

Understanding variations in downwelling longwave radiation using Brutsaert's equation

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Abstract

A dominant term in the surface energy balance and central to global warming is downwelling longwave radiation (R_{ld}). It is influenced by radiative properties of the atmospheric column, in particular by greenhouse gases, water vapour, clouds and differences in atmospheric heat storage. We use the semi-empirical equation derived by Brutsaert (1975) to identify the leading terms responsible for the spatial-temporal climatological variations in R_{ld} . This equation requires only near-surface observations of air temperature and humidity. We first evaluated this equation and its extension by Crawford and Duchon (1999) with observations from FLUXNET, the NASA-CERES dataset, and the ERA5 reanalysis. We found a strong spatiotemporal correlation between estimated R_{ld} and the datasets above, with r^2 ranging from 0.87 to 0.98 across the datasets for clear-sky and all-sky conditions. We then used the equations to show that changes in lower atmospheric heat storage explain more than 95% and around 73% of diurnal range and seasonal variations in R_{ld} , respectively, with the regional contribution decreasing with latitude. Seasonal changes in the emissivity of the atmosphere play a second role, which is controlled by anomalies in cloud cover at high latitudes but dominated by water vapor changes at mid-latitudes and subtropics, especially over monsoon regions. We also found that as aridity increases over the region, the contributions from changes in emissivity and lower atmospheric heat storage tend to offset each other (-40 W m^{-2} and $20\text{-}30 \text{ W m}^{-2}$, respectively), explaining the relatively small decrease in R_{ld} with aridity ($-(10\text{-}20) \text{ W/m}^{-2}$). These equations thus provide a solid physical basis for understanding the spatiotemporal variability of surface downwelling longwave radiation. This should help to better understand and interpret climatological changes, such as those associated with extreme events and global warming.

34 1 Introduction

35 In the global mean surface energy budget, downward longwave radiation (R_{ld}) is dominant surface energy
 36 input (333 W/m² in global mean and 306 W²/m over land), contributing around twice as much energy as
 37 absorbed solar radiation (161 W/m² in global mean and 184 W²/m over land) (Trenberth et al. 2009, Wild
 38 et al. 2015). This dominance holds over all regions in the climatological mean, although there are some
 39 clear variations in space and time (Figs. 1 and S1). It is central to global warming, reflecting the greenhouse
 40 effect of the atmosphere (Held and Soden 2000), and its variations have been suggested to be the main
 41 contributor to some regional warming amplifications, such as in the Arctic (Lee et al. 2017) and the Tibetan
 42 Plateau (Su et al. 2017). Therefore, it is important to understand the main sources of variations in this
 43 surface energy balance term, which can be seen in Figure 1.

44 The flux of downwelling longwave radiation is influenced by the radiative properties of the entire
 45 atmospheric column, i.e., water vapour, clouds, and greenhouse gases, but also by the heat stored in the
 46 atmosphere, i.e., the temperature at which radiation is emitted back to the surface. To obtain an estimate of
 47 this flux, Brutsaert (1975) used functional expressions for the typical temperature and humidity profiles of
 48 the lower troposphere together with radiative transfer equations and semiempirical relationships of the
 49 absorptivity by water vapor, integrated these vertically, and expressed the resulting flux R_{ld} in terms of near-
 50 surface air temperature and water vapour pressure for clear-sky conditions. He thereby derived a semi-
 51 empirical equation for R_{ld} for an effective clear sky emissivity (ε_{cs}) and the corresponding flux of
 52 downwelling longwave radiation ($R_{ld,cs}$):

$$\varepsilon_{cs} = 1.24(e_a/T_a)^{1/7}, \quad (1)$$

$$R_{ld,cs} = \varepsilon_{cs}\sigma T_a^4. \quad (2)$$

53 where σ is Stefan–Boltzmann constant ($\sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$), e_a is the 2m water vapor pressure (unit:
 54 millibars) and T_a is the 2m air temperature (unit: K). The latter two meteorological variables can easily be
 55 obtained or inferred from weather stations, so that the downwelling flux of longwave radiation can be
 56 estimated from weather station observations. Note that the ε_{cs} shown in equation 1 is largely insensitive to
 57 changes in T_a . As a result, emissivity does not have a direct dependence on T_a , except that higher
 58 temperature may also lead to higher values in e_a .

59 This equation was later extended to all-sky conditions that include the effects of cloud cover, among which
 60 Crawford and Duchon (1999) is a common extension (Alados et al. 2012; Duarte et al. 2006; Flerchinger
 61 et al. 2009). This extension diagnoses cloud cover fraction (f_c) as the fraction of incoming solar radiation
 62 at the surface (R_s) in relation to the potential solar radiation ($R_{s,pot}$), that is, the incoming flux at the top of
 63 the atmosphere. The emissivity for all-sky conditions, ε , is then calculated as the mix of the emissivities of
 64 clear-sky conditions (Eqn. (1), weighted by the cloud-free proportion, $(1 - f_c)$) and clouds with an
 65 emissivity of $\varepsilon_c = 1$ (weighted by the cloud fraction f_c). Using this emissivity, the estimation of
 66 downwelling longwave radiation is then done by

$$f_c = 1 - R_s/R_{s,pot}, \quad (3)$$

$$\varepsilon = f_c + (1 - f_c)\varepsilon_{cs}, \quad (4)$$

$$R_{ld} = \varepsilon\sigma T_a^4. \quad (5)$$

67 Previous studies have already verified Equations 4-5 to have a very good agreement with site measurements
 68 with the r^2 of 0.883 and RMSE of 15.367 W/m² (Duarte et al. 2006; Hatfield et al. 1983), especially when the
 69 temperature is higher than 0°C (Aase and Idso 1978; Satterlund 1979). Other studies have worked to
 70 calibrate and modify this estimate further to different regions (Malek 1997; Sridhar and Elliott 2002).

71 This expression for downwelling longwave radiation R_{ld} given by Eqn. (5) allows us to quantify the different
 72 contributions by cloud cover, f_c , water vapor concentrations, e_a (as a measure of the total water vapor content

73 of the atmospheric column), and air temperature, T_a (as a proxy for the heat storage within the lower
74 atmosphere, Panwar et al. 2022). With this, we can then attribute variations in R_{ld} to their physical causes.

