Answer to the Reviewers and resubmission of the manuscript "**New straightforward formulae for the settling speed of prolate spheroids in the atmosphere: theoretical background and implementation in AerSett v2.0.1.**"

May 2, 2024

Following the discussion on EGUsphere on our manuscript, we wish to submit the manuscript for *Geosci Model Dev.*. please find below the detailed answers to the Reviewers, and the description of the modifications that have been brought to the text. Following the suggestion of the Reviewers, we have added one new figure panel (Fig. 5c) and two new figures (Figs. 2-3). We also bring more precisions : explanations to several aspects in the presentation and/or description of our results.

Please find below the detailed answers to Reviewer 1 (7 pages) and to reviewer 2 (9 pages).

We hope that with these modifications the manuscript can be for for publication in GMD,

Best regards,

On behalf of the authors,

Sylvain MAILLER

Answer to Reviewer comment RC1 on the manuscript "**New straightforward formulae for the settling speed of prolate spheroids in the atmosphere: theoretical background and implementation in AerSett v2.0.2.**"

(doi: 10.5194/egusphere-2023-2637)

May 2, 2024

We are grateful to Reviewer Carlos Alvarez Zambrano for his careful reading of our manuscript and his insightful questions and suggestions. All the Reviewer comments have been addressed (the corresponding modifications in the manuscript are presented in blue in the present document). In particular, following the Reviewer's request to discuss more the relevance of slip-correction for small particles, we have added a new figure panel to the manuscript (Fig. 5c in the revised manuscript). We hope that with these modifications we have answered satisfactorily all the Reviewer comments.

Contents

1 Transcript of the Reviewer Comment RC1

1.1 Summary

In this paper, the authors deduced two equations for calculating the settling velocity of atmospheric particles with elongated spheroidal shapes, considering both horizontal and vertical orientations. The first formulation relies solely on theoretical reasoning. The second method is based on drag expressions derived from numerical simulations using computational fluid dynamics (CFD). Their findings indicate that these two formulations yield comparable results, with a deviation, based on the mean particle diameter, within 2% and 10% for particles falling horizontally. The authors also implemented their formulations into a Fortran-based model to calculate dust transport.

1.2 Overall Evaluation

The manuscript is well-written, and the authors have done a great job deducing the equations and providing explanations for the reasoning behind them. However, certain sections of the paper, including those related to the formulation deduction, could benefit from additional explanations and discussion. With the incorporation of extra clarifications and/or inclusion of details, in my opinion, this manuscript will ultimately make a good contribution to the atmospheric dust transport community. Below, I include some questions and comments that could enhance the quality of this paper.

1. I recommend that the authors provide a brief description of AerSett v2.0.2 in the Introduction section, as not everyone may be familiar with this module previously published by (almost) the same authors.
2. Line 67: I suggest changing the expression "might be tricky" to a more formal expression, such as "pose challenges."
3. Line 69: It would be advisable to include the definition of the aspect ratio, even though it is defined later in the document.
4. Abstract and Line 85: It is not clear if the authors implemented both formulations as mentioned in Line 85, or if they used the equation obtained from the first approach, as stated in the Abstract.
5. Equation 10: Define x in the D(x).
6. Equation 11: Is v_∞ the settling velocity for prolate spheroid-shaped particles? If so, what is the main difference with $U^{(\lambda,\phi)}$?
7. Section 2.3: Why is the slip correction factor needed? Is the correction being applied to the whole range of particle sizes? To determine the applicability of the slip-correction factor, the Knudsen number (Kn), the ratio of the mean free path to the particle diameter, needs to be observed. Depending on the calculated value of Kn, the correction may be relevant or not. However, the mean free path depends on the pressure, density, and dynamic viscosity of the air. This raises a question for the authors: do the calculations include variations in these air parameters, or was only a constant pressure considered? I recommend that the authors explore in detail the impact and applicability of the slip correction and include in the paper a discussion of for what particle sizes and/or air pressures the correction is important.
8. Equation 29: Define u in $F_{cg}(u)$.
9. Line 195: The authors state that Eq. 31 provides an accuracy better than 2.5%. However, it is not clear what was the reference used to calculate/compare the results of this equation.
10. Conclusions: I suggest that the authors expand the discussion of the limitations of this formulation. They can explore, for example: i) how other orientation values would change their findings. Although the authors stated that particles tend to fall horizontally, it is also known that during the particle lifespan, they change their orientation. ii) Are there any ideas on how to incorporate porosity into each particle for this new formulation?

2 Answers

2.1 Comment 1. Adding a description of AerSett v2.0.2

We agree that a description of the module was missing in the introduction, since this module is not (hopefully, not yet) well-known to the community. We have added the following sentences into the introduction, to present what was already don in Aersett v1.0, and what will be presented in v2.0.2:

.... This formulation has been implemented by the same authors in AerSett v1.0 (Mailler et al., 2023a), a Fortran module designed to be included easily in chemistry-transport models, and already included in Chimere v2023r1 (Menut et al., 2023).

.... We will also describe AerSett v2.0.2, a Fortran module designed to calculate accurately and effiently the settling speed of prolate particles oriented either horizontally or vertically in the atmosphere.

2.2 Comment 4. Did we implement both methods ?

Line 85 in the manuscript says that "In Section 4 we will present the implementation of both these methods in AerSett v2.0", but Abstract says that "we provide an implementation of the first of these methods in AerSett v2.0.2, a module written in Fortran.". The Reviewer is right in spotting an inconsistency here. The statement in the Abstract is correct, only the first of these method is implemented. In the end of section 3 of the manuscript, we explain why we consider that using the first formulation is more simple and accurate enough for atmospheric sciences.

We have corrected the introdution as follows:

In Section 4 we will present the implementation of the method described in Section 2 in AerSett v2.0.2, and we will give our conclusions in Section 5.

