Author Response to Reviews of Earth Surface Dynamics Manuscript egusphere-2023-320

Statistical characterization of erosion and sediment transport mechanics in shallow tidal environments. Part 2: suspended sediment dynamics

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Summary

The authors wish to thank the Editorial Board and the Reviewers for their overall constructive and insightful comments on our paper, which significantly improved the manuscript and its readability.
We carefully revised the introduction following the Reviewers' suggestions in order to better highlight how the proposed approach contributes to filling the knowledge gap in long-term morphodynamic modelling and to better emphasize its complementarity with the companion paper on erosion dynamics. Moreover, we have significantly expanded the Method section, as suggested by the Reviewers. This expansion includes the description of equations implemented in the numerical sediment transport model, as well as an extended discussion on the choice of the threshold value to apply the peak-over-threshold analysis to suspended sediment concentration time series.
Finally, we provided additional details about some modelling choices that were not properly justified in the previous version of the manuscript, such as the selection of the boundary conditions and the initial bed sediment composition. Reviewers' suggestions on the companion paper that could have been applied also to this manuscript have been implemented, such as details on the study area, wind climate, choice of synthetic descriptors and model performance.
Overall, in the new version of the manuscript, we consistently revised the main text and importantly expanded the Supplementary Information, by adding the detailed model description and figures S2 to S6.
In the following, we discuss in detail all Reviewers' comments and show how we have addressed them in the revised manuscript, referencing line numbers in the revised manuscript with the track changes.
Please note that the Reviewers' comments are in blue, our detailed responses are in black, and the text of the revised manuscript is framed.

Legend

RC: Reviewer Comment

AR: Author Response

☐ : Modified manuscript text

Note: References to reviewers' comments are indicated as RCx.x and numbered progressively.

Reply to Reviewer #1

RC1.1: This is an interesting paper combining a modeling approach and a statistical analysis of suspended sediment dynamics in the Venice Lagoon. The paper is well written, but the findings bring very little new insight, in comparison with its companion paper Part I on erosion dynamics. Although this is probably more of an editorial issue, I am questioning the relevance of making two papers out of this study. Indeed, both papers have basically the same structure, with very similar introduction and method sections. In addition, the results of both studies are highly correlated (see lines 300-305), which is not surprising as SSC dynamics (Part II) is itself highly correlated with erosion dynamics (Part I). To further support this, the authors keep referring to the companion paper on erosion dynamics to discuss their results on suspended sediment dynamics (section 3). … In conclusion, I don't deny the interest of this study, but I suggest to merge both companion papers into one.

AR: We thank the Reviewer for the overall positive and constructive comment on our manuscript. We must say that, while preparing the original version, we carefully examined the options of submitting one single manuscript or two companion papers. Let us better explain and justify the reasoning that led us to decide that the best option was represented by two companion papers.

As highlighted by the Reviewer, the objective of these two papers is to test the hypothesis and establish a theoretical framework for upscaling the effects of stochastic processes in the long-term morphodynamic modelling of shallow tidal environments. To this aim, both erosive and sediment transport dynamics obviously need to be taken into account and we deem that applying the same methodology to these two physical variables (namely bottom shear stress-BSS and suspended sediment concentration-SSC) makes undoubtedly the approach simpler, easier to be understood and more reasonable and justifiable. For this reason, the structure of these two manuscripts was intentionally kept similar.

Although the structure is similar and BSS and SSC are physically intertwined, the results are complementary and do not overlap. Indeed, we highlighted many differences between BSS and SSC dynamics, which surely deserve to be explained in detail. Following the Reviewer's suggestions, in the revised version, we further differentiate the two papers by rewriting the introductions to better highlight the complementarity of the two works (see our response to RC1.3 and RC2.1) and the Method sections to provide more details on the equations and the specific modelling choices (see our responses to RC1.2, 1.5, 1.6, and from RC2.2 to 2.6).

The choice of submitting two companion papers is also driven by the very practical reason of the manuscript length. A clear explanation of many details, which are necessary to understand the methodology we adopted, and a proper presentation of the analysis we performed to test the possibility to model both BSS and SSC dynamics as Poisson processes, require a quite long description and many visual elements (i.e. figures and tables). This is even clearer in the revised version of the two manuscripts, after the improvements suggested by the Reviewers. In total, the two revised manuscripts have 20 visual elements in the main text (10 for the BSS and 10 for the SSC) and 31 visual elements in the supplementary information (17 for the BSS and 14 for the SSC). Honestly, we deem that packing all this material into one single paper would jeopardize the readability of the manuscript because of the length and the need to continuously check the Supplementary information file, where too many visuals would necessarily be moved.