75 Here, our aim is to first evaluate this estimate for downwelling longwave radiation with current global
76 datasets at the continental scale. These variations are illustrated using the NASA-CERES (EBAF 4.1)
77 dataset (Loeb et al., 2018; Kato et al., 2018, NASA/LARC/SD/ASDC 2017) and the NASA-CERES
78 Syn1deg dataset (Doelling et al., 2013, 2016) in Figure 1 and are compared to variations in solar radiation.
79 It can be seen that the climatological distribution of R_{ld} is mostly associated with latitudes, while also
80 presenting some zonal variations, e.g., across western and eastern North America. In comparison, the
81 seasonal cycle of R_{ld} is less determined by latitudes (Fig. 1b). It has a larger magnitude over land than over
82 oceans, over arid regions than humid regions, and over cold regions more than over warm ones. Although
83 studies have revealed a close correlation between the variation of R_{ld} and other factors like air temperature,
84 water vapor, and CO_2 concentration (Wang and Liang 2009; Wei et al. 2021), here we go beyond
85 correlations and rather attribute these variations to the different terms in Eqns. (1)-(5) that represent
86 different radiative properties affecting R_{ld} .

87 To figure out the dominant driver for these spatiotemporal variations, we decompose changes in R_{ld} into its
88 components: cloud cover, f_c , heat storage changes of atmosphere as reflected by 2m air temperature, T_a ,
89 and air humidity, e_a , by performing the differentiation of these equations. We show that heat storage
90 changes predominantly shape the diurnal range and seasonal cycle of R_{ld} , while cloud cover variations play
91 a second role in most cases. In addition, the temporal variations of R_{ld} are less over the ocean than over
92 land, and less during winter than summer. On the other hand, the spatial variations of R_{ld} from arid to humid
93 regions is relatively small, which we will show is due to a compensating effect of corresponding changes
94 in atmospheric emissivity and heat storage.

95 Our paper is organized as follows: After briefly describing the datasets used in our evaluation in Section 2,
96 we first the estimate of R_{ld} from these equations at the global scale, using multiple datasets in Section 3.1.
97 After showing that the annual-mean and large-scale variations are well captured, we then use the equations
98 to decompose the temporal variations of R_{ld} in terms of its mean spatial and temporal variations and relate
99 these to their causes in Section 3.2. The spatial variations of R_{ld} are then further discussed in Section 3.3 in
100 terms of its relationship with aridity. We then close with a brief summary and broader implications.

101 2 Datasets

102 To test R_{ld} estimates, we use FLUXNET half-hour observations (Pastorello et al. 2020, half-hourly values,
103 189 sites, see Table S1 and Figure S2 for details), the NASA-CERES monthly satellite-based radiation
104 dataset (Doelling et al., 2013, 2016, monthly means, covering years 2001 to 2018), and the ERA5 monthly
105 reanalysis dataset (Hersbach et al. 2018, monthly means, covering years 1979 to 2021).

106 For each dataset, T_a , e_a , and f_c are needed as inputs for Eqs. (1)-(5), while R_{ld} data is used for the
107 comparison. Cloud cover f_c is calculated using Eq. (3) for all three datasets with incoming solar radiation
108 at the surface (R_s) and the potential solar radiation ($R_{s,pot}$). For NASA-CERES estimation, T_a from the
109 CPC Global Unified Temperature dataset (CPC Global Unified Temperature) is used as temperature
110 observation.

111 For all three datasets, water vapor pressure, e_a , is not directly given. It is calculated from the water vapor
112 deficit (VPD, FLUXNET) or dewpoint temperature (T_{dew} , ERA5) using Monteith and Unsworth (2008):

$$e_a = 6.1079 \times \exp(17.269T_{dew}/(237.3 + T_{dew})), \quad (6)$$

$$e_a = 6.1079 \times \exp(17.269T_a/(237.3 + T_a)) - VPD, \quad (7)$$

113 And the calculated e_a from ERA5 is also used in NASA-CERES estimation.

114 For the analysis of the spatial variations of R_{ld} along water availability, we use the aridity index ($AI = \frac{R}{LP}$)
 115 (Budyko 1958; UNCOD 1977). This index is calculated using the mean annual net radiation (R) taken from
 116 the NASA-CERES dataset, the mean annual net precipitation (P) taken from the CPC Global Unified
 117 Gauge-Based Analysis of Daily Precipitation data (Chen et al. 2008 and Xie et al. 2007, CPC Global Unified
 118 Gauge-Based Analysis of Daily Precipitation), and a latent heat of vaporization for water of $L =$
 119 2260 kJ/kg. A larger value of AI indicates stronger aridity.

120 **3 Results and discussion**

121 **3.1 Comparison to observed, satellite, and reanalysis data**

122 We first compared the estimates of R_{ld} at a point-by-point basis separately for clear-sky and all-sky
 123 conditions using Eqns. (2) and (5), respectively. This comparison is shown in Figure 2 using FLUXNET,
 124 CERES, and ERA5 data. The estimates correlate very well with r^2 of 0.92 and 0.87 for clear-sky and all-
 125 sky conditions, respectively, and RMSE values of 18.24 and 24.56 W m⁻². The slope of the linear
 126 regressions between the estimated and observed R_{ld} for FLUXNET are 1.03 and 1.02, with most data points
 127 being concentrated around the 1:1 line (Figs. 2a and 2b). Note that for all-sky conditions, the agreement is
 128 slightly less good, with a lower correlation coefficient and a larger RSME. The agreement with the NASA-
 129 CERES and ERA5 datasets are even better, with higher correlation coefficients and lower RSME.

130 Despite this high level of agreement of the estimates, we can see some systematic biases in the estimates
 131 for R_{ld} . These can be seen in Figure 3 and Figure S3, which show the spatial distribution of these biases
 132 and their variations against temperature and humidity. For clear-sky conditions, there appears to be a
 133 general underestimation in the high latitudes and, to some extent, in arid regions (Figs. 3c and 3e). Brutsaert
 134 (1975) already described that for very low temperatures and in arid conditions, there are better parameter
 135 values than those used in Eq. 1, with a larger coefficient than 1.24 and a different exponent. This can then
 136 lead to an underestimation of R_{ld} under low humidity (Figs. 3a, S3a, S3c). Moreover, B75 has not
 137 considered the gradual increase in emissivity as temperature decreases below freezing (Aase and Idso
 138 1978), thus explaining the underestimation under low temperature (Figs. 3b, S3b, S3b). The biases seen in
 139 Figure 3 are nevertheless notably smaller than the spatial-temporal variations shown in Figure 1. This means
 140 that these biases do not prevent us from using Brutsaert to attribute the causes for the seasonal variation
 141 and the spatial range of R_{ld} .

142 The biases for all-sky conditions generally share the distribution with that of clear-sky conditions, with a
 143 smaller magnitude (Figs. 3b, 3d and 3f), which are also small compared to the spatial-temporal variations.