2.3 Comment 5. Equation 10: Define x in the D(x).

Eq. 10 in the manuscript is as follows:

$$C_D\left(Re\right) = \frac{A^{\lambda,\phi}}{Re}\mathcal{D}\left(Re\right), \text{ with } \lim_{x\to 0^+}\mathcal{D}\left(x\right) = 1. \tag{10}$$

In this equation, x is the infinitesimal quantity going to zero in $\lim_{x\to 0^+}\mathcal{D}\left(x\right) = 1$, it is just a dummy variable name. However, introducing a dummy variable here is not indispensable, and may just induce confusion, therefore we have clarified Equation 10 as follows:

$$C_D\left(Re\right) = \frac{A^{\lambda,\phi}}{Re}\mathcal{D}\left(Re\right), \text{ with } \lim_{Re\to 0^+}\mathcal{D}\left(Re\right) = 1, \tag{10}$$

2.4 Comment 6.

Eq. 11 and the surrounding text are as follows:

"

$$v_\infty = \frac{4}{3}\frac{\left(\rho_p - \rho\right) g d_{eq}^2}{A^{\lambda,\phi}\mu\mathcal{D}\left(Re\right)} \tag{11}$$

$$= \frac{U^{\lambda,\phi}}{\mathcal{D}\left(Re\right)}, \tag{12}$$

where $U^{\lambda,\phi} = \frac{4}{3}\frac{(\rho_p-\rho)gd_{eq}^2}{A^{\lambda,\phi}\mu}$ is the settling velocity of a prolate spheroid with aspect ratio λ and orientation angle ϕ, under the Stokes law for prolate spheroids."

In these equations, v_∞ is the settling velocity for a prolate spheroid-shaped particle, and $\frac{U^{\lambda,\phi}}{\mathcal{D}(Re)}$ is the settling speed of the same particle *under the Stokes law*. More explicitly, v_∞ includes the large-particle drag correction, while $U^{\lambda,\phi}$ does not. Therefore, $U^{\lambda,\phi}$ has an exact analytic expression $U^{\lambda,\phi} = \frac{4}{3}\frac{(\rho_p-\rho)gd_{eq}^2}{A^{\lambda,\phi}\mu}$, already known from past theoretical works as detailed in the introduction, while v_∞ includes $\mathcal{D}\left(Re\right)$, a drag-correction term that accounts for deviations from the creeping-flow regime that occur for larger Reynolds number.

We have reformulated the sentence after Eq. 11 as follows:

where $U^{\lambda,\phi} = \frac{4}{3}\frac{(\rho_p-\rho)gd_{eq}^2}{A^{\lambda,\phi}\mu}$ is the settling velocity of a prolate spheroid with aspect ratio λ and orientation angle ϕ supposing that the Stokes law is verified exactly. On the other hand, v_∞ is the settling speed of the same prolate spheroid taking into account the deviations from the Stokes law, reflected in the $mathcalD\left(Re\right)$ drag function.

2.5 Comment 7. on the slip-correction factor

We agree that this discussion is important, however it has been done for the case of spherical particles in Mailler et al. (2023b) (their Section 5). The conclusions of this figure are not changed in any substantial way for prolate spheroidal particles. In short, the main point-by-point answer to your questions on this point are:

- The slip-correction is needed to take into account the fact that for the smallest particles, their size is comparable to the free mean path of air molecules so that air does not behave like a continuous fluid. We can develop this point in the introduction.
- yes, the correction is applied for the whole range of particle sizes. However, for particles with diameter $D > 10\,\mu$m, this correction is almost negligible (see Fig. 4 of Mailler et al. (2023b)).
- Regarding the atmospheric conditions used for this manuscript, only Figures 2-3-5 in the initial manuscript (and 4-5-7 in the revised manuscript) depend on particular atmospheric conditions. These figures have been produced with $P = 101325\,\mathrm{Pa}$ and $T = 298.15\,\mathrm{K}$. These precisions have been brought in the revised version of the manuscript (Section 2.4)
- Regarding the influence of atmospheric pressure, temperature and viscosity, Fig. 4 of Mailler et al. (2023b) shows that the impact of both the slip-correction and the large-particle drag correction on the settling speed for spherical particles, as a function of particle size and of atmospheric pressure (temperature and viscosity being calculated from pressure using the US Standard Atmosphere).
- We feel that Fig. 4 of Mailler et al. (2023b), which is a pressure-diameter diagram, gives an indication as of for which diameters and pressures are slip-correction and/or large-particle drag corrections relevant. We agree that this part of the conclusions of Mailler et al. (2023b) needs to be reminded to the Reader in a future version of this manuscript, probably in the introduction. The present manuscript complements this already existing discussion by discussing for which particles eccentricity correction may become substantial (for which we answer in the conclusion that differences begin to be substantial for aspect ration greater than 2).