To conclude, we deem that the option of two companion papers offers the chance to clearly communicate our findings (compared to that of one single paper) and, at the same time, to highlight the strong link between our analysis of BSS and SSC dynamics (that may be missed with two separate papers in different journals). For all these reasons, we deem that merging these two papers into one would not be the optimal solution.

RC1.2: In the companion paper, the choice of a peak over threshold analysis is very natural, as erosion processes are physically triggered when the bed shear stress exceeds a threshold value. Here, the choice of such an analysis is less obvious, and determining an SSC threshold is highly arbitrary. Although the authors justify quite elegantly their choice of threshold value (line 212), they should at least discuss the sensitivity of their results and conclusions to this threshold value.

AR: We thank the Reviewer for his/her suggestion. We agree that the choice of the threshold value C_0 needs to be explained more in detail. To this aim, we have completely rewritten the subsection where we introduce the Peak-Over-Threshold analysis as follows:

(line 324) ~~Sediment transport dynamics in tidal environments are the results of the complex interplay among hydrodynamic, biologic, and geomorphologic processes. This interplay between different factors can be fully framed only by taking into account both its deterministic and stochastic components. As an example, Carniello et al. (2011) argued that morphological dynamics in the Venice Lagoon are mostly linked to a few severe resuspension events induced by wind waves, whose dynamics are markedly stochastic in the present configuration (D'Alpaos et al., 2013; Carniello et al., 2016). Measurements confirm that high SSC events are also important sediment suppliers for salt marshes (Tognin et al., 2021)~~.

~~In the present work, we used the peak-over-threshold theory (POT) (Balkema and de Haan, 1974) to analyze temporal and spatial dynamics of the total SSC at any location within each selected configuration of the Venice lagoon.~~

~~First, a minimum-intensity threshold, C_0, was chosen to identify the set of over-threshold events from the modelled SSC record, and then a statistical analysis of interarrival times, durations and intensities of the exceedances of the threshold was carried out.~~

The interplay among the different drivers that control suspended sediment dynamics in shallow tidal environments can be fully framed only by taking into account also its stochastic components, associated with wind waves and storm surges, which are largely responsible for the morphodynamic evolution of these systems (Carniello et al., 2011; Tognin et al., 2021).

To this aim, in the present work, we statistically characterize the spatial and temporal dynamics of resuspension events by applying the peak-over-threshold theory (POT) (Balkema and de Haan, 1974) to the one-year-long time series of SSC computed with the numerical model described above for the different configurations of the Venice Lagoon.

Before applying the POT analysis, the SSC time series provided by the numerical simulations were low-pass filtered by applying a moving average procedure with a time window of 6 hours, in order to preserve the tide-induced modulation of the signal but, at the same time, to remove artificial upcrossing and downcrossing of the threshold, generated by short-term fluctuations. This pre-processing procedure prevents the identification of a false dependence of subsequent over-threshold events due to spurious fluctuations.

Once a proper threshold, C_0, is chosen, the POT identifies three different random variables: interarrival times, durations and intensities of the exceedances of the threshold. The interarrival time is defined as the time interval between two consecutive upcrossings of the threshold, the duration of the events is the time elapsed between any upcrossing and the subsequent downcrossing of the threshold, and, finally, the intensity is calculated as the largest exceedance of the threshold in the time-lapse between an upcrossing and the subsequent downcrossing. These random variables are characterized by their probability density functions and the corresponding moments for any location in all the considered configurations of the Venice Lagoon, in order to provide a complete description of the SSC pattern. *Besides synthetically characterising over-threshold events, these three variables can be combined to compute more complex metrics to describe SSC dynamics (e.g. the volume of sediment reworked in a selected time frame).*

The nature of the stochastic processes can be determined by the analysis of the interarrival times distribution. Indeed, resuspension events can be mathematically modelled as a Poisson process if the interarrival times between subsequent exceedances of the threshold, C_0, are independent and exponentially distributed random variables *(Cramér and Leadbetter, 1967; Gallager, 2013).*

Moreover, the memorylessness of the Poisson process guarantees that the number of events observed in disjoint subperiods is an independent, Poisson-distributed random variable (Gallager, 2013). When the sequence of random events that define a 1-D Poisson process along the time axis can be associated with a vector of random marks that defines the duration and intensity of each over-threshold event, the process can be defined as a marked Poisson process. The distribution of these marks does not affect the chance to model the process as Poissonian, which, indeed, relies only on the exponentiality of interarrival times. However, when also duration and intensity are exponentially distributed, the set-up of a stochastic framework can be further simplified. In order to assess that over-threshold SSC events can be modelled as a marked Poisson process, we performed the Kolmogorov-Smirnov (KS) goodness of fit test on the distribution of the interarrival times, intensity and duration of over-threshold events.