144 Overall, this evaluation shows that the expressions given by Eqns. (1) - (5) are very well suited to describe
 145 the spatiotemporal variations of R_{ld} for current climatological conditions.

146

147 **3.2 Attribution of diurnal and seasonal variations**

148 We next use Eqns. (1) - (5) to attribute temporal variations of R_{ld} to their physical causes. To do so, we can
 149 express changes ΔR_{ld} as a function of changes in water vapor, Δe_a , cloud cover, Δf_c , and air temperature,
 150 ΔT_a . The functional dependence is derived from the equations by differentiation and applying the chain
 151 rule. In a first step, we express a change ΔR_{ld} by the partial contributions $\Delta R_{ld,\varepsilon}$ and $\Delta R_{ld,T}$, that are due to
 152 changes in emissivity, $\Delta \varepsilon$, and due to changes in atmospheric heat storage that are associated with a change
 153 in air temperature ΔT_a :

$$\Delta R_{ld} = \Delta R_{ld,\varepsilon} + \Delta R_{ld,T} = \frac{\partial R_{ld}}{\partial \varepsilon} \Delta \varepsilon + \frac{\partial R_{ld}}{\partial T_a} \Delta T_a = \sigma \bar{T}_a^{-4} \Delta \varepsilon + 4\sigma \bar{\varepsilon} \bar{T}_a^{-3} \Delta T_a. \quad (8)$$

154 The 2 terms at the right side of Eq. 8 are $\Delta R_{ld,\varepsilon}$ and $\Delta R_{ld,T}$, respectively.

155 The contribution $\Delta R_{ld,\varepsilon}$ is further decomposed into contributions $\Delta R_{ld,f_c}$, $\Delta R_{ld,e_a}$, and $\Delta R_{ld,T_a}$ due to
 156 variations in clouds, Δf_c , air humidity, Δe_a , and surface temperature, ΔT_a . We obtain:

$$\begin{aligned}
 \Delta R_{ld,\varepsilon} &= \sigma \bar{T}_a^{-4} \Delta \varepsilon \approx \sigma \bar{T}_a^{-4} \times \frac{\partial \varepsilon}{\partial f_c} \Delta f_c + \sigma \bar{T}_a^{-4} \times \frac{\partial \varepsilon}{\partial e_a} \Delta e_a + \sigma \bar{T}_a^{-4} \times \frac{\partial \varepsilon}{\partial T_a} \Delta T_a \\
 &= \sigma \bar{T}_a^{-4} \times \left(1 - 1.24 \left(\frac{\bar{e}_a}{\bar{T}_a} \right)^{\frac{1}{7}} \right) \Delta f_c + \sigma \bar{T}_a^{-4} \times \frac{1.24}{7} \frac{(1 - \bar{f}_c)}{(\bar{e}_a)^{\frac{6}{7}} (\bar{T}_a)^{\frac{1}{7}}} \Delta e_a \\
 &\quad + \sigma \bar{T}_a^{-4} \times \left(-\frac{1.24}{7} \right) \times \frac{(1 - \bar{f}_c) (\bar{e}_a)^{\frac{1}{7}}}{(\bar{T}_a)^{\frac{8}{7}}} \times \Delta T_a.
 \end{aligned} \tag{9}$$

157 The 3 terms at the right side of Eq. 9 are $\Delta R_{ld,f_c}$, $\Delta R_{ld,e_a}$, and $\Delta R_{ld,T_a}$, respectively.

158 Note that the third term is of less magnitude compared with the other two terms (e.g. in terms of the seasonal
 159 range as shown in Fig. 5f), which is hence not focused in this work.

160 We next applied this approach to the diurnal deviations ΔR_{ld} from the daily mean using the FLUXNET
 161 dataset. This decomposition is shown in Figure 4 in aggregated form across the FLUXNET sites for whole
 162 year (Fig. 4a), the Northern hemisphere summer (Fig. 4b) and winter seasons (Fig. 4c). More than 95% of
 163 the diurnal variations (of about $\pm 20 \text{ W m}^{-2}$) are caused by diurnal changes in air temperature, while
 164 variations in emissivity play practically no role (Fig. S4). Diurnal changes in air temperature reflect
 165 variations in heat storage of the atmospheric boundary layer. This is consistent with the notion that diurnal
 166 variations in solar radiation over land are buffered primarily by the lower atmosphere, rather than below
 167 the surface as it is the case for open water bodies and the ocean (Kleidon and Renner 2017). Since most of
 168 the stations in the FLUXNET dataset are located in the midlatitudes of the Northern hemisphere, the
 169 variations are consistently larger in summer due to the greater solar input (Fig. 4b) than in winter (Fig. 4c).

170 Figure 5 shows the same kind of decomposition, but for seasonal variations in R_{ld} in the NASA-CERES
 171 dataset, which is the difference between the maximum and minimum of monthly R_{ld} data. Generally, areas
 172 with relatively low annual-mean R_{ld} , e.g. the high latitude regions of North America and northeastern
 173 Eurasia, have the largest seasonal cycle (Fig. 1). The decomposition shows that this variation is mostly due
 174 to the seasonal variation in atmospheric heat storage ($\Delta R_{ld,T}$), with a portion of around 73% on a global
 175 scale, and the rest are attributed to the seasonal changes in water vapor (24%) and cloud cover (12%).
 176 Notably, seasonal variations in emissivity play a greater role than atmospheric heat storage in changing R_{ld}
 177 in tropical areas, especially over the monsoon region. This is predominantly due to the strong seasonal
 178 fluctuations in water vapor levels and cloud-cover (Figs. 5d-5f).

179 The aggregation to the global scale across land and ocean is shown in Fig. S5, where the deviations are
 180 calculated as the difference of the monthly means to the annual mean. Figs. S5 show that the seasonal
 181 variations of R_{ld} is generally less over the ocean than on the land, an effect that can also be seen in Fig. 1.
 182 The decomposition shows that these variations are mostly caused by changes in lower atmospheric heat
 183 storage, with a slight modulation by emissivity changes. This can, again, be largely explained by the effect
 184 described above for the diurnal variations (Kleidon and Renner 2017). Over the land, the changes in
 185 radiation are majorly buffered by the heat storage in the lower atmosphere by the variations in convective
 186 boundary layer height. However, over marine areas, solar radiation penetrates the transparent water bodies,
 187 the heat storage of which hence buffers the season cycle of the radiation over the ocean. Since the heat
 188 storage of the water body is larger than that of the lower atmospheric boundary layer, the buffering effect
 189 is consequently larger, which leads to the less seasonal cycle of the surface temperature and R_{ld} over the
 190 ocean.