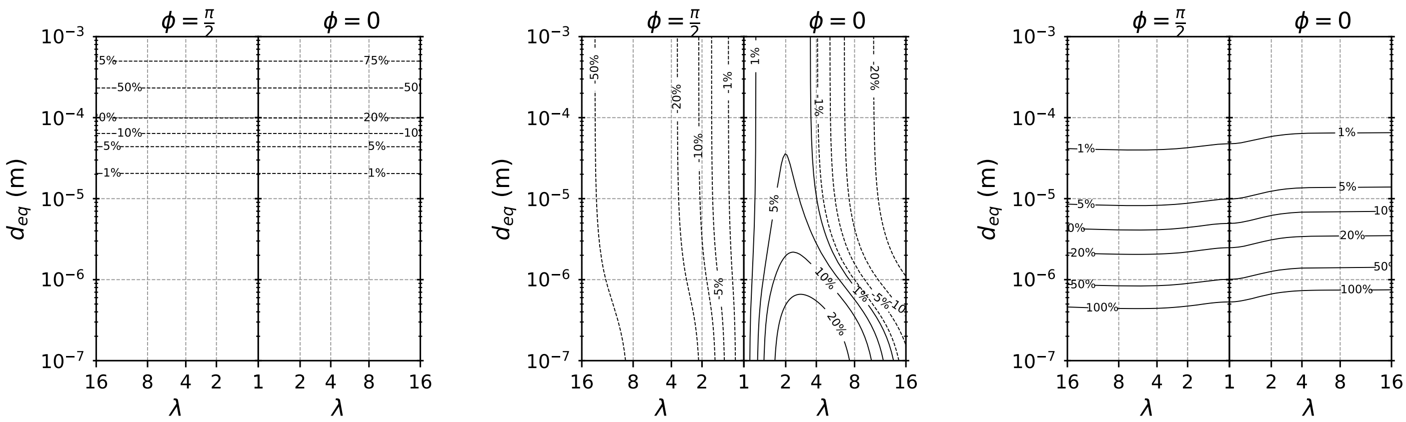
After our initial answer(previous paragraph), the Reviewer reiterates that:
" I propose that the authors thoroughly investigate the impact and relevance of the slip correction, incorporating a detailed discussion in the paper regarding the particle sizes and/or air pressures for which the correction holds significance. "

A new figure (Fig. 5c) giving the magnitude of slip-correction as a function of particle equivalent diameter and eccentricity has been added, and discussed in the manuscript:
" Figure 5 shows the effect of large-particle correction (Fig. 5a), eccentricity correction effects (Fig. 5b) and slip correction (Fig. 5c) on the settling speed, showing that the large-particle correction begins to be significant ($< -5\%$) for particles with $d_{eq} > 30\,\mu\mathrm{m}$. On the contrary, slip correction is significant ($> 5\%$) only for particles with $d_{eq} < 3-5\,\mu\mathrm{m}$, depending on particle eccentricity. For lower pressure values ($p \simeq 200\,\mathrm{hPa}$) representative of the higher troposphere or lower stratosphere, slip correction increases due to the longer mean-free path for air particles in thinner air. At these altitudes, slip-correction reaches 5% for particles with $d_{eq} < 8-15\,\mu\mathrm{m}$ (not shown), while large-particle corrections also reaches -5% for particles with $d_{eq} > 30\,\mu\mathrm{m}$ (not shown).

Figure 1 (not included in the revised manuscript for conciseness) is the same as Fig. 5a-c in the manuscript but for conditions representative of the tropopause following NOAA/NASA/USAF (1976). At these pressure and temperature conditions, we find that slip-correction is significant for $d_{eq} < 8-15\,\mu\mathrm{m}$, and large-particle correction for $d_{eq} > 30\,\mu\mathrm{m}$. The conclusions from this additional analysis are mentioned in the revised manuscript (see blue text above).

2.6 Comment 8. define u in $F_{cg}(u)$

u is just a dummy variable here, it has no meaning outside of Eq. 29. However, letter u may suggest a speed, in particular in the context of this manuscript, which can mislead the reader.



(a) (b) (c)

Figure 1: (a) Large-particle correction $\frac{\widetilde{v}_\infty - \widetilde{U}^{\lambda,\phi}}{\widetilde{U}^{\lambda,\phi}}$ in % (contours); (b) eccentricity correction $\frac{\widetilde{v}_\infty(\lambda;d_{eq}) - \widetilde{v}_\infty(\lambda=0;d_{eq})}{\widetilde{v}_\infty(\lambda=0;d_{eq})}$; and (c) slip-correction $\frac{\widetilde{v}_\infty(\lambda;d_{eq}) - v_\infty(\lambda;d_{eq})}{v_\infty(\lambda;d_{eq})}$. The three panels are in % (contours). The figure is produced for standard atmospheric conditions at the tropopause according to NOAA/NASA/USAF (1976): ($p = 22632.1\,\mathrm{Pa}$, $T = 216.65\,\mathrm{K}$))

In the revised version, we have substituted u (which may suggest a speed) by x. x is the standard textbook notation for a dummy variable in the definition of a function, which will hopefully reduce the probability of misunderstanding.

2.7 Comment 9. Where does the 2.5% accuracy come from ?

In line 195 and around, the following statement is made, for which the Reviewer asks for precisions.

" Eq. 18 with C_D as expressed in Eq. 27 yields:

$$\mathcal{S} = \left(F_{cg}\left(R \cdot \mathcal{S}\right)\right)^{-1}. \tag{30}$$

An equivalent fixed-point equation has been solved in Mailler et al. (2023b) (their Eqs. 13 and 16), yielding the following approximated expression for $\mathcal{S}(R)$:

$$\mathcal{S}(R) = 1 - \left[1 + \left(\frac{R}{4.880}\right)^{-0.4335}\right]^{-1.905}, \tag{31}$$

which holds with an accuracy better than 2.5% for the $Re < 1000$."

The justification of this statement is at the core of Mailler et al. (2023b). The assertion of the 2.5% accuracy is to be understood as the loss of accuracy when solving Eq. 30 using explicit expression 31 to obtain the solution right away instead of performing an iterative resolution of Eq. 30.

In the revised manuscript, we have clarified te sentence as follows:

" As discussed in Mailler et al. (2023b), using this explicit formula instead of numerically resolving Eq. 30 induces a loss of less than 2.5% in accuracy for $Re < 1000$, which is not critical since, the uncertainty of the Clift-Gauvin formula itself (and of other comparable drag-coefficient formulations) is around 7% when compared to experimental measurements (Goossens, 2019)."