In the POT analysis, the threshold value plays a critical role and its choice deserves careful attention. In the case of erosion dynamics (D'Alpaos et al., 2023), the identification of the threshold with the critical shear stress for erosion seems to be quite straightforward and has the advantage of preserving also the physical meaning of the process. Instead, when dealing with SSC, the absence of a clear physical threshold mechanism may make the identification of the threshold value less direct. The present analysis aims to characterize the bulk effect of morphologically meaningful SSC events, rather than to describe only the extreme events, and, simultaneously, to remove the weak resuspension events induced by periodic tidal currents that can be described as a recurrent, deterministic process. From this point of view, the choice of a threshold value, C_0, *that identifies morphologically significant over-threshold SSC events, has to consider two opposite requirements. On the one hand, stochastic sediment concentration generated by storm-induced wind waves can be distinguished from tide-modulated daily concentration only if* C_0 *is large enough. On the other hand, too high values of* C_0 *either require a long, computationally prohibitive simulated time series or can lead to a non-informative analysis because of the large number of events unaccounted for. These observations narrow the range in which the threshold can be selected. The lower boundary is set by the SSC observed in the absence of wind and, therefore, associated exclusively with the tide. While the upper boundary has to be maintained well below the maximum observed values to consider all the morphologically meaningful events. In the specific case of the Venice Lagoon, to satisfy these requirements, the* C_0 *value has to fall between 30 and 60 mg* l^{-1}, *as suggested by in-situ SSC measurements (Carniello et al., 2012, 2014).*

The sensitivity analysis performed on the present-day configuration of the Venice Lagoon (Carniello et al., 2016) suggests that the chance to model SSC events as a Poisson process is weakly affected by the specific threshold value in the above range. Indeed, using threshold values equal to 30, 40, 50 or 60 mg l^{-1} *hardly* *changes the areas where interarrival times are not exponentially distributed and, therefore, wind-induced SSC cannot be described as a Poisson process (Figure S6). On the basis of these observations and to allow the comparison among the different configurations, in the present analysis, we used a constant threshold, C_0, equal to 40 mg l^{-1}.*

~~According to the extreme value theory, a Poisson process emerges from a stochastic signal whenever enough high censoring threshold is chosen (Cramér and Leadbetter, 1967). However, as this present analysis is designed to remove only the weak resuspension events induced by periodic tidal currents, the critical threshold is well below the maximum observed values. As a consequence, the aim of the proposed analysis is to characterize the bulk effect of morphologically meaningful SSC events, rather than to describe the extreme events. Notwithstanding the increasing popularity of Poisson processes for the analytical modelling of the long-term evolution of geophysical processes controlled by stochastic drivers in hydrological and geomorphological sciences (e.g., Rodriguez-Iturbe et al., 1987; D'Odorico and Fagherazzi, 2003; Botter et al., 2013; Park et al., 2014; Bertassello et al., 2018), only in the last few years this approach has been adopted for tidal systems (D'Alpaos et al., 2013; Carniello et al., 2016) and the applications portray an encouraging framework.~~

For the Reviewer's convenience, we report here Figure S6 added to the Supplementary information showing the results of the KS test using different C_0 thresholds.