191 In summary, what our decomposition shows is that most temporal variations in R_{ld} in current, climatological
 192 conditions are explained by heat storage changes within the lower atmosphere.

193 **3.3 Attribution of geographic variations with aridity**

194 Last, we applied the decomposition to the climatological variations in R_{ld} along with differences in mean
 195 water availability. Water availability was characterized by Budyko’s aridity index (AI), with values $AI < 1$
 196 representing humid regions, and larger values reflecting increased aridity. The spatial distribution of AI is
 197 shown in Fig. S6. Here, the deviations ΔR_{ld} are calculated with respect to the annual mean over land. The
 198 different contributions to the deviations are shown in Fig. 6, as well as the delineation along the aridity
 199 index (Figs. 6e - f).

200 The decomposition of the spatial distribution of the climatological means shows that the variations are
 201 largely caused by differences in lower atmospheric heat storage as well (Fig. 6a). The contribution due to
 202 variations in emissivity has a smaller magnitude (Fig. 6b), and is dominated by changes in cloud cover (Fig.
 203 6c) and changes in water vapor (Fig. 6d) at high- and mid- latitudes respectively.

204 These variations are evaluated with respect to the aridity index in Figs. 6e, 6f and S7. While there is a large
 205 spread, as seen in the quantiles, there is a small, but consistent trend towards lower values of R_{ld} in more
 206 arid regions, with a magnitude of about $-10\sim 20 \text{ W m}^{-2}$ across the entire aridity index spectrum (black
 207 dashed line in Figs. 6e and 6f). We also notice a shift in the contributions, with emissivity contributing less
 208 and lower atmospheric heat storage contributing more with increased values of AI. The decreasing
 209 contributions in emissivity of about $-20\sim 40 \text{ W m}^{-2}$ is caused by reductions in cloud cover and water vapor
 210 (Figs. 6f), which can be attributed to the common presence of high-pressure systems in subtropical arid
 211 areas (Zampieri et al. 2009) and less monsoon there. The decreasing contribution by lower atmospheric
 212 emissivity is compensated for by an increased contribution of about $+10\sim 20 \text{ W m}^{-2}$ by atmospheric heat
 213 storage that is caused by the generally warmer mean temperatures in arid regions.

214 **4. Discussion and Conclusions**

215 We found that the semiempirical equations of Brutsaert (1975) and Crawford and Duchon (1999) work very
 216 well to estimate the downwelling flux of longwave radiation by comparing these to estimates from
 217 observation, satellite, and reanalysis datasets, with r^2 ranging from 0.87 to 0.98 across the datasets for clear-
 218 sky and all-sky conditions. We then showed that one can use these equations to decompose this flux into
 219 different components, and relate changes to differences in cloud cover, water vapor, and lower atmospheric
 220 heat storage. We found that most diurnal changes in downwelling longwave radiation are caused by
 221 differences in lower atmospheric heat storage that are reflected in differences in surface air temperature,
 222 with the changes in atmospheric emissivity playing the secondary role. The dominance of surface air
 223 temperature can be also observed in the seasonal ranges of R_{ld} , except in tropical monsoon regions due to
 224 large variations in water vapor and cloud-cover. As for the spatial variation, from arid to humid region, the
 225 increasing lower atmospheric heat storage and decreasing atmospheric emissivity have an offsetting effect
 226 on the R_{ld} variation, thus leading to relatively subtle changes in R_{ld} along with aridity index.

227 Relating our decomposition to radiative kernel helps to gain a more comprehensive understanding of
 228 variations in R_{ld} . Referring to the sensitivity in the downwelling longwave radiation for an incremental
 229 change in an atmospheric property (e.g., T_a , f_c , and e_a), radiative kernel has been used to attribute R_{ld}
 230 changes, based on numerically calculation with radiative transfer code (Previdi 2010 and Vargas Zeppetello
 231 et al. 2019) or partial differentiating with explicit formula for R_{ld} (Shakespeare and Roderick, 2022).
 232 Following Shakespeare and Roderick (2022), the approximate radiative kernel of T_a , f_c , and e_a are calculated

233 based on Eqs. 8-9 (i.e., $\frac{\partial R_{ld}}{\partial T} = 4\sigma\bar{\epsilon}\bar{T}_a^{-3}$, $\frac{\partial R_{ld}}{\partial f_c} = \sigma\bar{T}_a^{-4} \times \left(1 - 1.24 \left(\frac{e_a}{T_a}\right)^{\frac{1}{7}}\right)$, and $\frac{\partial R_{ld}}{\partial e_a} = \sigma\bar{T}_a^{-4} \times \frac{1.24}{7} \frac{(1-f_c)}{(e_a)^{\frac{6}{7}}(T_a)^{\frac{1}{7}}}$)

234 and shown in the left panel of Fig. S8. As shown in Fig S8a, the sensitivity of R_{ld} to T_a peaks in the tropics
 235 with a maximum of around $5 \text{ W/m}^2/\text{K}$ and decreases at higher latitudes, which is generally consistent with
 236 Shakespeare & Roderick (2022). Moreover, the seasonal cycle of the atmospheric properties themselves
 237 are shown in the right panel of Figure S8, which reveals that the spatial distribution of the contribution of

238 T_a , e_a , and f_c to the seasonal variations in R_{ld} (Figure 5) is dominated by the seasonal changes of the air
239 properties (Figs. S8b, S8d, and S8f) instead of the sensitivity of R_{ld} to them (Figs. S8a, S8c, and S8e).

240 These equations can then be applied to different aspects of climate research. For instance, the values of
241 downwelling longwave radiation are often missing in FLUXNET data (Table S2), and these equations can
242 be used to fill the gaps with air temperature and humidity observations. We can also use these equations to
243 better understand the physical mechanisms for temperature change due to extreme events. For instance,
244 Park et al. (2015) and Alekseev et al. (2019) found that an enhancement of downwelling longwave radiation
245 in the Arctic is found to be preceded by the advection of moisture and heat. The equations by Brutsaert
246 (1975) and Crawford and Duchon (1999) can then be used to quantify the individual contributions by the
247 advection of heat and moisture (Tian et al. 2022). Another example is the attribution of differences in
248 temperature magnitudes across humid and arid regions (Ghausi et al., 2023). Du et al. (2020) used these
249 equations to explain why global warming was stronger during clear-sky conditions in observations in China
250 due to the greater sensitivity of clear-sky emissivity to a change in water vapor. This was then used to
251 explain the observed, stronger global warming in the arid regions of China, which have less clouds and a
252 higher frequency of clear-sky conditions than the humid regions. Furthermore, while the empirical
253 coefficient of 1.24 in Eq. (1) may change due to emissivity changes from greenhouse gases, this formulation
254 can nevertheless provide a useful basis in terms of the interannual changes of R_{ld} , which is shown in Fig.
255 S9. As shown in Fig. S9a, R_{ld} increases in most of the land regions, at an average rate of $0.64 \text{ W/m}^2/\text{decade}$,
256 with the contribution of increased temperature, increased water vapor, and decreased cloud cover
257 contributing 0.46 , 0.28 , $-0.10 \text{ W/m}^2/\text{decade}$, respectively. Furthermore, it can be observed in Figs. S9d-S9i
258 that the temperature effect is generally around $0.5 \text{ W/m}^2/\text{decade}$, while the influence of emissivity is
259 significantly dominant in the monsoon region, which is majorly due to the interannual changes in water
260 vapor.

