Answer to Reviewer comment RC2 on the manuscript "**New straightforward formulae for the settling speed of prolate spheroids in the atmosphere: theoretical background and implementation in AerSett v2.0.2.**"

(doi: 10.5194/egusphere-2023-2637)

March 22, 2024

We are grateful to Anonymous Reviewer 2 for their careful reading of our manuscript and their insightful questions and suggestions. Their comment seem to be written from a fluid mechanicist point of view, which makes it a particularly useful apport to this discussion, since the manuscript was prepared from an atmospheric physics point of view. We are particularly grateful to the Reviewer to bring to our attention that what we had called the "pseudo-Reynolds number" in the present study and in Mailler et al. (2023) is already known as the Archimedes number. This information will permit us to alleviate and clarify the redaction of our manuscript.

Contents

1 Transcript of the Reviewer Comment RC2

The authors revisit two published results (Mallios et al., 2020; Sanjeevi et al., 2022) for the drag coefficient on prolate spheroids in vertical and horizontal orientations settling in an incompressible Newtonian fluid. I would be happy to recommend the manuscript it the authors address my concerns.

1.1 Major

1.1.1 Use of dimensionless quantities

First of all, I am confused whether why the authors need to explicitly calculate the velocity if the drag coefficients are already calculated as functions of two governing dimensionless quantities: Reynolds numbers and aspect ratios. Usually in simulations, equations of motion are made dimensionless and these drag coefficients can then be directly used and there is usually no need of calculating velocities. If the authors can include a justification, that would be better for the readers. Usually in fluid dynamics literature, analyses and calculations are performed in dimensionless forms and as a result one can make use of the drag coefficients themselves and one does not need to worry about the dependence of nondimensional particle velocity and its variation with d_{eq}. The goal of making governing variables dimensionless is to encode information in a compact form which can then be easily used in calculations. I do not think such an effort to explicitly calculate the "steady state" particle velocity as a function of d_{eq} is needed in the first place if one solves dimensionless equations of motion.

1.1.2 Plot drag coefficients as functions of Re

Instead of presenting their results a functions of d_{eq}, I urge the authors to first plot the drag coefficients as functions of Re for different λ using the results of Mallios et al. (2020) and Sanjeevi et al. (2022) simultaneously in a single plot. This would clearly showcase the ranges of Re these results can respectively be used and the range for which they are consistent. This eliminates the need to worry about exact values of d_{eq}.

1.1.3 Values of ρ, ρ_p and μ. Applicability to liquids.

What are the values of ρ, ρ_p, and μ did the authors use for their calculations? Does ρ and μ correspond to values of air? I think these results should equally be valid even in the case when prolate particles are settling in liquids given their Re are in the range when the expressions given by Mallios et al. (2020) and Sanjeevi et al. (2022)are valid. Why do then the authors focus only on atmosphere?

1.1.4 Stokes number

In addition to Re and λ, in the case when $Re > 1$, [the] Stokes number also becomes important. The authors should comment on why they ignored it. They should also include a discussion addressing the time evolution of particle speed in the cases when particles have a finite Re.

1.1.5 Orientation of particles

It is well known that prolate particles (spheroidal particles in general see Ardekani et al. (2016)) rotate as they settle under gravity and attain a steady state orientation such that their broadside is horizontal if their R is lower than a critical value. At R above this critical value, they undergo orientation instability such that they do not have a steady a steady state orientation. In such a case, how useful are the assumptions about $\phi = 0$ and $\pi/2$?

1.1.6 Stratification of the atmosphere

It is well known that the earth's atmosphere has density and viscosity stratification (see Magnaudet and Mercier (2020); More and Ardekani (2023)) In such a case how useful are these calculations? Shouldn't one need to include effects of stratification and time dependence then?

1.1.7 clearly exclude liquid droplets

Aerosols can also mean liquid droplets suspended in air. The authors should clearly state that by aerosol they strictly focus on solid particles in air as in the case of liquid droplets, the surface boundary conditions are different and the calculations presented are not valid.

2 Minor

1. Eq 15 should have μ^2 in the denominator. This definition of R is equivalent to something called Archimedes number

2. Line 233: typo should be "expression"

3 Answers

3.1 Major

3.1.1 Use of dimensionless quantities

Comment "First of all, I am confused whether why the authors need to explicitly calculate the velocity if the drag coefficients are already calculated as functions of two governing dimensionless quantities: Reynolds numbers and aspect ratios. Usually in simulations, equations of motion are made dimensionless and these drag coefficients can then be directly used and there is usually no need of calculating velocities. If the authors can include a justification, that would be better for the readers."