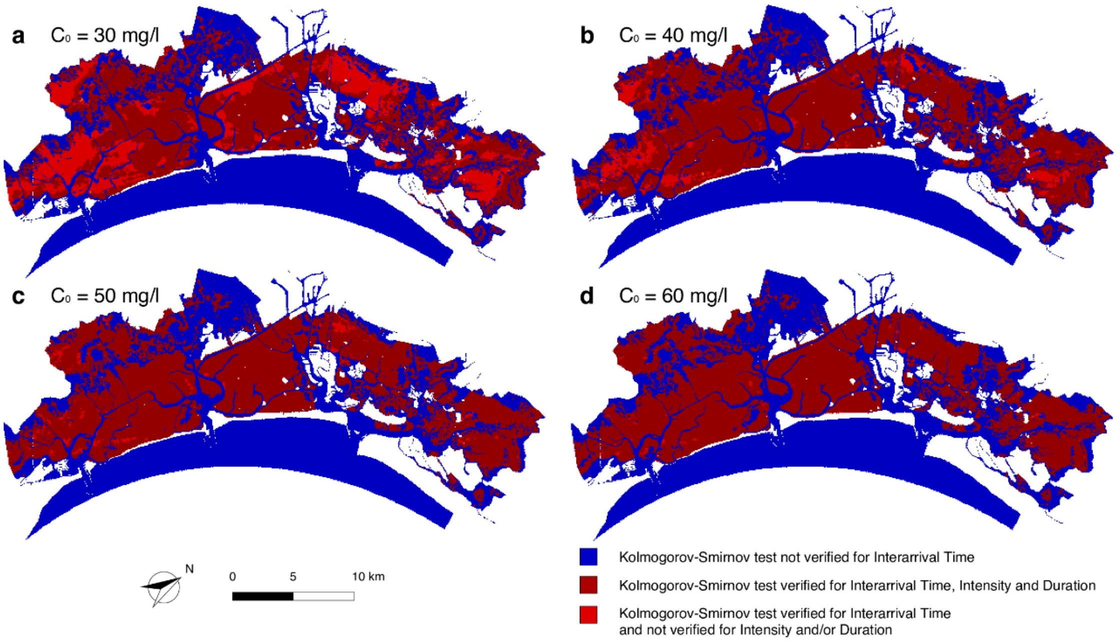

Figure S6. Sensitivity analysis of the threshold C_0. Spatial distribution of Kolmogorov-Smirnov (KS) test at significance level ($\alpha = 0.05$) for different values of the threshold, C_0: (a) 30 mg/l; (b) 40 mg/l; (c) 50 mg/l; (d) 60 mg/l. In the maps we can distinguish areas where the KS test is: not verified (dark blue); verified for all the considered stochastic variables (interarrival time, intensity over the threshold and duration) (dark red); verified for the interarrival time and not for intensity and/or duration (light red). Maps show little to no influence of the threshold value within the selected range on the possibility to model over-threshold SSC events as a Poisson process.

RC1.3: Something that intrigue me is hidden in lines 310-311. I am wondering if the results of this paper can be combined with the results of the companion paper to better constrained the erosion coefficient "e" (equation 3, companion paper). The value of this parameter is highly uncertain, given the values encountered in the literature range over more than one order of magnitude. If that is possible, that would be a very interesting result of this study.

AR: We appreciate the Reviewer's insightful observation on this point because the estimation of the erosion coefficient "e" is ideally one of the first validation steps that can be done by applying the statistically-based model we aim to develop, once the possibility to describe erosion and resuspension events as Poisson processes has been verified.

As we have now better clarified in the companion paper on BSS dynamics (see line 412 of the revised manuscript with the track changes), the calibration of the parameter "e" could not be performed solely on the basis of erosion dynamics because the erosion work represents the total potential erosion and thus completely disregards the possible settling of sediment carried in suspension, once the hydrodynamic conditions are favourable to deposition. The statistical characterization of SSC dynamics we derived in this paper aims to complete the framework proposed to describe BSS and, thus, to properly model the net bed evolution (i.e. calibrate also the parameter "e").

However, to correctly perform the analysis suggested by the Reviewer, a further step is still required: the set-up of the stochastic model to consider both erosion and resuspension dynamics. This may seem a very trivial and straightforward point, but, instead, it requires a careful and detailed explanation and validation, in which the point suggested by the Reviewer surely play a fundamental role. Even when the stochastic model will be available, directly comparing subsequent morphological configurations of the Venice Lagoon to "calibrate" the erosion coefficient "e" will be questionable, because the morphological evolution of the lagoon over the last century was deeply affected by human interventions (see our response to RC1.4 and RC2.7). Very likely the comparison between the result of the model and one of the lagoon configurations will highlight the effects of the anthropogenic interventions (i.e., excavation of large navigable channels, dredging, etc), which are not described by the statistically-based model.