261 It is worth noting that several effects on R_{ld} variations are not included in B75 and C&D99, e.g., the well-
262 mixed greenhouse gas concentrations (Shakespeare and Roderick, 2022), large aerosol particles (Zhou and
263 Savijärvi, 2013), and cloud base (Viúdez-Mora et al. 2015). Although rarely influencing the diurnal change,
264 seasonal cycles, and spatial distribution, these terms need attention when the interannual trend of R_{ld} is
265 investigated under global warming, which can be implied by the difference between Figs. S9a and S9b. In
266 addition, B75 in conjunction with C&D99 is adopted in this work to decompose the R_{ld} variations in
267 different spatial-temporal scales, considering its solid physical foundations and the relatively less
268 computation consumption. Further analysis can be performed based on other estimations, e.g. Prata 1996,
269 which shows consistency with reanalysis data (Allan et al. 2004). The cloud effect can be also detected
270 using the difference between all-sky and clear-sky R_{ld} (Allan 2011; Ghausi et al., 2022). Moreover, datasets
271 that are more focused on radiation and energy budget can be used to test the robustness of the results, e.g.,
272 BSRN (Driemel et al. 2018) and GEBA (Wild et al. 2017).

273 We conclude that the equations by Brutsaert (1975) and Crawford and Duchon (1999) are still very useful
274 in advancing our understanding of surface temperature changes. Our evaluation has shown how well these
275 equations estimate this flux, and our application to the decomposition of different contributions has shown
276 its utility in understanding the causes of its variation. These equations should help us to better understand
277 aspects of climate variability, extreme events, and global warming, linking these to the mechanistic
278 contributions by downwelling longwave radiation.

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285 **Author contributions**

286 YLT, SAG, and AK conceived and designed the analysis, with inputs from DZ and GW. YLT performed
287 the analysis and discussed the results with all authors. YLT and AK wrote the paper.

288 **Competing interests**

289 The contact author has declared that none of the authors has any competing interests.

290 **Data availability**

291 The data used in this study was downloaded from the links provided with the references. No new data was
292 created.

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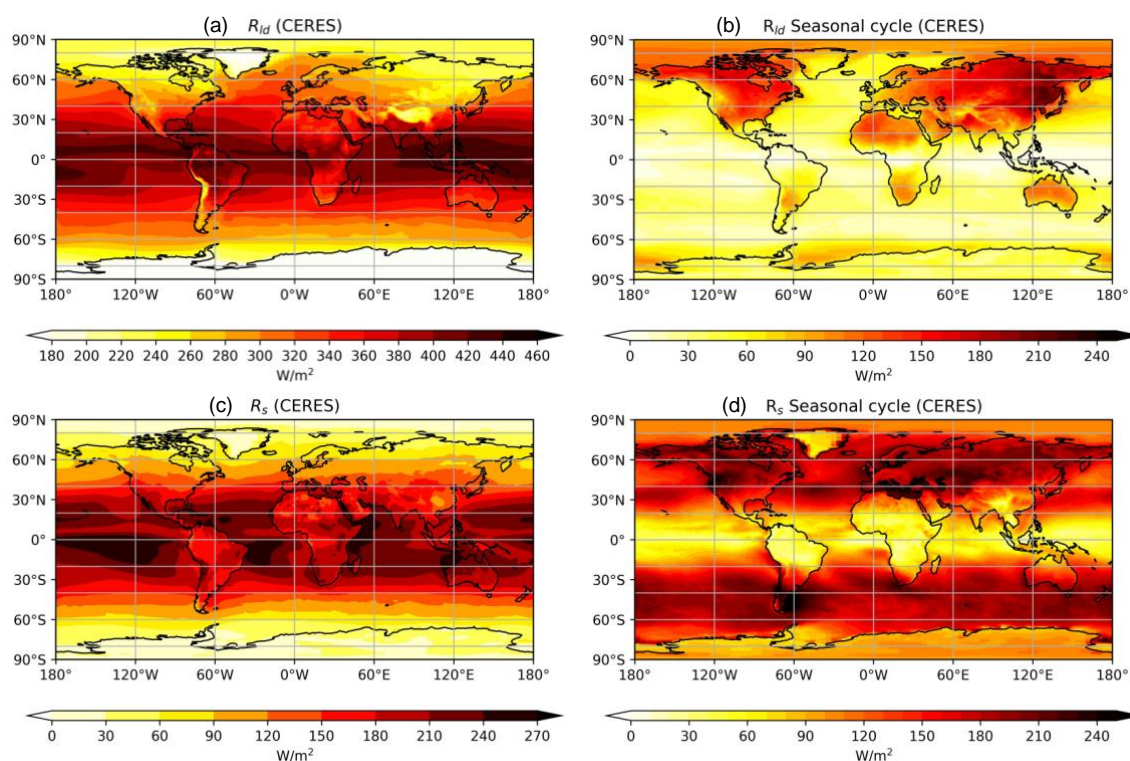
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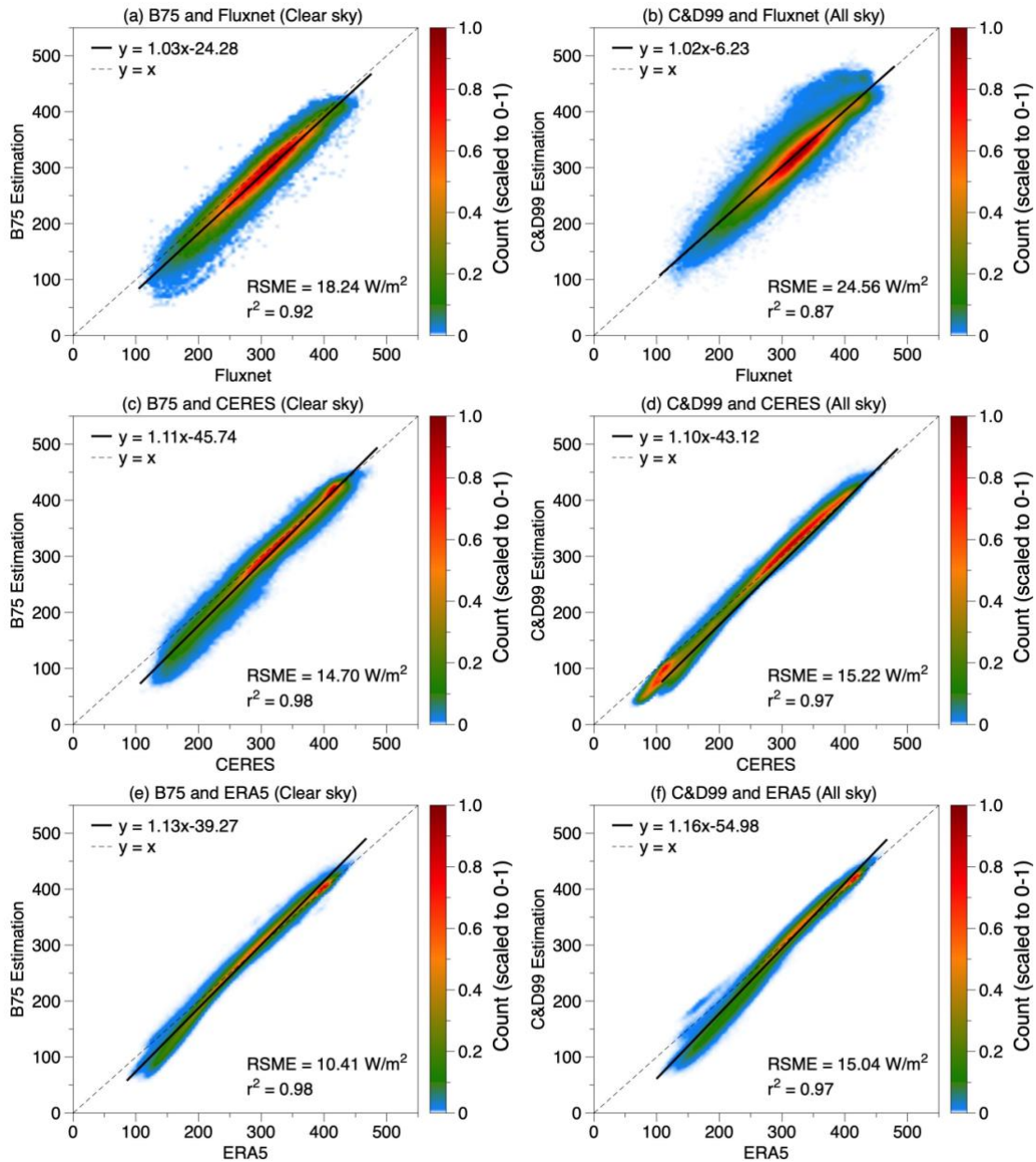
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460 **Figures**

