The authors provide a general framework for stochastic perturbations of PDEs dependent on a chosen association of the components of the PDE with a differential form, and some further parameters. They show that with the correct choice, one may deduce the SALT and LU perturbations out of this framework which makes this an interesting result worthy of publication.

I would suggest a minor improvement on the presentation is in order before publication. In particular, I believe that a compact summary of the procedure making explicit which choices are to be made by the user, what guides those choices, and the determined steps from these choices, should be included in the introduction. This is partly done in Lines 77 to 80, but it should be made clear in this bullet point summary that (at least as I understand) θ is non-uniquely chosen, T is non-uniquely chosen, and the rest of the procedure is determined from these choices. A comment on how these choices can be made, as in what needs to be followed in making these choices, and what guides the choices, should be included in this summary. Moreover there should be a similar discussion in the introduction as to how the SALT and LU schemes are recovered in this setting; is there a natural way to choose θ and T based on the principles of these schemes, which genuinely gives an alternative derivation of the SALT and LU SPDEs, or are you just equating coefficients knowing what outcome you want to achieve and selecting them accordingly? This difference is significant in the purpose of the paper.

Some specific points are given below:

- 1. In line 2/3, is 'asserting that the difference...' the content of Brenier's Theorem, or is this something which you are implementing in light of Brenier's Theorem? If the former then you should remove the comma at the end of line 2, if the latter then you should give a brief comment before that comma on what Brenier's Theorem is.
- 2. Similarly in line 4 I assume that 'perturbations are demonstrated...' is what you achieve in the paper, though this should be made more clear what as it could be understood to be part of Brenier's work. Thus you should write 'we demonstrate that perturbations...'.
- 3. Line 4 should read 'an SPDE'.
- 4. Line 5/6, what is the reference to 'Resseguier et al' here? If this is another setting like LU and SALT then 'both' should not be used as it applies to all three. If not, then what is this relevance of this paper here?
- 5. Of course in Line 9 the copyright statement needs to be presented.
- 6. Line 12 should read 'academic studies'
- 7. Line 13 'estimate of a state variable'
- 8. Line 20 a reference should be given for the 'curse of dimensionality'.
- 9. Line 26 'addition of noise'
- 10. Line 28 'we refer'
- 11. Line 36 'multiplicative noise'
- 12. Line 43/44, 'slowly-varying' and 'fast-varying' should be consistent e.g. replace 'slowly' with 'slow' or 'fast' with 'quickly'.

- 13. Line 46 'homogenization methods... may also lead to violation of energy conservation', can you comment on why this is the case, for example what the mentioned workarounds are protecting against?
- 14. Line 52 'another'
- 15. Line 88 'In summary' and 'provides with the perspective that' should be reworded.
- 16. Line 95 'we reserve this for future study'.
- 17. Line 101 'A final conclusion and discussion is given...'
- 18. Line 105 'if p is shown in both upper and lower indices'
- 19. Line 147 'an SPDE'
- 20. Line 162 'a detailed definition'
- 21. Line 162 'a Taylor Expansion and Itô's Lemma can be used'
- 22. Line 168 'a Taylor Expansion'
- 23. Line 195 What is this \mathcal{F} ? What is the space of f and space of θ ? There needs to be more precision about what is happening here, and how this \mathcal{F} is known to exist or how it is chosen/can be chosen.
- 24. Line 203 should read 'As the physical PDE (20) is deterministic'
- 25. Lines 231, 233 should end in full stops, and this implies in general for equations which are followed by a new sentence (starting with a capital letter).
- 26. Line 256 'Again an advection...'. Also 'for an n-form'
- 27. Line 259 'provides an'
- 28. Line 271 'is remeniscent of'
- 29. Line 273 'from the cross...'. Also $It\bar{o}$ should be replaced by Itô.
- 30. Line 284 needs to be reworded
- 31. Line 347 I now understand the reference to Resseguier to be as a secondary reference for Location Uncertainty, though this should be made clear.
- 32. Line 356 and throughout should say 'Itô'.
- 33. Line 388 'already outlined'
- 34. Line 393, 'terms, the LU equation'
- 35. Line 428 'used as an advection velocity in the SALT'
- 36. Line 533 'needs to be'