In conclusion, we deem that the presentation of this model and related detailed analysis are beyond the aim of this study and cannot fit into one single paper, but we better highlighted the role of the SSC dynamics and its complementarity with erosion dynamics in the overall picture of the stochastic model we aim to develop, by modifying the text in several points as follows:

> (line 1) A proper understanding of *sediment resuspension and transport processes*~~dynamics, critically including resuspension and deposition processes of suspended sediments,~~ is key to the morphodynamics of shallow tidal environments. *However, a complete spatial and temporal coverage of suspended sediment concentration (SSC) to describe these processes is hardly available, preventing the effective representation of depositional dynamics in long-term modelling approaches.* ~~Aiming to account for deposition mechanics in a synthetic theoretical framework introduced to model erosion dynamics (D'Alpaos et al., 2023), here we investigate suspended sediment dynamics.~~ *Aiming to couple erosion and deposition dynamics in a unique synthetic theoretical framework, here we investigate SSC dynamics following a similar approach to that adopted for erosion (D'Alpaos et al., 2023)*.
>
> (line 23) *Although erosion and resuspension are intimately intertwined, erosion alone does not suffice to describe also SSC because of the non-local dynamics due*

to advection and dispersion processes. The statistical characterization of SSC events completes the framework introduced for erosion mechanics and together they represent a promising tool to generate synthetic, yet realistic, time series of shear stress and SSC for the long-term modelling of tidal environments.

(line 85) *To explicitly model the effects of stochastic, morphologically-meaningful events as well as their temporal succession, a possible alternative would be to directly consider the physical processes responsible for the morphological evolution (i.e. erosion, transport and deposition of sediment) instead of upscaling the bed level changes. From this perspective, a synthetic, statically-based model represents a particularly promising framework to reduce the computation burden associated with the explicit description of these processes through the use of independent Monte Carlo realizations. Notwithstanding the increasing popularity of statistically-based approaches for the long-term modelling in hydrological and geomorphological sciences (e.g., Rodriguez-Iturbe et al., 1987; D'Odorico and Fagherazzi, 2003; Botter et al., 2013; Park et al., 2014), applications to tidal systems are still quite unusual (D'Alpaos et al., 2013; Carniello et al., 2016).*
In order to explicitly describe sediment transport and bed evolution in a statistically-based framework, two different complementary processes need to be characterized: bottom shear stress (BSS), which can be considered a proxy for erosion, and suspended sediment concentration (SSC), which represents a measure of the sediment potentially available for deposition. To this goal, the characterization of BSS is provided by D'Alpaos et al. (2023). Here we aim to complete the proposed framework by statistically characterizing SSC and testing the possibility to describe suspended sediment dynamics as a Poisson process in long-term morphodynamic models.

(line 552) This confirms that resuspension events can be modelled as a 3-D marked Poisson process with marks (intensity and duration) mutually dependent but independent on the interarrival times in all the historical configurations of the Venice Lagoon. *Moreover, a comparison with the analysis of over-threshold BSS events shows that interarrival times, intensities and durations of both BSS and SSC events are mutually related but are complementary features because of the non-local dynamics due to advection and dispersion processes. These findings, together with those obtained for BSS events (D'Alpaos et al., 2023), provide the basis to develop a theoretical framework for generating synthetic, yet statistically realistic, forcings to be used in the long-term morphodynamic modelling of shallow tidal environments, in general, and for the Venice Lagoon, in particular.*

RC1.4: Lines 97-98: This somewhat contradicts results from the companion paper about erosion work, which increased then decreased over time, due to increase inter arrival time.

AR: We think that this apparent contradiction may be due to the excessive conciseness in the description of the temporal succession of the morphological modification of the Venice Lagoon in the last century in the first version of the manuscript.

Indeed, after the strong erosion experienced between 1930 and 1970, the sediment loss displays a relative slowdown because it reached a plateau due to the more intense hydrodynamic forcing required to rework bed sediment at an increasing water depth, resulting from the erosion process (Carniello et al., 2009; D'Alpaos, 2010a; Finotello et al., 2023). From this point of view, this confirms and does not contradict our results about erosion work in the companion paper.