Answer Classically, fluid mechanics give the expression of the drag coefficient as a function of Re. In dimensional quantities, this is equivalent to giving the force as a function of the speed, which is enough to solve the equation of motion for the particle. For spheres, several such formulations are discussed in Goossens (2019). For spheroids, Sanjeevi et al. (2022) gives the drag, lift and torque coefficients as a function of Re, of particle aspect ratio and particle orientation.

The reason why this approach is not satisfying for atmospheric science is that the Reynolds number is not known beforehand. Of course, it would be possible to numerically solve the equation of motion for the settling particle until its speed stabilizes, thereby obtaining its terminal fall speed. This would be very time-consuming for atmospheric science in which this calculation would have to be repeated in each model cell and for each possible particle diameter and density. Another, more tractable alternative, is to perform an iterative numerical resolution to calculate the speed as a function of the force. This boils down to the numerical resolution of a non-linear equation, which can be done by dichotomy or any other method, which also has a substantial computational cost.

Another alternative is, as we have done for spherical bodies in Mailler et al. (2023), to use dimensionless quantities to perform this numerical resolution once and for all, and find a direct, approximate expression for the Reynolds number as a function of the Archimedes number. We believe that this approach is particularly suitable for atmospheric sciences for the following reasons:

1. The known parameters of the problem are the size and shape of the particle, its density, and the thermodynamic properties of the carrying fluid (air).

2. What is unknown and needed is the settling speed of the particle, which is an important factor in determining its atmospheric advection and lifetime.

3. Due to the small size of the particles and their lack of inertia, their settling speed is reached almost instantly (compared to their atmospheric lifetime or to the time they need to move towards another atmospheric layer with substantially different characteristics)

Comment: Usually in fluid dynamics literature, analyses and calculations are performed in dimensionless forms and as a result one can make use of the drag coefficients themselves and one does not need to worry about the dependence of nondimensional particle velocity and its variation with d_{eq}. The goal of making governing variables dimensionless is to encode information in a compact form which can then be easily used in calculations.

Answer We agree on the use of non-dimensional variables to "encode information in a compact form". This is why in the present study for spheroids, as in Mailler et al. (2023) for sphere, the method we apply is to build a function giving a non-dimensional speed $\mathcal{S}$ (*a priori* unknown in our atmospheric science problem) as a function of the Archimedes number, known *a priori* in our problem of atmospheric physics (Eq. 31 in the submitted manuscript, for the Mallios et al. (2020) formulation).

Eq. 31 therefore "encodes information in a compact form", and results in Eq. 32 when dimensions are restored (or Eq. 33 if slip-correction is needed).

Comment: I do not think such an effort to explicitly calculate the "steady state" particle velocity as a function of d_{eq} is needed in the first place if one solves dimensionless equations of motion.

Answer: As said above, we believe that the approach we describe above is minimizing the effort for atmospheric modellers, because we give an explicit formula for the needed quantity (settling velocity) as a function of known quantities (size, shape and density of the particle, thermodynamic properties of the fluid), **without** solving the equation of motion which, as said above, would be too tedious and costly for operational use.

3.1.2 Plot drag coefficients as functions of Re

Comment: Instead of presenting their results a functions of d_{eq}, I urge the authors to first plot the drag coefficients as functions of Re for different λ using the results of Mallios et al. (2020) and Sanjeevi et al. (2022) simultaneously in a single plot. This would clearly showcase the ranges of Re these results can respectively be used and the range for which they are consistent. This eliminates the need to worry about exact values of d_{eq}.

Answer: We agree that such plots would be a more classical way to compare the two approaches in terms $C_D = f(Re)$ profiles. We produced such plots (Figs. 2-1) and will include them and discuss them in the revised version.

Fig. 1 shows that the agreement between both formulations is excellent for small particles (Re=1) , and that a good agreement persists even for big particles in the horizontal orientation (up to Re=300), but for the vertical orientation substantial disagreement arises between the two formulations, with Mallios et al. (2020) giving stronger drag coefficients than Sanjeevi et al. (2022) for this orientation. This is consistent with Fig. 6 in the manuscript, which shows strong differences between both approaches for the vertical orientation and the largest particles (strong Reynolds number). As already noted in the manuscript, the substantial discrepancies between Sanjeevi et al. (2022) and Mallios et al. (2020) in the case of large particles oriented vertically are not a problem because, as mentioned already in the manuscript and by the Reviewer, large particles (with strong Reynolds number) tend to fall with a horizontal orientation.

