

Review 3

The authors present a theoretical experiment to estimate thermal diffusivity within the debris layer of a debris-covered glacier. The topic of thermal properties within debris-cover is timely and relevant given the many other studies that have been published in the past few years. Overall, the paper is well thought through and lays out the structure and approach used. I think that the writing style could be improved to be more formal as is typical in a journal article, and attention to detail is needed for many sentences throughout. There are many minor edits that I suggest throughout, and in general the text should be made much clearer as it is difficult to follow in places. The entire paper should be read with an eye towards grammar and citation formatting, as there are many (beyond the comments I made) that are incorrect. I think the overall suggestions are useful for future work, and should be an informative paper, but the structure and cleanliness of the paper needs to be improved before publication.

We thank the reviewer for their review and critical comments about the manuscript and work. We have substantially restructured and rewritten the manuscript, improving the organisation and flow, and ensuring no repetition. We have added a clear aims and experiments section and checked all language and formatting carefully.

I think that the paper clearly states the justification to focus its analysis on the Conway and Rasmussen (2000) paper, which I feel accounts for many of the comments of a prior reviewer. To address this, it might be useful to provide more of a comparison between the newer Laha et al (2022) paper and the Conway and Rasmussen (2000) paper to further support the need for this study. I think that the value of this paper comes from its theoretical approach instead of using entire seasons of existing data from other papers.

We agree in the that value of our study, but in response to reviewer 1 have added a clearer statement of purpose to address this point:

“As the existing, and limited, sets of field data used to provide generalized values for the effective thermal conductivity of unmeasured glacier sites have been analysed based upon the simple Conway and Rasmussen method, rather than the later methods developed by Laha and others, there is value in a deeper exploration of the limitations to interpreting values derived by this method, in particular to better understand the meaning of comparison between sites at regional and global scales (e.g. Rowan et al., 2021; Miles et al., 2022). For example, the measurement parameters for temporal or spatial sampling intervals, thermistor spacings, and debris depths used in the application of the standard method presented by Conway and Rasmussen (2000) are selected ad hoc and differ from measurement site to measurement site (e.g. Juen et al., 2013; Chand and Kayastha, 2018; Rowan et al., 2021), and uncertainty estimates associated with this are missing. This means that baseline literature values that are subject to

onward use on the literature may be differently influenced by sensor, installation and numerical truncation errors. This study explores the effect of the chosen temporal and vertical spatial temperature sampling interval and other systematic measurement errors originating in the measurement setup on the derived thermal diffusivity values. To explore the capabilities and limitations of this approach we apply this method to artificially generated data with a known value of thermal diffusivity, which allows us to individually quantify systematic and statistical errors by error source. We additionally present an online tool to allow interactive analysis of these combined errors for a given dataset and a best practice guideline on how to minimize the systematic errors inherent in the methods of Conway and Rasmussen (2000)."

We also now discuss the relative merits of the approaches in the discussion section, including the applications or use cases for which each is best.

The methods used are well outlined overall and supported by the figures and online tool provided in the paper. There are a couple paragraphs in the methods that could be more useful in the introduction or discussion (L87, L110) Please see my in-line comments regarding minor edits to certain paragraphs and sentences.

We have reorganised the sectioning, and moved these parts, and streamlined the introduction in places.

It is important for readers to understand that this is a theoretical model and will of course not account for every eventuality that might occur in the "real debris" layer on a glacier, but I think a major benefit of this study is the online tool to explore the errors interactively. This tool was easy to use and provides the theoretical knowledge to explore thermal diffusivity within a debris-layer. The assumptions that are made regarding density, conduction, and specific heat capacity are reasonable considering the constraints of a numerical study. The results show that this model holds up against specific instances of real data which helps validate the model being presented here. However, there should be more in the discussion or conclusion about the limitations of this study and more specific reasoning for the best practice guidelines that it provides.

Thank you, we emphasise this in the new section explaining the experiments performed and also discuss the limitations more fully, making clear distinctions between understanding the numerical behaviour in response to theoretical sampling and implementations. We also emphasise the tool more within the discussion text.

