Response to referee #1 on egusphere-2023-2731

General comments:

This paper presents interesting results from a field survey of mountain glaciers at Colle Gnifetti using a phase-sensitive radio echo-sounder (pRES). The authors obtained the results with a layer-optimized SAR processing method, which is special in that it performs coherent summation over the synthetic aperture along an optimally estimated linear slope. The echograms from pRES revealed deeper internal reflection horizons (IRHs) that were not visible in the results from previous surveys with pulsed GPR. This data set is thus valuable for studying snow accumulation, ice flow and ice core chronologies in this area. The paper also provides valuable insights into different reflection mechanisms of the glacial IRHs at different depths by comparing radar data and ice core records. The algorithm of the tailored layer-optimized processing is an important part of the paper and aims to improve the detectability of IRHs. So, it is necessary to accurately assess the effectiveness of this algorithm in terms of the improvement in IRH detectability by comparing it with other methods that people have routinely been using in radar data processing. However, this paper does not have this kind of comparison and analysis, which is my major concern. The paper will be improved well for publication if this concern can be addressed.

> **Author response**:
> Dear referee #1,
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> we thank you for your time to review our manuscript and your constructive feedback. Following your comments, we will provide a comparison of our implementation of LO-SAR processing to a moving averaging filter and to LO-SAR processing as implemented by Castelletti et al. (2019) as a supplement. In the manuscript we will highlight the differences and advantages compared to (Castelletti et al., 2019) more clearly. Finally, we include a short discussion about the selection of the slope iteration range based on the limit for the occurrence of destructive interference.
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> Kind regards,
> Falk Oraschewski
> on behalf of all co-authors.

Specific comments, questions, and suggestions:

1. For the across-saddle and upstream profiles, it is desirable to see an echogram for each profile that is generated by applying a moving averaging filter of the same aperture length on each trace in Fig. 5 (c) and Fig. 5 (e). This averaging is like unfocused SAR processing without along-track decimation, which is much faster than the proposed LO-SAR processing. Because the phase shift due to slope has been corrected in reference phase substruction and thus the corrected phase is constant along the slope, this averaging is also similar to the coherent integration performed in the reference by Castelletti et al. (2019), i.e., the summation is not along the slope. By comparing these two echograms with Fig. 5 (d) and Fig. 5 (f),

the improvement on IRH detectability by the proposed LO-SAR can be qualitatively and quantitatively analyzed and discussed.

> **Author response**: We agree that a direct comparison with existing processing methods will help to demonstrate the advantage of the proposed LO-SAR processing as implemented here for mobile pRES data. A figure with this comparison will be provided as a supplement to our manuscript.
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> In summary the advantages are as follows:
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> 1. Moving averaging that includes the phase (and corresponds to unfocused SAR processing) will result in patterns of destructive interference even at moderate slopes (see following comment). Alternatively, non-coherent moving averaging could also be applied by only taking the power into account. But in this case the full potential of the phase-coherent signal cannot be used, which particularly lies in the desired extinction of spatially incoherent signals of non-specular reflections and of background noise.
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> 2. Our implementation of LO-SAR processing does not substantially differ from the implementation by Castelletti et al. (2019), but provides some improvements in the processed radargram. To first highlight the differences, Castelletti et al. coherently sum along a range bin over some aperture length whereby they iterate over a range of phase shifts to find the optimal signal-to-noise ratio (SNR). Thereby, their approach provides the englacial slopes as a byproduct computed from the optimal phase shift. Their implementation to our data would correspond to applying LO-SAR processing on the raw signal before the reference phase correction by finding a phase-shift that cancels out the horizontal phase-gradient.
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> Our approach can be understood as an implementation of the LO-SAR processing by Castelletti et al. tailored to mobile pRES data. In this regard, we see a central contribution in demonstrating that the corrected phase is constant along IRHs which can be used to directly extract their slopes by iterating over a range of slopes. We describe this in the manuscript in detail not least because it helps to clarify common misunderstandings regarding the information content of the corrected phase signal and thereby has relevance for the interpretation of other types of pRES data. Only in a second step the obtained slopes are then used to perform the coherent summation, which has two advantages: (i) The slopes can be filtered prior to the coherent summation to remove outliers, for example occurring at the transition between two reflections. (ii) Coherent summation can be performed directly along the englacial slope, which for any point on the radargram (whether there is an IRH or not) reduces the integration of signal power from other nearby IRHs. In combination this results in some improvements, as for example a better resolution of closely spaced IRHs. We will revise the LO-SAR processing section of the manuscript to clearer highlight these differences.