To avoid confusion, we modified the paragraph as follows:

> (line 144) The Venice Lagoon (Figure 1) underwent different morphological changes over the last four centuries, in particular due to anthropogenic modifications (Carinello et al., 2009; D'Alpaos, 2010; Finotello et al., 2023). From the beginning of the fifteenth century, the main rivers (Brenta, Piave, and Sile) were gradually diverted in order to flow directly into the sea and prevent the lagoon from silting up, but this triggered *the present-day* sediment starvation condition. ~~Later, during the last century,~~The inlets were provided with jetties *between 1839 and 1934* and deep navigation channels were excavated to connect the inner harbour with the sea *between 1925 and 1970* (D'Alpaos, 2010). The jetties deeply changed the hydrodynamics at the inlets establishing an asymmetric hydrodynamic behaviour responsible for a net export of sediment toward the sea *after their construction* (Martini et al., 2004; Finotello et al., 2023), especially during severe storm events, which are responsible for the resuspension of large sediment volumes (Carniello et al., 2012). In general, these modifications, *together with sea level rise,* heavily influenced sediment transport triggering strong erosion processes *in the following period*. ~~that were further aggravated by sea-level rise.~~ The net sediment loss clearly emerges from the comparison among the different surveys of the Venice Lagoon, which show a generalized deepening of tidal flats and subtidal platforms as well as a reduction of salt-marsh area (Carniello et al., 2009). Indeed, in the last century, the average tidal-flat bottom elevation lowered from -0.51 m to -1.49 m above mean sea level (a.m.s.l.), while the salt-marsh area progressively shrank from 164.36 km^2 to 42.99 km^2 (Tommasini et al., 2019). *This erosive trend displays a relative slowdown in the last 30 years because of the larger hydrodynamic forcing required to rework bed sediment at an increasing water depth (Finotello et al., 2023).*

RC1.5: Lines 124-127: Provide the Strickler equation.

RC1.6: Lines 128-136: Provide the equation of the BSS induced by wind waves and the equation of the maximum (total?) BSS.

AR: We merged together our responses to RC1.5 and 1.6 because these two observations are closely linked. Following Reviewer's suggestions, we modified the text as follows:

> (line 181) The hydrodynamic module solves the 2-D shallow water equations using a semi-implicit staggered finite element method based on Galerkin's approach (Defina, 2000). The equations are suitably rewritten in order to deal with flooding and drying processes in morphologically irregular domains. ~~The Strickler equation is used to evaluate the bottom shear stress induced by currents, τ_c considering the case of turbulent flow over a rough wall.~~ *Moreover,* ~~Further,~~ the hydrodynamic module provides the flow field characteristic *used* ~~requested~~ by the wind-wave module to simulate the generation and propagation of wind waves.
>
> The wind-wave module (Carniello et al., 2011) solves the wave action conservation equation parametrized using the zero-order moment of the wave action spectrum in the frequency domain (Holthuijsen et al., 1989). ~~The peak wave period is related to the local wind speed and water depth, and this empirical correlation function is used to determine the spatial and temporal distribution of the wave period~~. *The spatial and temporal patterns of wave period are computed using an empirical function relating the mean peak wave period to the local wind speed and water depth* (Young and Verhagen, 1996; Breugem and Holthuijsen, 2007; Carniello et al., 2011). ~~The bottom shear stress induced by wind waves, τ_{ww}, is computed as a function of the maximum horizontal orbital velocity at the bottom, which is related to the significant wave height through the linear theory.~~
>
> *The WWTM provides both current- and wave-induced bottom shear stresses. The bottom shear stress induced by currents, τ_{tc}, is computed using the Strickler formulation, which, in the case of a turbulent flow over a rough wall, reads (Defina, 2000)*
>
> $$\tau_{tc} = \rho g Y \left(\frac{|q|}{K_s^2 H^{10/3}}\right) q \qquad (1)$$
>
> *where ρ is water density, g is the gravity acceleration, Y is the effective water depth (i.e. the actual volume of water per unit area), $\boldsymbol{q}$ is the flow rate per unit width, K_s is the Strickler roughness coefficient, and H is an equivalent water depth accounting for ground irregularities (Defina, 2000).*
>
> *The bottom shear stress induced by wind waves, τ_{ww}, is computed as a function of the total horizontal orbital velocity at the bottom, u_m, and the wave friction factor, f_w, as follows*
>
> $$\tau_{ww} = \frac{1}{2} \rho \, f_w \, u_m^2 \qquad (2)$$
>
> *The bottom orbital velocity, u_m, is evaluated by applying the linear theory and is also used, together with the wave period and median grain size, to compute the wave friction factor (Soulsby, 1997). Because of the non-linear interaction between the wave and current boundary layers, the total bottom shear stress, τ_{wc}, is enhanced beyond the linear addition of the current- and wave-driven stresses.*
>
> *To account for this process, in the WWTM the empirical formulation suggested by Soulsby (1995, 1997) is adopted:*

$$\tau_{\mathrm{wc}} = \tau_{tc} + \tau_{wc}\left[1 + 1.2\left(\frac{\tau_{ww}}{\tau_{ww}+\tau_{tc}}\right)\right] \qquad (3)$$

