return levels for different return periods) which are almost unchanged.

Impact of the jittering on the statistical distribution of the breakdown coefficient. *The jittering process leads to significantly modify the distribution of W and, in turn, it can significantly modify the scaling properties of the different parameters of the cascade generator.*

This was especially highlighted for the non-zero subdivision probability. *In the Figure 1a of the manuscript (also shown below in Figure 4), the non-zero subdivision probability p_x varies from 0 - for very low rainfall intensities to 1 - for very large ones.*

- *With the initial series, the precipitation intensity-p_x relationship is found to depend on the temporal scale (see the different curves in Fig. 1a). However, dots reported in the figure for small intensities correspond to small rainfall amounts, which are highly contaminated with the resolution issue as many of these small rainfall amounts are 0.1, 0.2, 0.3 values. This is especially critical for the smallest temporal scales.*
- *When jittered, the differences between the different time scales – mentioned above for instance for the intensity-p_x relationship, is reduced suggesting that an important part of the dependency on temporal scale is mostly due to the measurement precision artifact (see Figure 4a and c). When we disregard all data smaller than 0.8 mm (that are numerous, especially for fine temporal scales), the different dots on the x-axis in the figure with a p_x value of 0 disappear. This confirms the relevance of one unique scaling model for p_x as a function of intensity only. This is the reason why we considered Model B, which by construction disregards possible relationships with temporal scale.*

This artifact issue obviously deserves more attention in all works with MRC. It would require too long explanation and was not integrated in the manuscript. We will nevertheless mention it more clearly in the discussion.

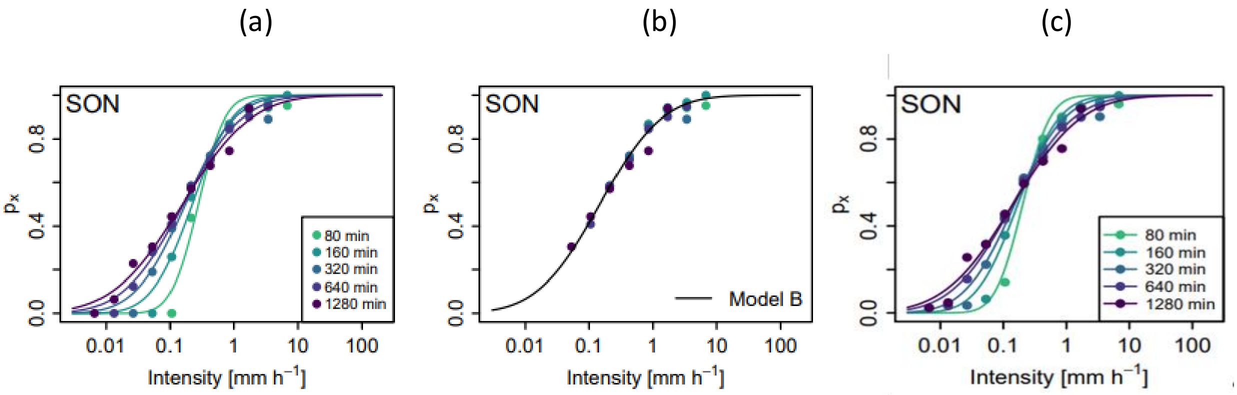

Figure 4. Non-zero subdivisions probability px as estimated on observation data from Zurich station for each intensity class and temporal scale. (a) Same model A as in the manuscript, (b) model B, a threshold of 0.8mm is applied to discard px (same figure as in Supplementary Material) and (c) same analysis as in panel (a) but this time a jittering procedure is applied to the data before performing the estimation.

RC1.9 Table 1: As I stated above Model B has a total of 5 parameters when parameters I_0 and I_1 are included. Similarly, Model B+ has 7 parameters.

> **AC1.9** *Please see our answer to RC1.6.*

RC1.10 Figure 1: "SON" should be defined. I assume it is Sep – Nov? Also, the caption says "winter", whereas SON would be autumn.

> **AC1.10** *Thank you for spotting this error. Yes, SON means September to November corresponding to the autumn season. Similarly, DJF means December to February, corresponding to the winter season. This will be clarified.*

RC1.11 Figure 3: Panel (a) takes effort to interpret. I have a few comments:

1. What is "x"? Is it W? W_1? W+? For clarity, please replace x by what is represent.

2. An ECDF should go from 0 to 1 but it is not obvious that each individual curve does that. For example, the Z =1 curve appears to have a value F(x) ~= 0.5 at x = 1, but does the Z = 1 curve jump to a value of F(x) = 1 very close to x = 1?