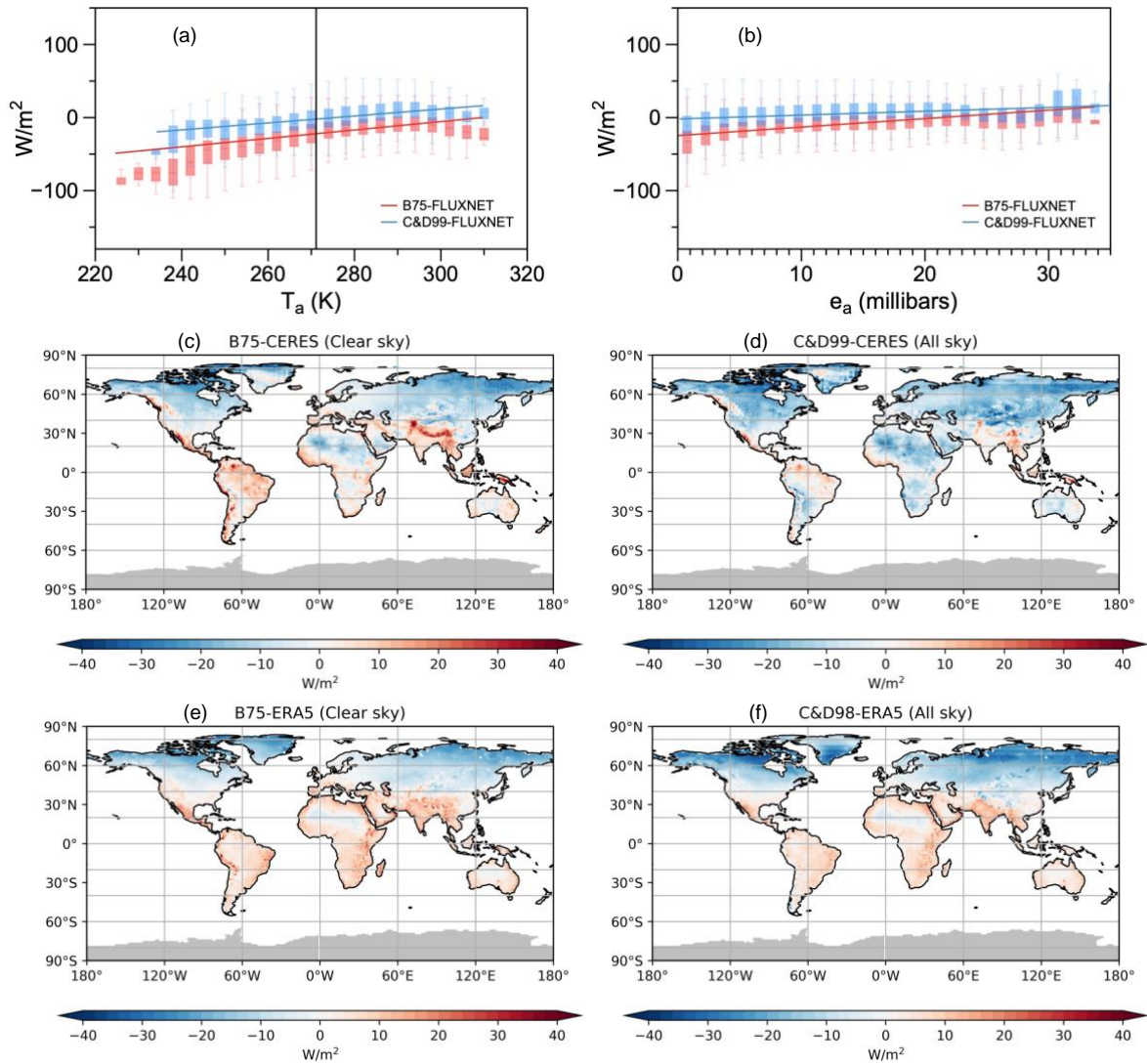


461
 462 **Figure 1.** Spatial distribution of (a, c) the climatological mean and (b, d) the seasonal amplitude of
 463 downward longwave radiation and absorbed solar radiation at the surface respectively from the NASA-
 464 CERES dataset. The seasonal amplitude is calculated as the difference between the maximum and minimum
 465 monthly data.



