Dear reviewer,

We are again very grateful for the thorough review of our manuscript and the constructive comments, which contributed greatly to the quality of the revised manuscript. Below we respond to all the comments.

Comment: Revision by the authors in response to the previous review has been reasonably made in most cases and mathematical explanations and model confirmation and comparison are enhanced in my opinion. While I still found a couple of parts/equations difficult to understand, it is most likely that those can be easily addressed by the authors. I think after reflecting those the manuscript looks acceptable for GMD.

Response: We are glad that the reviewer appreciated our efforts to improve the manuscript and that he considers it to be acceptable for publication in GMD after minor revisions.

Comment: Eq. 25. Should right-hand side requires a factor of 1/V to be multiplied with?

Response: The reviewer is correct, and we made the change.

Comment: L175. Same as Eq. 25. The first definition of mu_x is understandable, but the second one with summation symbol is hard to understand especially given the definition in Table 1 and equations in L81 and 82. The second equation may require 1/V to be multiplied with on the right hand side? Eq. 26 remains valid regardless of above two points.

Response: Also, here, the reviewer is correct, and we made the change.

Comment: Eq. 28. Again to be consistent with previous definition of mu, C should not be here?

Response: This has now also been corrected. It was done correctly in the code and applications, so this mistake does not affect other parts of the manuscript.

Comment: L192. I cannot understand why r = 0 in Eq. 26 leads to R = P. Should this be like r = constant?

Response: For production, Eq. 26 is treated as an indefinite integral. When $r(\chi) = 0$ the result P is simply the constant of integration, which is correct for the zeroth-order kinetics considered. It would be incorrect to define r = constant, as this would yield a result in the form of $A\chi + B$ and, therefore, correspond to first-order kinetics.

We now clarify this in the text: "Representing the production of new material with a zeroth-order kinetic term (r = 0) results in R = P as the constant of integration when treating the integral in equation 27 as indefinite."

Comment: L201-202. "As these reactions do not discriminate with respect to age, the moments will not change." For this r(x) has to be no longer a function of x as r = -kC; otherwise, r(x) = -kC(x) and dphi/dt|R will be still a function of x and need distribution integration for its calculation? Regardless of whether this is correct or not, please clarify about parameterization of r(x) too.

Response: If the reaction does not discriminate by age, then $f(\chi)$ is not a function of χ (more precisely, it is a constant function with $df/d\chi=0$). The reaction rate $r(\chi)$ still is a function of χ: it is proportional to the concentration $C(\chi)$. Thus, the shape of the distribution $C(\chi)$ remains unchanged since the reaction $r(\chi)$ has the same shape. The magnitude can change (changing the total concentration C), but this does not affect the moments.

Comment: Appendix: Eq. A10. I still could not figure out how the first term on the right-hand side of Eq. A10 is derived from Eq. A9 and divergent theorem.

Response: The first term of equation A8 is

$$T = \frac{\partial \phi}{\partial \mu}\left(\frac{D_l C_L}{\delta_x}\frac{\partial \mu}{\partial x}_l - \frac{D_r C_R}{\delta_x}\frac{\partial \mu}{\partial x}_r\right)$$

The equations

$$C_L = C_l - \frac{1}{2}\frac{\partial C}{\partial x}\delta_x \qquad \text{(A9a)}$$

$$C_R = C_r + \frac{1}{2}\frac{\partial C}{\partial x}\delta_x \qquad \text{(A9b)}$$

are inserted, yielding

$$T = \frac{\partial \phi}{\partial \mu}\left(\frac{D_l C_l}{\delta_x}\frac{\partial \mu}{\partial x}_l - \frac{D_r C_r}{\delta_x}\frac{\partial \mu}{\partial x}_r\right) + \frac{\partial \phi}{\partial \mu}\left(-0.5 D_l \frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}_l - 0.5 D_r \frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}_r\right)$$

When we take the limit, for the first term, we obtain

$$\lim_{\delta x \to 0} \frac{\partial \phi}{\partial \mu}\left(\frac{D_l C_l}{\delta_x}\frac{\partial \mu}{\partial x}_l - \frac{D_r C_r}{\delta_x}\frac{\partial \mu}{\partial x}_r\right) = -\frac{\partial \phi}{\partial \mu}\left[\frac{\partial}{\partial x}\left(DC\frac{\partial \mu}{\partial x}\right)\right]$$

For infinitesimally small x, the terms inside the brackets of the second term may be added, yielding

$$\frac{\partial \phi}{\partial \mu}\left(-0.5 D_l \frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}_l - 0.5 D_r \frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}_r\right) = -D\frac{\partial \phi}{\partial \mu}\frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}$$

Hence,

$$\lim_{\delta x \to 0} T = -\frac{\partial \phi}{\partial \mu}\left[\frac{\partial}{\partial x}\left(DC\frac{\partial \mu}{\partial x}\right) + D\frac{\partial C}{\partial x}\frac{\partial \mu}{\partial x}\right]$$

which is the first term of A10.

Since these steps do not introduce new ideas, we do not want to write them out in the manuscript. However, the description may not have been entirely accurate, as we do not have to apply the divergence theorem, but instead, we take the limit to 0. We have improved the text in the revised manuscript.

Comment: Supplement: 1.1. It is very hard to imagine what kinds of vectors the authors are using. Definition of x is given but xL and xR are assumed to be obvious and not defined. Individual x vector contains ages of individual n particles? If so and x0 is x at time 0, it is confusing as xL and xR appear in definition of x0.

Response: The text's description of χ_L and χ_R was not sufficiently clear. These vectors contain the ages of all particles at the domain's boundaries. It is correct that χ_0 is χ at time 0. We revised the text in the supplement and now show the equations for the boundary conditions (equations S2 and S3).