3. Lastly, although plotting the ECDF is convenient in that several curves can be plotted in one panel, I think it would be much easier to interpret histograms of W for various classes of Z. Notable differences between winter and summer might also be more obvious.

> **AC1.11** *Thank you for the detailed comments on these panels.*
>
> - *x here referred to W. We will replace x by W for simplicity as suggested.*
> - *Sure, each ECDF should go from 0 to 1 and it is also the case here. We agree that in the figure this is unnoticeable due to the superposition of different curves and axis. It is also true that for "z=1" the ECDF jumps from around 0.5 for values close to 1, to 1, when x gets 1. The same behaviour can be noticed for "z=0" for very small x. This effect is due to the intermittency of the precipitation process that is reflected on the observed cascade weights with a considerable number of x = 0 or x = 1.*
> - *The distribution of positive weights can indeed significantly differ between Winter and Summer and these differences may be slightly difficult to appreciate with ECDFs. Please find below in Figure 4 histograms of observed weights that correspond to different classes of Z-index. On the left are shown histograms for Winter (December to February) and on the right for Summer (June to August). The above assumption is supported by the histograms, even though we believe that the differences can be better observed by looking at the ECDFs than at histograms in our case. It is due to the considerable number of weights equal to 0 or 1, that get more visual attention than the distribution of positive weights, 0<W<1.*