2.8 Comment 10. Expand the conclusions and discuss the limitations

We agree that the discussion could be enhanced and in particular the limitations of the present approach could be discussed further. Two points in particular are suggested by the Reviewer.

intermediate orientations

We have added the following piece of text to the manuscript:

From methods based on mechanics and statistical physics, Mallios et al. (2021) have determined the PDFs for particle's attack angle as a function of their aspect ratio and of the other characteristics of the particle and of the fluid (assuming particles shaped as prolate spheroids). Based on these PDFs the authors have calculated the average attack angle of particles with different sizes. They showed that particles with sizes less than $\simeq 2\,\mu\text{m}$ are in principle randomly oriented, while particles with sizes larger than $\simeq 20\,\mu\text{m}$ tend to fall with an essentially horizontal orientation. Therefore, in their present formulation, our results only permit the direct calculation of the settling speed of giant dust particles with $d_{eq} > 20\,\mu\text{m}$, assuming that their orientation is essentially horizontal. In principle, the results presented here are based on non-dimensional relationships and should be valid also for rigid prolate bodies settling in liquids, in the same ranges of Reynolds tested here (from $Re \ll 1$ to $Re \simeq 300$). In geosciences, this could be of interest for the settling of sediments in lakes or oceans, for example.

A future line of work is to find theoretical and/or heuristic ways to extend our findings to the intermediate orientations and to obtain an expression of the instant settling speed for each possible attack angle. Then, this expression could be integrated on all attack angles (weighted by the PDF of the attack angle) to obtain the resulting average settling speed for a given particle depending on particle's shape and fluid's characteristics, and for all possible sizes of atmospheric aerosols. This shall be the main topic of a future work, which is currently under progress.

porosity

We have added the following piece of text to the manuscript:

"The present method could be easily extended to particles made of porous materials by considering that a particle made of a material of density ρ_m having porosity ϕ can be treated as a dense material with effective density $\rho_p = \rho_m\,(1-\phi)$."

Apart from the limitations mentioned by the Reviewer, we also include a short mention of the possible extension of the present method to other shapes:

"Other limitations of the present work include the assumption of prolate spheroidal shape for the dust particles. Taking into the fact that expressions comparable to Eqs. 8-9 exist for the case of oblate spheroids, we see no particular obstacles in generalizing the approach developed in Mallios et al. (2020) and in the present article to the case of prolate spheroids. The case of triaxial spheroids or other, more irregular shapes, is still out of reach with the methods developed here."

On behalf of the all the authors,

Sylvain Mailler

References

Goossens, W. R.: Review of the empirical correlations for the drag coefficient of rigid spheres, Powder Tech., 352, 350–359, https://doi.org/10.1016/j.powtec.2019.04.075, 2019.

Mailler, S., Menut, L., Cholakian, A., and Pennel, R.: AerSett v1.0, https://doi.org/10.5281/zenodo.7535115, 2023a.

Mailler, S., Menut, L., Cholakian, A., and Pennel, R.: AerSett v1.0: a simple and straightforward model for the settling speed of big spherical atmospheric aerosols, Geosci. Model Dev., 16, 1119–1127, https://doi.org/10.5194/gmd-16-1119-2023, 2023b.

Mallios, S. A., Drakaki, E., and Amiridis, V.: Effects of dust particle sphericity and orientation on their gravitational settling in the earth's atmosphere, J. Aerosol Sci., 150, 105 634, https://doi.org/10.1016/j.jaerosci.2020.105634, 2020.

Mallios, S. A., Daskalopoulou, V., and Amiridis, V.: Orientation of non spherical prolate dust particles moving vertically in the Earth's atmosphere, J. Aerosol Sci., 151, 105 657, https://doi.org/10.1016/j.jaerosci.2020.105657, 2021.

Menut, L., Cholakian, A., Pennel, R., Siour, G., Mailler, S., Valari, M., Lugon, L., and Meurdesoif, Y.: Chimere v2023r1, https://doi.org/10.14768/7aae366b-662c-4889-900f-2d2a6101ec35, 2023.

NOAA/NASA/USAF: U.S Standard Atmosphere 1976, Tech. Rep. NASA-TM-X-74335, NOAA-S/T-76-1562, NASA/NOAA, URL https://hdl.handle.net/11245/1.523366, 1976.

Answer to Reviewer comment RC2 on the manuscript "**New straightforward formulae for the settling speed of prolate spheroids in the atmosphere: theoretical background and implementation in AerSett v2.0.2.**"

(doi: 10.5194/egusphere-2023-2637)

May 2, 2024

We are grateful to Anonymous Reviewer 2 for their careful reading of our manuscript and their insightful questions and suggestions. Their comment seem to be written from a fluid mechanicist point of view, which makes it a particularly useful apport to this discussion, since the manuscript was prepared from an atmospheric physics point of view. We are particularly grateful to the Reviewer to bring to our attention that what we had called the "pseudo-Reynolds number" in the present study and in Mailler et al. (2023) is already known as the Archimedes number. This information will permit us to alleviate and clarify the redaction of our manuscript.

We feel that we have answered all of the Reviewer comments and concerns in the discussion and/or by additions in the revised version of the Manuscript. All the modifications brought to the manuscript following comments by the Reviewer have been highlighted in blue in the present document.

Contents

1 Transcript of the Reviewer Comment RC2

The authors revisit two published results (Mallios et al., 2020; Sanjeevi et al., 2022) for the drag coefficient on prolate spheroids in vertical and horizontal orientations settling in an incompressible Newtonian fluid. I would be happy to recommend the manuscript it the authors address my concerns.

1.1 Major

1.1.1 Use of dimensionless quantities

First of all, I am confused whether why the authors need to explicitly calculate the velocity if the drag coefficients are already calculated as functions of two governing dimensionless quantities: Reynolds numbers and aspect ratios. Usually in simulations, equations of motion are made dimensionless and these drag coefficients can then be directly used and there is usually no need of calculating velocities. If the authors can include a justification, that would be better for the readers. Usually in fluid dynamics literature, analyses and calculations are performed in dimensionless forms and as a result one can make use of the drag coefficients themselves and one does not need to worry about the dependence of nondimensional particle velocity and its variation with d_{eq}. The goal of making governing variables dimensionless is to encode information in a compact form which can then be easily used in calculations. I do not think such an effort to explicitly calculate the "steady state" particle velocity as a function of d_{eq} is needed in the first place if one solves dimensionless equations of motion.