To further highlight the differences between the two companion papers, we reported in the Method section more details on the sediment transport model, which is exclusively used in the analysis of SSC and not for that of BSS. The modified version of the manuscript now reads:

(line 207) *The sediment transport and bed evolution module (STABEM, Carniello et al., 2012) is based on the solution of the advection-diffusion equation and Exner's equation:*

$$\frac{\partial C_i Y}{\partial t} + \nabla \cdot (\boldsymbol{q} C_i) - \nabla \cdot (\boldsymbol{D_h} \nabla C_i) = E_i - D_i \qquad i = s, m \qquad (4)$$

$$(1-n)\frac{\partial z_b}{\partial t} = \sum_i (D_i - E_i) \qquad (5)$$

where C is the depth-averaged sediment concentration, $\boldsymbol{D_h}(x, y, t)$ represents the space- and time-dependent 2-D diffusion tensor, E and D are the entrainment and deposition rate of bed sediment, z_b is the bed elevation and n is the bed porosity, assumed equal to 0.4. The subscript i refers to the sediment classes, that in shallow tidal environments are typically represented by non-cohesive (sand - s) and cohesive (mud - m) sediment. The relative local content of mud (p_m) can be used to mark off the transition between the cohesive or non-cohesive nature of the mixture and determines the critical value of the bottom shear stress. To discriminate between non-cohesive and cohesive behaviours, the threshold value of mud content p_{mc} is set equal to 10% (van Ledden et al, 2004).
The deposition rate of pure sand, D_s, is given by

$$D_s = w_s r_0 C_s \qquad (6)$$

where w_s is the sand settling velocity and r_0 is the ratio of near-bed to depth-averaged concentration, which is assumed constant and equal to 1.4 (Parker et al., 1987).
The deposition rate of pure cohesive mud, D_m, is computed using Krone's formula:

$$D_m = w_m C_m \max\{0; 1 - \tau_{wc}/\tau_d\} \qquad (7)$$

where w_m is the mud settling velocity, τ_{wc} is the bottom shear stress, and τ_d is the critical shear stress for deposition. The settling velocities, w_s and w_m, are computed using the formulation proposed by van Rijn (1984) for solitary particles in clear and still water, thus not incorporating flocculation effects that are negligible for particle diameters larger than 20 µm (Mehta et al., 1989). The critical shear stress for deposition, τ_d, largely varies among different tidal systems and, for the Venice Lagoon, we set $\tau_d = 1$ Pa on the basis of field measurements (Amos et al, 2004).
Both sand and mud erosion rates strongly depend on the cohesive nature of the mixture. The erosion rate for pure sand, E_s, is described by the van Rijn (1984)

formulation when the mixture is non-cohesive ($p_m \leq p_{mc}$) and by the Partheniades' formula for cohesive mixtures ($p_m > p_{mc}$):

$$E_s = \begin{cases} (1-p_m) w_s \cdot 1.5 \left(\frac{D_{50}/Y}{D_*^{0.3}}\right) T^{1.5} & \text{for } p_m \leq p_{mc} \\ (1-p_m) \cdot M_c T & \text{for } p_m > p_{mc} \end{cases} \quad (8)$$

The erosion rate for pure mud, E_s, is described by the formulation proposed by van Ledden et al. (2004) for non-cohesive mixtures ($p_m \leq p_{mc}$) and by the Partheniades' formula for cohesive mixtures ($p_m > p_{mc}$):

$$E_m = \begin{cases} \frac{p_m}{1-p_m} \cdot M_{nc} T & \text{for } p_m \leq p_{mc} \\ p_m \cdot M_c T & \text{for } p_m > p_{mc} \end{cases} \quad (9)$$

In Eqs. 8 and 9, D_* denotes the dimensionless grain size and it is computed as $D_* = D_{50}[(s-1)g/\nu^2]^{1/3}$, where s is the sediment-specific density and ν is the water kinematic viscosity; T is the transport parameter; M_{nc} and M_c are the specific entrainments for non-cohesive and cohesive mixtures, respectively, which can be computed as (van Rijn, 1984; van Ledden et al, 2004):

$$M_{nc} = \alpha \frac{\sqrt{(s-1) g D_{50}}}{D_*^{0.9}}$$
$$M_c = \left(\frac{M_{nc}}{M_m} \cdot \frac{1}{1-p_{mc}}\right)^{\frac{1-p_m}{1-p_{mc}}} \cdot M_m \quad (10)$$

where M_m is the specific entrainment for pure mud and it is set equal to $5 \cdot 10^{-2}$ g m s^{-1} and the parameter α is equal to $1 \cdot 10^{-5}$ (Carniello et al., 2012).