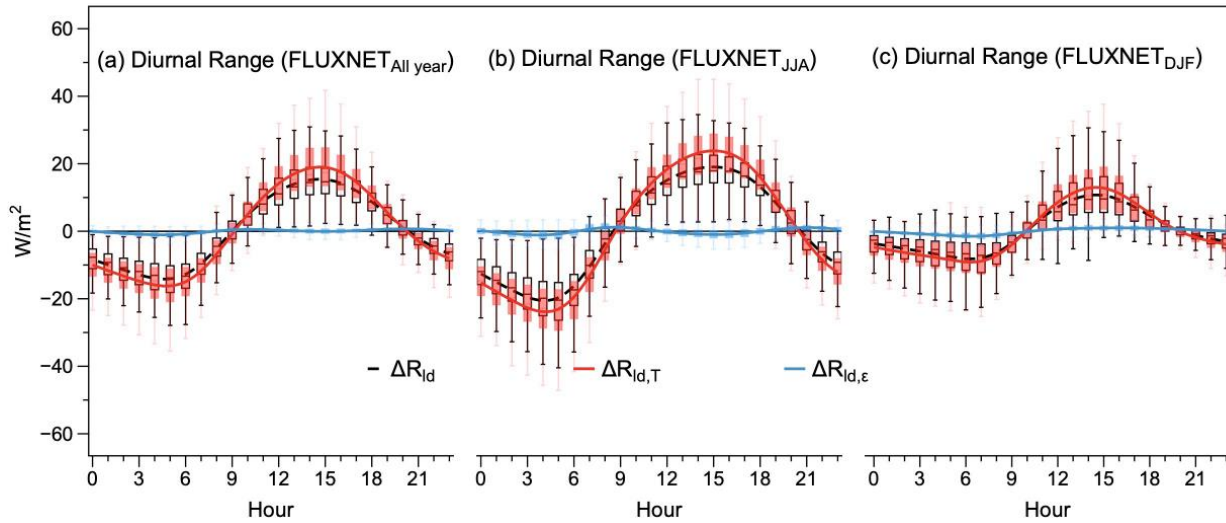
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467 Figure 2. Comparison of Rld estimated by Brutsaert (1975) (a, c, e) for clear-sky conditions and by
 468 Crawford and Duchon (1999) (b, d, f) for all-sky conditions using (a, b) FLUXNET hourly data of 189
 469 sites, (c, d) NASA-CERES monthly data of 1°×1° from 2001 to 2018 and (e, f) ERA5 monthly data of
 470 resolution of 1°×1° from 1979 to 2021. Colors indicate the density of the data points and is scaled to values
 471 between 0 - 1.