1.1.2 Plot drag coefficients as functions of Re

Instead of presenting their results a functions of d_{eq}, I urge the authors to first plot the drag coefficients as functions of Re for different λ using the results of Mallios et al. (2020) and Sanjeevi et al. (2022) simultaneously in a single plot. This would clearly showcase the ranges of Re these results can respectively be used and the range for which they are consistent. This eliminates the need to worry about exact values of d_{eq}.

1.1.3 Values of ρ, ρ_p and μ. Applicability to liquids.

What are the values of ρ, ρ_p, and μ did the authors use for their calculations? Does ρ and μ correspond to values of air? I think these results should equally be valid even in the case when prolate particles are settling in liquids given their Re are in the range when the expressions given by Mallios et al. (2020) and Sanjeevi et al. (2022)are valid. Why do then the authors focus only on atmosphere?

1.1.4 Stokes number

In addition to Re and λ, in the case when $Re > 1$, [the] Stokes number also becomes important. The authors should comment on why they ignored it. They should also include a discussion addressing the time evolution of particle speed in the cases when particles have a finite Re.

1.1.5 Orientation of particles

It is well known that prolate particles (spheroidal particles in general see Ardekani et al. (2016)) rotate as they settle under gravity and attain a steady state orientation such that their broadside is horizontal if their R is lower than a critical value. At R above this critical value, they undergo orientation instability such that they do not have a steady a steady state orientation. In such a case, how useful are the assumptions about $\phi = 0$ and $\pi/2$?

1.1.6 Stratification of the atmosphere

It is well known that the earth's atmosphere has density and viscosity stratification (see Magnaudet and Mercier (2020); More and Ardekani (2023)) In such a case how useful are these calculations? Shouldn't one need to include effects of stratification and time dependence then?

1.1.7 clearly exclude liquid droplets

Aerosols can also mean liquid droplets suspended in air. The authors should clearly state that by aerosol they strictly focus on solid particles in air as in the case of liquid droplets, the surface boundary conditions are different and the calculations presented are not valid.

2 Minor

1. Eq 15 should have μ^2 in the denominator. This definition of R is equivalent to something called Archimedes number
2. Line 233: typo should be "expression"

3 Answers

3.1 Major

3.1.1 Use of dimensionless quantities

Comment "First of all, I am confused whether why the authors need to explicitly calculate the velocity if the drag coefficients are already calculated as functions of two governing dimensionless quantities: Reynolds numbers and aspect ratios. Usually in simulations, equations of motion are made dimensionless and these drag coefficients can then be directly used and there is usually no need of calculating velocities. If the authors can include a justification, that would be better for the readers."

Answer Classically, fluid mechanics give the expression of the drag coefficient as a function of Re. In dimensional quantities, this is equivalent to giving the force as a function of the speed, which is enough to solve the equation of motion for the particle. For spheres, several such formulations are discussed in Goossens (2019). For spheroids, Sanjeevi et al. (2022) gives the drag, lift and torque coefficients as a function of Re, of particle aspect ratio and particle orientation.

The reason why this approach is not satisfying for atmospheric science is that the Reynolds number is not known beforehand. Of course, it would be possible to numerically solve the equation of motion for the settling particle until its speed stabilizes, thereby obtaining its terminal fall speed. This would be very time-consuming for atmospheric science in which this calculation would have to be repeated in each model cell and for each possible particle diameter and density. Another, more tractable alternative, is to perform an iterative numerical resolution to calculate the speed as a function of the force. This boils down to the numerical resolution of a non-linear equation, which can be done by dichotomy or any other method, which also has a substantial computational cost.

Another alternative is, as we have done for spherical bodies in Mailler et al. (2023), to use dimensionless quantities to perform this numerical resolution once and for all, and find a direct, approximate expression for the Reynolds number as a function of the Archimedes number. We believe that this approach is particularly suitable for atmospheric sciences for the following reasons:

1. The known parameters of the problem are the size and shape of the particle, its density, and the thermodynamic properties of the carrying fluid (air).
2. What is unknown and needed is the settling speed of the particle, which is an important factor in determining its atmospheric advection and lifetime.
3. Due to the small size of the particles and their lack of inertia, their settling speed is reached almost instantly (compared to their atmospheric lifetime or to the time they need to move towards another atmospheric layer with substantially different characteristics)

We have discussed this problem in the revised version of the introduction by adding the following paragraph:

In the current work, we try to expand this formulation in the case of non spherical solid particles, focusing on prolate spheroids. As in Mailler et al. (2023), the point of this study is to obtain a direct and computationnally efficient method for the calculation of the settling speed as a function of known parameters (characteristics of the flow and of the particle). This problem is reciprocal of the classical problem in fluid mechanics (calculating the force as a function of the speed). In atmospheric science, the characteristics of the particle, including the gravity force it is submitted to, are known, while the settling speed is not known a priori, making this classical approach impractical for our problem.

Comment: Usually in fluid dynamics literature, analyses and calculations are performed in dimensionless forms and as a result one can make use of the drag coefficients themselves and one does not need to worry about the dependence of nondimensional particle velocity and its variation with d_{eq}. The goal of making governing variables dimensionless is to encode information in a compact form which can then be easily used in calculations.

Answer We agree on the use of non-dimensional variables to "encode information in a compact form". This is why in the present study for spheroids, as in Mailler et al. (2023) for sphere, the method we apply is to build a function giving a non-dimensional speed $\mathcal{S}$ (*a priori* unknown in our atmospheric science problem) as a function of the Archimedes number, known *a priori* in our problem of atmospheric physics (Eq. 31 in the submitted manuscript, for the Mallios et al. (2020) formulation).