As it is intended, this paper focuses its analysis on the Conway and Rasmussen paper and identifies limitations and error sources based on that method of estimating thermal diffusivity values. This paper provides value to the literature because of how frequently the Conway and Rasmussen method is used and oftentimes, without deeper consideration for its error sources and limitations. This

paper does not aim to analyze other new methods as discussed in Laha et al., and Petersen et al., and I think this is okay due to the relative lack of use of these newer methods. While the guidelines and suggestions that this paper makes are based on a theoretical model, there needs to be more consistency in the field-based methods used in this discipline, so this paper provides some suggestions for how to do that.

Thanks, we agree, and have aimed to clarify this with a better purpose statement within the manuscript.

The best practice guidelines provided are helpful considerations for future field studies that aim to measure thermal diffusivity within the debris-layer. The debris-covered glacier literature needs to have more consistent methods of measuring thermal diffusivity to compare findings across different field sites. The guidelines this paper presents should be used in the future by other field-based studies and these future studies should consider the limitations and error sources that are discussed here. Please just add to these guidelines and explain the reasoning for these guidelines in a clear and structured manner.

We have expanded upon our reasoning, and in particular highlight the reasons for contrasting recommendations for different purposes (e.g. Laha and others (2022) seek a method to best determine sub debris ablation rate from thermistor measurements directly, while we explore methods used to determine thermal conductivity values for application in generalised surface energy balance models of ablation beneath debris cover).

Line by Line comments - also in line on PDF.

L57: Specify vertical spacing and provide slightly more reasoning for why you are exploring these variables in the sentences before. Also discuss why horizontal spatial variability is not focused on in this study.

We clarify this and highlight the potential for real-world fluxes to differ from the idealised 1D heat equation, restating that we aim to show the consequences only of the calculation implementation, not the degree to which the real world meets the assumptions of the method.

L87: This paragraph could be better in the intro or discussion sections.

Moved, and streamlined

L93: Grammatical issues. A period or comma is needed, and the spacing is strange.

Addressed in the full rewrite of the manuscript.

L103: Be more specific in terms of “they.”

Addressed in the full rewrite of the manuscript.

L103-109: Also adding another sentence here to improve the justification of this approach would help convince readers this is a valuable approach you are taking. Provide more comparison from Laha et al. and Petersen et al.

Agreed, we provide more information on the benefits of each method and their appropriate usage, in principle and in existing literature.

EQ 3, 4, 5: The O in these equations is not defined and is not consistent across these three Eqs.

These equations were removed when this section was moved into a more streamlined introduction section.

L177: Figures need to be cited in line, and when cited need to be clearly referencing that given figure.

Done, figure captions and in-text citations of them have been checked throughout.

Figure 3 caption: data 3, 4, 5 are not shown on the graphic – I think I know which data you are referring to, but please clarify and provide the same description as the values in the figure.

Done, figure captions and in-text citations of them have been checked throughout.

L182: Another figure citation missing.

Done, figure captions and in-text citations of them have been checked throughout.

Figure 4 caption: “timeseries” will need a space.

Done, figure captions and in-text citations of them have been checked throughout.

L184: Figure 6 is being referenced here and it seems a bit out of order if you are indeed citing that figure. Please make sure every figure is referenced.

Done, figure captions and in-text citations of them have been checked throughout.

L206-208: Provide a little more clarity on these two resampling methods. Method 1 is clear, but method 2 is less so, provide more detail here to avoid confusion.

Thanks, we clarify that it is averaging over a time interval and explain that we choose to investigate that as in some of the reported field data temperatures (e.g.

Rowan et al, 2021) for Khumbu glacier were collected as time averages rather than samples.

L208: Figure reference missing.

Done, figure captions and in-text citations of them have been checked throughout.

L218: Make this entire paragraph more clear.

Paragraph has been rephrased

L272: "purly" to purely

Done

Eqs. 16-20: Why are these equations in the results? Shouldn't these be in the methods section and discussed there?

Removed or moved.

L290: This paragraph in discussion needs to be re-written or at least made more clear. It is confusing and doesn't read smoothly.

Paragraph has been rephrased

L316: remove comma after "true, "

Done