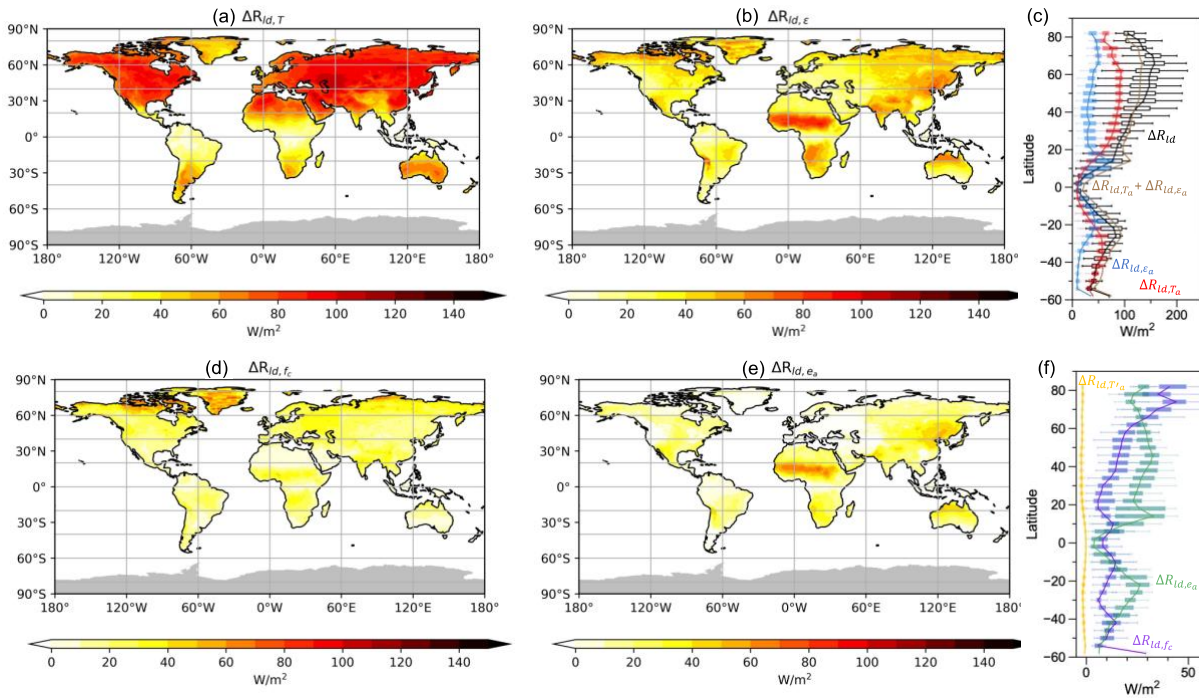


472

473 **Figure 3.** Biases in the estimates for multi-year mean R_{ld} for FLUXNET data of 189 sites against (a) air
 474 temperature and (b) water vapor pressure. Distribution of biases in the estimates for multi-year mean R_{ld}
 475 for (c, d) NASA-CERES data from 2001 to 2018 and (e, f) ERA reanalysis from 1979 to 2021 for (c, e)
 476 clear-sky and (d, f) all-sky conditions over land. Grey shading indicates missing values.
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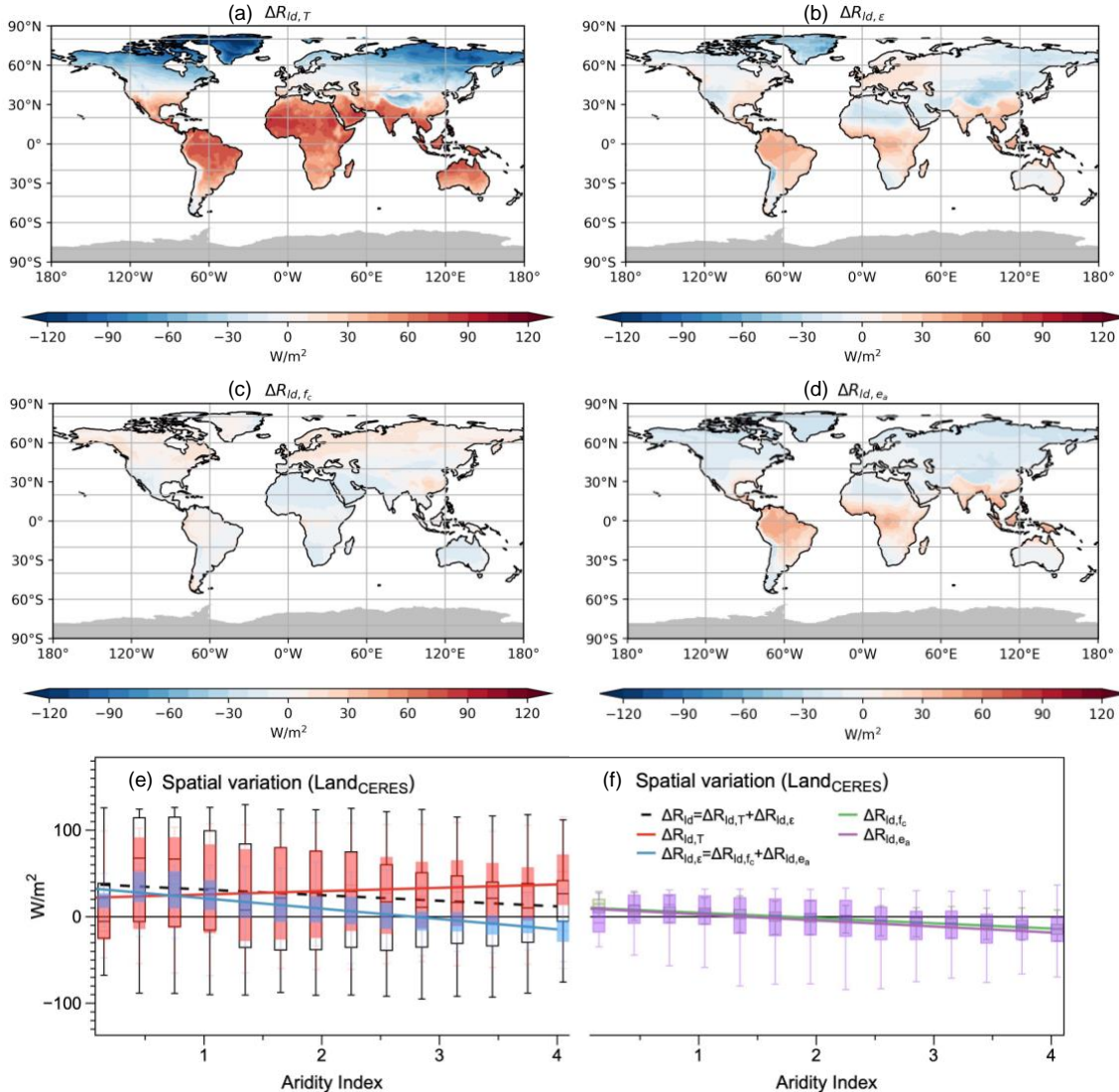


478
 479 **Figure 4.** The multi-year average diurnal variations in R_{ld} (black dashed line) and its decomposition into
 480 contributions by changes in emissivity (blue, $\Delta R_{ld,\epsilon}$) and lower atmospheric heat storage (red, $\Delta R_{ld,T}$) in
 481 the FLUXNET dataset aggregated over 189 sites for (a) the whole year, (b) June-August, and (c) December
 482 – February. The box shows the variation among the 189 sites. The upper and lower whiskers indicate 95th
 483 and 5th percentiles, upper boundary, median line, and lower boundary of the box indicate the 75th, 50th, and
 484 25th quantiles, respectively. For each site and each day, the daily mean value is removed, with the deviations
 485 shown. Regression lines are based on site-mean or grid-mean value using LOESS regression.
 486



487
 488 **Figure 5.** Decompositions of the mean seasonal variation (Δ , difference between the maximum and
 489 minimum monthly data at each grid) of R_{ld} in the NASA-CERES dataset into contributions by (a) lower
 490 atmospheric heat storage ($\Delta R_{ld,T}$) and (b) emissivity ($\Delta R_{ld,\epsilon}$), and (c) their latitudinal variations.
 491 Decomposed of $\Delta R_{ld,\epsilon}$ into contributions by variations in (d) cloud cover ($\Delta R_{ld,fc}$) and (e) humidity

492 $(\Delta R_{ld,e_a})$, (f) their latitudinal variations. In Figs. a, b, d, e, grey shading indicates missing values. In Figs. c
 493 and f, the box shows the variation among the land grids at the same latitude, while the solid line is their
 494 mean. The upper and lower whisker indicate 95th and 5th percentiles, upper boundary, median line, and
 495 lower boundary of the box indicate the 75th, 50th, 25th quantiles, respectively.
 496



497 **Figure 6.** Decompositions of the multiyear-mean spatial variation of R_{ld} (deviations of the multiyear-mean
 498 value for each grid from the land-mean value) in the NASA-CERES dataset into contributions by (a) lower
 499 atmospheric heat storage ($\Delta R_{ld,T}$) and (b) emissivity ($\Delta R_{ld,\epsilon}$). Decomposition of $\Delta R_{ld,\epsilon}$ into contributions
 500 by (c) variations in cloud cover ($\Delta R_{ld,f_c}$) and (d) humidity ($\Delta R_{ld,e_a}$). In Figs. a-d, grey shading indicates
 501 missing values. In Figs. e and f, the box shows the variation among the land grids with the same aridity.
 502 The upper and lower whisker indicate 95th and 5th percentiles, upper boundary, median line, and lower
 503 boundary of the box indicate the 75th, 50th, 25th quantiles, respectively.
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