Eq. 31 therefore "encodes information in a compact form" by giving a mathematical relationship between two non-dimensional quantities - the Archimedes number related to the force, and the $\mathcal{S}$ function related to the speed. This formulation results in Eq. 32 when dimensions are restored (or Eq. 33 if slip-correction is needed).

We feel that replacing the "pseudo-Reynolds number" by its correct name, the well-known Archimedes number, as suggested by the Reviewer, may help the reader to realize that our apprach is essentially based on finding relationships between non-dimensional quantities, as requested by the Reviewer.

Comment: I do not think such an effort to explicitly calculate the "steady state" particle velocity as a function of d_{eq} is needed in the first place if one solves dimensionless equations of motion.

Answer: As said above, we believe that the approach we describe above is minimizing the effort for atmospheric modellers, because we give an explicit formula for the needed quantity (settling velocity) as a function of known quantities (size, shape and density of the particle, thermodynamic properties of the fluid), **without** solving the equation of motion which, as said above, would be too tedious and costly for operational use. As an illustration of this computational efficiency, Table 2 shows that using the approach developed here for the calculation of the settling speed reduces the calculation time by about a factor 4 (for particles with $D > 10\,\mu\mathrm{m}$ for which a large-particle correction is needed). In the revised version, this is clarified by the following additionnal paragraph (already cited above):

In the current work, we try to expand this formulation in the case of non spherical solid particles, focusing on prolate spheroids. As in Mailler et al. (2023), the point of this study is to obtain a direct and computationnally efficient method for the calculation of the settling speed as a function of known parameters (characteristics of the flow and of the particle). This problem is reciprocal of the classical problem in fluid mechanics (calculating the force as a function of the speed). In atmospheric science, the characteristics of the particle, including the gravity force it is submitted to, are known, while the settling speed is not known a priori, making this classical approach impractical for our problem.

3.1.2 Plot drag coefficients as functions of Re

Comment: Instead of presenting their results a functions of d_{eq}, I urge the authors to first plot the drag coefficients as functions of Re for different λ using the results of Mallios et al. (2020) and Sanjeevi et al. (2022) simultaneously in a single plot. This would clearly showcase the ranges of Re these results can respectively be used and the range for which they are consistent. This eliminates the need to worry about exact values of d_{eq}.

Answer: We agree that such plots are a classical way to compare the two approaches in terms $C_D = f\left(Re\right)$ profiles.

We have produced such plots and included them in the revised version of the article (Figs. 2-3). These plots are discussed in Section 2.4 in the revised manuscript.

3.1.3 Values of ρ, ρ_p and μ. Applicability to liquids.

Comment: What are the values of ρ, ρ_p, and μ did the authors use for their calculations? Does ρ and μ

correspond to values of air?

Answer: Yes, as also noted by the other Reviewer, these important precisions were missing in the initial manuscript. The have been added in the revised version:

Both panels of Fig. 4 as well as all the subsequent figures in the study have been produced using standard atmospheric consitions for air ($p = 101325\,\mathrm{hPa}$ and $T = 298.15\,\mathrm{K}$). Dynamic viscosity μ has been calculated following the US Standard Atmosphere (NOAA/NASA/USAF, 1976):

$$\mu = \frac{\beta T^{\frac{3}{2}}}{T+S}, \tag{1}$$

where $\beta = 1.458 \times 10^{-6}\,\mathrm{kg\,s^{-1}\,m^{-1}\,K^{-\frac{1}{2}}}$ and $S = 110.4\,\mathrm{K}$. In these conditions of temperature and pressure and with the molar mass of dry air $M_a = 28.9644 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}$ (also from the US Standard Atmosphere), the density of air is $\rho = 1.18\,\mathrm{kg\,m^{-3}}$.

Comment: I think these results should equally be valid even in the case when prolate particles are settling in liquids given their Re are in the range when the expressions given by Mallios et al. (2020) and Sanjeevi et al. (2022)are valid. Why do then the authors focus only on atmosphere?

Answer: The reason we focus on atmosphere only is subjective, due to the fact that all co-authors work in institutes for atmospheric science. We are not necessarily aware of the possible specificities of other fields. In principle, we agree that the same principles and results should be applicable to particles settling in liquids. In particular, a possible field of application of our work in geophysics could be the settling of particles in lakes and oceans, where the physical problem to solve is comparable (estimate the settling velocity of a particle with known shape, size and density in water with known physical properties). For that, one would need to study the typical shape, size and density of oceanic particles to see what is the typical Reynolds number, and if our method applies. The following sentences have been added to the conclusions:

In principle, the results presented here are based on non-dimensional relationships and should be valid also for rigid prolate bodies settling in liquids, in the same ranges of Reynolds tested here (from $Re \ll 1$ to $Re \simeq 300$). In geosciences, this could be of interest for the settling of sediments in lakes or oceans, for example.

3.1.4 Stokes number

Comment In addition to Re and λ, in the case when $Re > 1$, [the] Stokes number also becomes important. The authors should comment on why they ignored it. They should also include a discussion addressing the time evolution of particle speed in the cases when particles have a finite Re.

Answer: The Stokes number characterizes a particle advected by a fluid flow. It can be defined as: 0

$$Stk = \frac{\tau \cdot U}{L}, \tag{2}$$

where τ is a characteristic response time for the particle speed, U a characteristic speed for the flow, and L a characteristic length of the flow. In our system though, it is difficult to define clearly the characteristic length or time for atmospheric flow, and therefore to give a clear expression of the Stokes number. Usually, in atmospheric science, it is considered that, except for the action of gravity, the response time of particles is sufficiently short so that they can be considered to follow passively the trajectories of the air parcels that carry them. Some relevant exceptions to this occur:

1. in the presence of obstacles or vegetation (*e.g.* Pleim et al. (2022)), in which case it is important to determine whether the atmospheric aerosol may be intercepted by vegetation in what is called “dry deposition”

2. in the presence of falling rain drops (*e.g.* Cherrier et al. (2017)), in which case it is important to determine whether the atmospheric aerosol may collide with a raindrop (“below-cloud scavenging”).