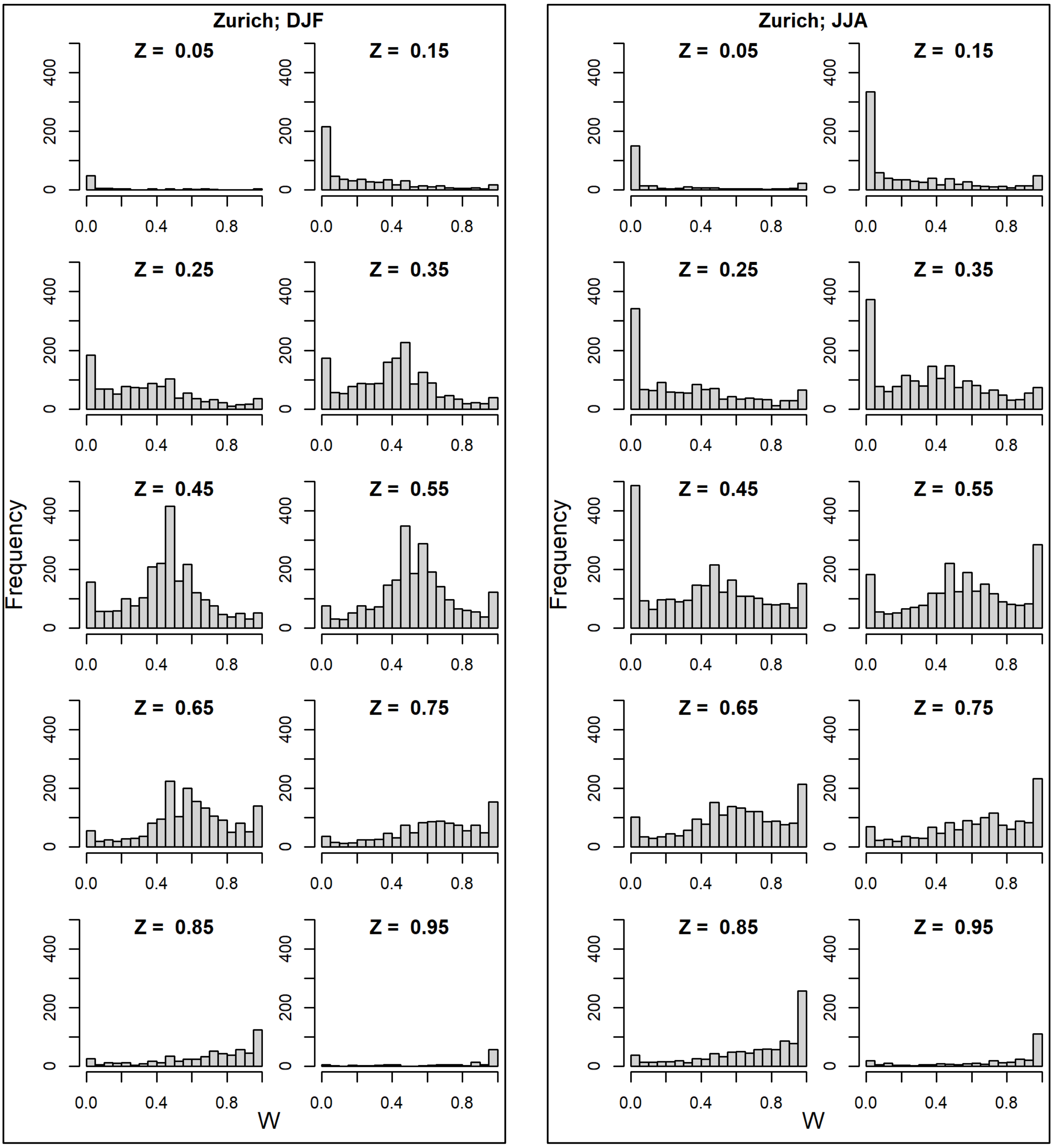

Figure 4. Histograms of observed breakdown coefficients for each class of asymmetry index Z. In the left panel are the results for winter (DJF) while on the right are the results for summer (JJA). Please note that the weights calculated for precipitation amounts smaller than 0.8 mm are discarded from this analysis.

RC1.12 Figure 6: Consider using a log-log scale.

> **AC1.12** *This is a great suggestion as with the current presentation more visual attention is given to very high return levels, which usually occur in summer. Please find below the analogue figure (Figure 5) but in log-log scale. These results correspond to 5-year and 20-year return levels at 40-minute temporal scale. The following representation will replace the one in Figure 6.*

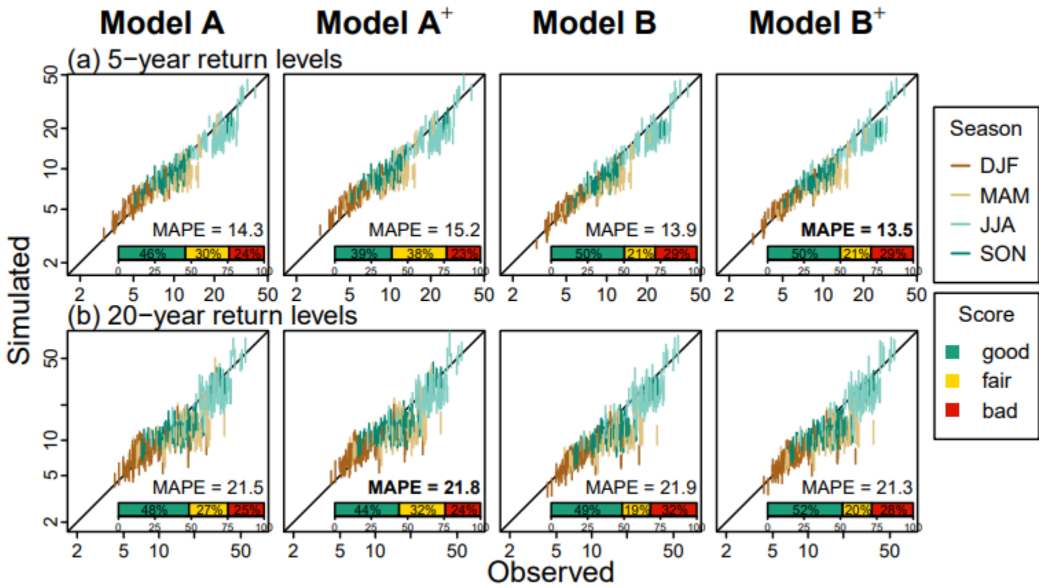

*Figure 5. **(Log-scale)** Observed versus simulated return levels at the 40-minute temporal resolution for (a) 5-year and (b) 20-year return periods, for each model and at each site. Same results as in Figure 6 of the manuscript but log-log scale is used for plotting.*

RC1.13 229: Replace "confronted" with "compared".

AC1.13 *Thank you for this suggestion. We will account for it.*

RC1.14 474: "…reveals actually not obvious…" Typo?

AC1.14 *Thank you for pointing out this mistake. We will remove the word "actually".*

References

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