The transport parameter, T, is usually defined as $T = \max\{0; \tau_{wc}/\tau_c - 1\}$ where τ_c is the critical shear stress for erosion and can be assumed to vary monotonically between the critical value for pure sand, τ_{cs}, and the critical value for pure mud, τ_{cm}, depending on the mud content (van Ledden et al, 2004):

$$\tau_c = \begin{cases} (1+p_m)\tau_{cs} & \text{for } p_m \leq p_{mc} \\ \frac{\tau_{cs}(1+p_{mc})-\tau_{cm}}{1-p_{mc}}(1-p_m) + \tau_{cm} & \text{for } p_m > p_{mc} \end{cases} \quad (11)$$

However, this classic definition of the transport parameter describes a sharp transition between T = 0 and $T = \tau_{wc}/\tau_c - 1$ that does not take into account the spatial and temporal variability of both τ_{wc} and τ_c. Indeed, in real tidal systems, the bottom shear stress slightly varies owing to the non-uniform flow velocity, wave characteristics and small-scale bottom heterogeneity, while the critical shear stress is also affected by the random grain exposure and bed composition in time and space. Hence, following the stochastic approach suggested by Grass (1970), both the total bottom shear stress, τ_{wc}, and the critical shear stress for erosion, τ_c, are treated as random variables (τ_{wc}', and τ_c', respectively) with lognormal distributions, and their expected values are those calculated by WWTM and STABEM. Consequently, the erosion rate depends on the probability that τ_{wc}' exceeds τ_c' (Carniello et al., 2012). The result of this stochastic approach is a smooth transition between T = 0 and $T = \tau_{wc}/\tau_c - 1$. The comparison with SSC *field measurements shows a much better agreement of the stochastic approach compared to that of the classic formulation (Supplementary information and Figure S3). Finally, erosion and deposition rates of sand and mud result in a variation of bed level and composition through time, which is computed using Eq. 5 and updating the local mud content.*

(line 297) *To correctly model SSC as well as bed evolution, the knowledge of the bed sediment composition is crucial. Sufficiently detailed, spatially-distributed grain-size data are available for the present-day configuration of the Venice Lagoon (Amos et al., 2004; Umgiesser et al., 2006). Using this dataset, Carniello et al. (2012) empirically related the median grain size* D_{50} *to the local bottom elevation and the distance from the inlets:*

$$D_f = \begin{cases} \max\{300;\ 50(-h_f - 0.8)^{0.75}\} & \text{if } h_f \le 1 \text{ m a.m.s.l.} \\ 15 & \text{if } h_f > 1 \text{ m a.m.s.l.} \end{cases} \quad (12)$$

$$D_{50} = D_{hf} + 100e^{-0.0097L^3} \quad (13)$$

where h_f *is the bottom elevation in m a.m.s.l.; L is the linear distance from the closer inlet in km;* D_{50} *and* D_{hf} *are the grain diameter μm. This relationship describes a coarsening of the sediment grain size distribution at deeper locations (i.e. channels) and at shorter distances from the sea. Because bottom elevation and the distance from the inlet are the two main parameters describing the spatial variation in sediment grain size, we assume that this relationship holds independently on the specific morphological configuration of the Venice Lagoon and we used Eqs 12 and 13 to compute the distribution of median grain size* D_{50} *in all the six selected historical configurations.*

The spatial distribution of mud content, p_m*, is then computed as a combination of the local* D_{50} *and the typical grain size of mud and sand fractions (Umgiesser et al., 2006)*

$$p_m = 1 - \frac{\ln(D_{50}/D_m)}{\ln(D_s/D_m)} \quad (14)$$

where D_m *and* D_s *are the typical grain size of mud and sand, respectively. Analysing the grain size distribution measured in the Venice Lagoon (Amos et al., 2004; Umgiesser et al., 2006), we set* $D_m = 20\mu m$ *and* $D_s = 200\mu m$.

RC1.7: Line 153: Missing prime for the total BSS?

AR: Thank you for noting. Added (see also modified text in the previous response).