Other than that, the Stokes number is generally not relevant in the atmospheric science, due to the absence of obstacles in the atmospheric flow.

D (m)	τ (s)
10^{-5}	8.0×10^{-4}
10^{-4}	8.0×10^{-2}
4.5×10^{-4}	1.6

Table 1: Response time of the speed of particles as a function of their diameter for $\rho_p = 2650\,\mathrm{kg\,m^{-3}}$, in standard atmospheric conditions ($T = 298.15\,\mathrm{K}$, $p = 101325\,\mathrm{Pa}$)

However, the question of the Reviewer regarding whether we have to consider the time evolution of particle speed is relevant. For this, in the Stokes case and for a spherical particle, the response time of the particle is given by:

$$\tau = \frac{\rho_p D^2}{18\mu} \tag{3}$$

we can calculate τ as follows for a spherical particle with density $\rho_p = 2650\,\mathrm{kg\,m^{-3}}$ evolving in dry air in standard atmospheric conditions:

Clearly, all particles with diameter $D < 10^{-4}\,\mathrm{m}$ follow the flow with a lag that is very short relative to any relevant timescale for atmospheric motion (except the previously noted cases of interaction with falling raindrops or with vegetation). $D < 10^{-4}\,\mathrm{m}$ is already extremely large for an atmospheric particle, such particles are scarcely observed in the atmosphere. $4.5 \times 10^{-4}\,\mathrm{m}$ in Table 1 as the maximal value of diameter tested has been chosen because this is the size of the largest atmospheric particle collected by van der Does et al. (2018) in their *in situ* observations of giant dust particles. For such a giant particle, the response time of the particle may become non-negligible in some cases of strong turbulent motion. It is also to be noted that the calculation of τ done for Table 1 is done assuming a Stokes regime, but the biggest particles strongly deviate from the Stokes regime: the drag force and its derivative with speed become more reduced. Even taking that into account, the reaction time for these extremely large particles particles to adjust to the motion of the flow is at worst of a few seconds. This is short comparable to the typical time-scales of atmospheric motion, and also short compared to the time steps of GCMs or chemistry-transport models (typically, from one minute to a few minutes).

3.1.5 Orientation of particles

Comment: It is well known that prolate particles (spheroidal particles in general see Ardekani et al. (2016)) rotate as they settle under gravity and attain a steady state orientation such that their broadside is horizontal if their R is lower than a critical value. At R above this critical value, they undergo orientation instability such that they do not have a steady a steady state orientation. In such a case, how useful are the assumptions about $\phi = 0$ and $\pi/2$?

Answer: We agree on the observations made by the Reviewer in this comment and the fact that they limit the possible use of our results in their present form. However, results with $\phi = \pi/2$ can be directly used because, as said by the Reviewer and also shown in Mallios et al. (2021), the large prolate particles tend fall with this orientation. This aspect is, in part, already discussed in the manuscript, based on Mallios et al. (2021), which derives a probability distribution function for particle orientation depending on the physical parameters of the problem. For smaller particles, orientation becomes more evenly distributed (and finally random for the smallest ones), as discussed in Mallios et al. (2021).

We are currently working to derive expressions for the settling speed valid for any particular orientation, check them against the Sanjeevi et al. (2022) CFD simulations (which also give data for intermediate orientation), and integrate them over the PDFs of particle orientation given by Mallios et al. (2021). Only this ongoing work will hopefully answer completely and satisfactorily this Reviewer comment.

This limitation is now discussed in the Conclusion of the manuscript: A future line of work is to find theoretical and/or heuristic ways to extend our findings to the intermediate orientations and to obtain an expression of the instant settling speed for each possible attack angle. Then, this expression could be integrated on all attack angles (weighted by the PDF of the attack angle) to obtain the resulting average settling speed for a given particle depending on particle's shape and fluid's characteristics, and for all possible

T (K)	μ (Pa s)
273.15	1.72×10^{-5}
283.15	1.77×10^{-5}
298.15	1.84×10^{-5}

Table 2: Dynamic viscosity of air as a function of temperature

sizes of atmospheric aerosols. This shall be the main topic of a future work, which is currently under progress.

3.1.6 Stratification of the atmosphere

Comment: It is well known that the earth's atmosphere has density and viscosity stratification (see Magnaudet and Mercier (2020); More and Ardekani (2023)) In such a case how useful are these calculations? Shouldn't one need to include effects of stratification and time dependence then?

Answer: It is true and well-known that the atmosphere is stratified. Density and dynamic viscosity are the key atmospheric variables that affect the settling of particles. Density of air essentially follows an exponential decrease with altitude, with a scale height $H \simeq 8\times10^3$ m. This is an extremely smooth evolution.

Dynamic viscosity of air is a direct function of its temperature:

$$\mu = \frac{\beta T^{\frac{3}{2}}}{T + S}, \tag{4}$$

where $\beta = 1.458 \times 10^{-6}\,\mathrm{kg\,s^{-1}\,m^{-1}\,K^{-\frac{1}{2}}}$ and $S = 110.4\,\mathrm{K}$. The evolution of μ with temperature for selected T values is shown on Table 2, which clearly shows that even for strong variations of temperature (10 K), μ varies only by a couple of percent, so even if we suppose a very sharp stratification of the atmosphere where the temperature changes by 10 K over, say, 100 m, the resulting variation in dynamic viscosity will not be substantial: even in the worst case of an extremely big particle with $D = 4.5 \times 10^{-4}$ m, the response time of the particle speed will be a couple of seconds to adjust its settling speed to the environment (see Tab. 1). During this couple of seconds, with a settling speed $U \simeq 6.4\,\mathrm{m\,s^{-1}}$ obtained with our formulae, the particle will travel over, say, 50 meters, a distance over which the variation of air density is negligible, and that of μ will be, in the worst case, a couple of percent (Tab. 2). Of course, such a giant diameter is a worst-case only. Particles with $D \simeq 10^{-4}$ m typical of giant dust will have a speed $U \simeq 1.5\,\mathrm{m\,s^{-1}}$, and during its response time of about 0.1 s (see Table 1) it will travel a dozen of centimeters, a distance across which atmospheric temperature and density does not have relevant variations.