2. In 5.2 for discussions on slope estimation, the authors mentioned that "strong reflections spread out over several range bins, across which the raw phase tends to be stable", and according to Fig. 5 (b), the "inclination relative to the surface is quantified during the LO-SAR processing and attains values of about 10° at both ends". For the aperture length of 5

meters used in the processing, this inclination corresponds to only two range bins from one end of the aperture to the other end, therefore there might be no significant difference in performing the summation along the slope and not along the slope if the reflected power does not have big difference neither within two range bins. It would be helpful to demonstrate at what aperture length and at what slope angle the improvement of SNR for IRH detectability can be expected from the proposed LO-SAR.

> **Author response**: The two cited statements are not in contradiction with the improvement by LO-SAR processing, because the first statement refers to the raw phase, whereas the summation is applied to the corrected phase. The raw phase is approximately stable in the vertical across a reflection, but varies horizontally (as the horizontal gradient of the raw phase is proportional to the slope of the reflection). The corrected phase, on the other hand, strongly varies in the vertical and in the horizontal is only constant along the slope of the reflection (Fig. 4b). Accordingly, horizontal summation that does not take (even small) slopes into account will result in destructive interference using either of them.
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> The limit for the occurrence of destructive interference can be estimated from the phase offset between range bins of $\frac{3}{p}\pi$ (Eq. 4). With a zero-padding of $p = 8$ some points along the aperture length will be completely out of phase when the gradient of the summation line deviates from the correct reflection slope by 3 range bins (after zero-padding) along the aperture length. For an aperture length of 5 m this corresponds to a slope of $\sim 2°$ which is present at large parts of the radargram.
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> In addition, note that the first statement is already true without zero-padding, but more pronounced in, and here used while referring to, the zero-padded signal for which with $p = 8$ a slope of $10°$ would correspond to 16 range bins. Nevertheless, zero-padding does not generally affect the destructive interference limit as the phase offset vs. total range is independent of it.

3. The iteration range of slopes used was from $-30°$ to $30°$ in steps of $0.2°$, much larger than the slop range observed in the data. Some discussion about smart selection of this range may be included in section 5.2 for reducing the computation intensity of the proposed LO-SAR.

> **Author response**: Thank you for this nice suggestion. We will include a short discussion about the selection of the iteration range for the slopes into the manuscript. This selection essentially depends on two factors: the step size should lie well below the limit for the occurrence of destructive interference (see above), and the maximum range should cover the range of slopes that are to be expected. We decided to use a rather wide slope range to not *a priori* prevent the detection of potential stronger inclined reflections in the near-basal ice. In our case compute time was not a limiting factor, because the profile length is short, but in other applications a better choice of the iteration limits will surely be useful.

4. What was the data logger sampling rate used during data collection? 40 kHz as discussed in section 5.1?

Author response: Yes, we used the sampling rate of the pRES data logger of 40 kHz. Technically, the ADC on the VAB board of the pRES samples at 80kHz and then two consecutive samples are averaged to give a 40kHz output which is written to the SD card. The pRES can be set to use the full 80kHz sampling rate, but because the AF filter is attenuating most of the additional high frequency input anyway, this would not be of much help.

Technical corrections and suggestions:

1. In Fig. 2 (a), the two legends are hard to distinguish, consider changing one of them with dashed line. The two red line segments for antennas need to explicitly be mentioned either in the figure caption or in text.

   Author response: We agree regarding the legends and will adapt the line style. The two red segments originate from another use of the graphic antenna models and will be removed.

2. In Fig. 3, missing information flow direction arrow along the line between the block for "Pre-processed mobile pRES data" and the block for "Moving median filtering of S".

   Author response: These arrows were supposed to indicate that the "Estimated slopes S" and the "Power and phase of traces in synthetic aperture length" are used together in the "Coherent summation of power along S". We will include additional arrow heads to make clear that information is not directly flowing between "Pre-processed mobile pRES data" and "Moving median filtering of S"

3. Revise "by the weighting the values" to "by weighting the values" at line 166 on page 8.

   Author response: Fixed.

4. It is suggested to mark the crossover between the cross-saddle profile and the GPR profile, the crossover between the cross-saddle profile and the upstream profile towards KCC, and the crossover between the upstream profile towards KCC and the GPR profile with A, B and C respectively in Fig. 1, and according to mark those vertical white lines in Fig. 5 with A, B and C. It is much easier this way to identify these crossovers.

   Author response: Thank you for this suggestion, we agree that this improves the clarity of the crossover points and will adopt it.

5. The location of the white line to the right in Fig. 5 (a) does not match with the location of the crossover between the upstream profile towards KCC and the GPR profile in Fig. 1 which is at the very end of the GPS profile.

Author response: Thank you for catching this mistake. Indeed, there was an error in the plotting range of Fig. 5, which we have fixed.

References

Castelletti, D., Schroeder, D. M., Mantelli, E., and Hilger, A.: Layer Optimized SAR Processing and Slope Estimation in Radar Sounder Data, Journal of Glaciology, 65, 983–988, https://doi.org/10.1017/jog.2019.72, 2019.