As a result, the vertical scales of atmospheric stratification leave more than enough time for settling particles to adjust rapidly their vertical speed to its steady-state value, even for the biggest atmospheric particles.

3.1.7 Clearly exclude liquid droplets

Comment: Aerosols can also mean liquid droplets suspended in air. The authors should clearly state that by aerosol they strictly focus on solid particles in air as in the case of liquid droplets, the surface boundary conditions are different and the calculations presented are not valid.

Answer: Many aerosols in the atmosphere are, indeed, liquid. While big hydrometeors can typically be deformed by their interaction with air, liquid aerosol particles tend to be spherical due to surface tension. Liquid particles with a prolate shape have no reason to exist in the atmosphere, and in the discussion of our results we essentially discuss solid particles of mineral dusts.

The restriction of this approach to solid aerosols has been mentioned in several places in the revised manuscript (Abstract, Introduction, Conclusion)

3.2 Minor

3.2.1 Archimedes number

Comment: Eq 15 should have μ^2 in the denominator. This definition of R is equivalent to something called

Archimedes number

Answer: Thank you for having identified this typo. Fortunately, in all the later occurences of this number, the denominator was correctly written with μ^2.

It is correct that the non-dimensional number that we had defined as “the Reynolds number of a sphere having the same volume as the prolate spheroid and obeying the Stokes law” (under the influence of gravity) and named “pseudo-Reynolds number” in Mailler et al. (2023) and in the initial manuscript, is none other than the *Archimedes number*. It is always helpful to name things as they should be named and we are extremely grateful to the Reviewer for permitting us to have this non-dimensional number correctly.

We use use “Archimedes number” throughout the revised manuscript, and note it Ar instead of R.

3.2.2 Typo line 233

Answer: Expresison has been corrected to expression

On behalf of the all the authors,

Sylvain Mailler

References

Ardekani, M. N., Costa, P., Breugem, W. P., and Brandt, L.: Numerical study of the sedimentation of spheroidal particles, Int. J. Multiphase Flow, 87, 16–34, https://doi.org/10.1016/j.ijmultiphaseflow.2016.08.005, 2016.

Cherrier, G., Belut, E., Gerardin, F., Tanière, A., and Rimbert, N.: Aerosol particles scavenging by a droplet: Microphysical modeling in the Greenfield gap, Atmos. Environ., 166, 519–530, https://doi.org/10.1016/j.atmosenv.2017.07.052, 2017.

Goossens, W. R.: Review of the empirical correlations for the drag coefficient of rigid spheres, Powder Tech., 352, 350–359, https://doi.org/10.1016/j.powtec.2019.04.075, 2019.

Magnaudet, J. and Mercier, M. J.: Particles, Drops, and Bubbles Moving Across Sharp Interfaces and Stratified Layers, Annu. Rev. Fluid Mech., 52, 61–91, https://doi.org/10.1146/annurev-fluid-010719-060139, 2020.

Mailler, S., Menut, L., Cholakian, A., and Pennel, R.: AerSett v1.0: a simple and straightforward model for the settling speed of big spherical atmospheric aerosols, Geosci. Model Dev., 16, 1119–1127, https://doi.org/10.5194/gmd-16-1119-2023, 2023.

Mallios, S. A., Drakaki, E., and Amiridis, V.: Effects of dust particle sphericity and orientation on their gravitational settling in the earth’s atmosphere, J. Aerosol Sci., 150, 105 634, https://doi.org/10.1016/j.jaerosci.2020.105634, 2020.

Mallios, S. A., Daskalopoulou, V., and Amiridis, V.: Orientation of non spherical prolate dust particles moving vertically in the Earth’s atmosphere, J. Aerosol Sci., 151, 105 657, https://doi.org/10.1016/j.jaerosci.2020.105657, 2021.

More, R. V. and Ardekani, A. M.: Motion in Stratified Fluids, Annu. Rev. Fluid Mech., 55, 157–192, https://doi.org/10.1146/annurev-fluid-120720-011132, 2023.

NOAA/NASA/USAF: U.S Standard Atmosphere 1976, Tech. Rep. NASA-TM-X-74335, NOAA-S/T-76-1562, NASA/NOAA, URL `https://hdl.handle.net/11245/1.523366`, 1976.

Pleim, J. E., Ran, L., Saylor, R. D., Willison, J., and Binkowski, F. S.: A New Aerosol Dry Deposition Model for Air Quality and Climate Modeling, J. Adv. Model. Earth Sy., 14, e2022MS003 050, https://doi.org/10.1029/2022MS003050, 2022.

Sanjeevi, S. K., Dietiker, J. F., and Padding, J. T.: Accurate hydrodynamic force and torque correlations for prolate spheroids from Stokes regime to high Reynolds numbers, Chemical Engineering Journal, 444, 136 325, https://doi.org/10.1016/j.cej.2022.136325, 2022.

van der Does, M., Knippertz, P., Zschenderlein, P., Giles Harrison, R., and Stuut, J.-B. W.: The mysterious long-range transport of giant mineral dust particles, Sci. Adv., https://doi.org/10.1126/sciadv.aau2768, 2018.