Figure 2 (which represents $\frac{1}{C_D}$ instead of C_D to avoid masking the substantial differences at high Re) yields the same conclusion, with the additional result that for spherical particles ($\lambda = 1$) the formulae of Mallios et al. (2020) and Sanjeevi et al. (2022) give results that are extremely consistent throughout the range that is relevant for atmospheric aerosol.

3.1.3 Values of ρ, ρ_p and μ. Applicability to liquids.

Comment: What are the values of ρ, ρ_p, and μ did the authors use for their calculations? Does ρ and μ correspond to values of air?

Answer: Yes, as also noted by the other Reviewer, these important precisions are missing. The values of ρ and μ are that of dry air in standard atmospheric conditions ($p = 101325\,\mathrm{Pa}$, $T = 298.15\,\mathrm{K}$). ρ and μ are calculated from these conditions using the standard preconisations from NOAA/NASA/USAF (1976).

This will be clearly explained in the revised version.

Comment: I think these results should equally be valid even in the case when prolate particles are settling in liquids given their Re are in the range when the expressions given by Mallios et al. (2020) and Sanjeevi et al. (2022)are valid. Why do then the authors focus only on atmosphere?

Answer: The reason we focus on atmosphere only is subjective, due to the fact that all co-authors work in institutes for atmospheric science. We are not necessarily aware of the possible specificities of other fields. In principle, we agree that the same principles and results should be applicable to particles settling in liquids. In particular, a possible field of application of our work in geophysics could be the settling of particles in lakes and oceans, where the physical problem to solve is comparable (estimate the settling velocity of a particle with known shape, size and density in water with known physical properties). For that, one would need to study the typical shape, size and density of oceanic particles to see what is the typical Reynolds number, and if our method applies. Nevertheless, we will mention this possibility in the Conclusion section of the revised manuscript.

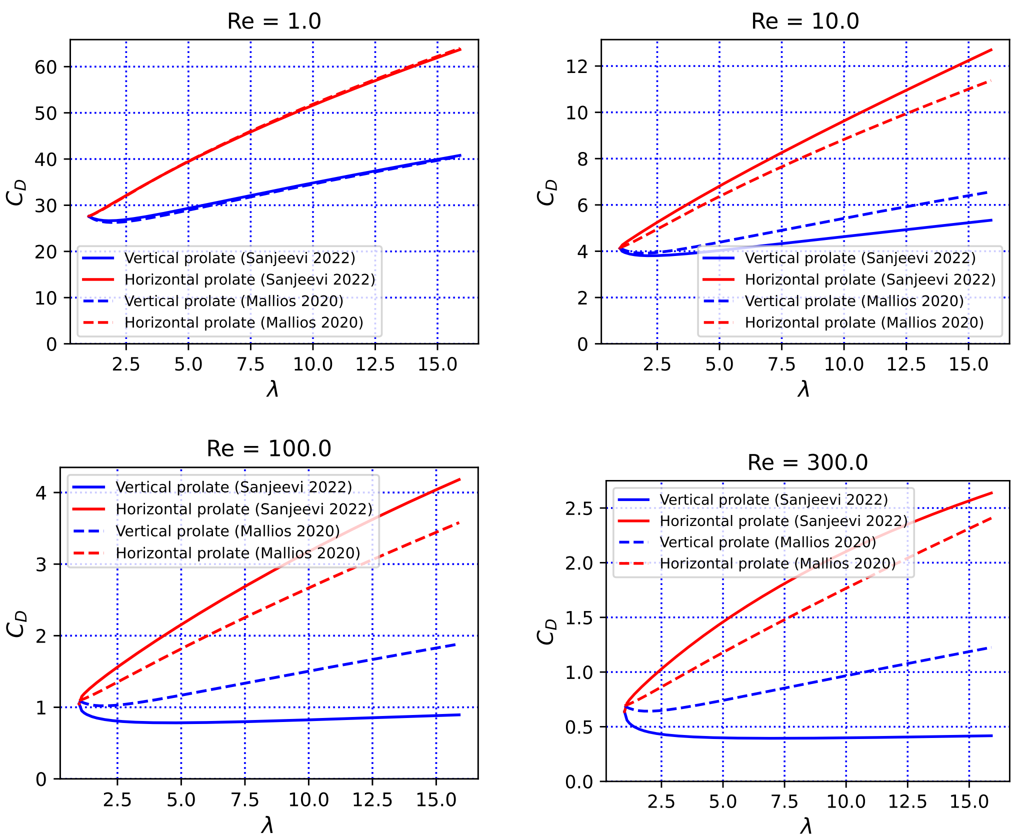

Figure 1: C_D as a function of λ for 4 selected values of Re

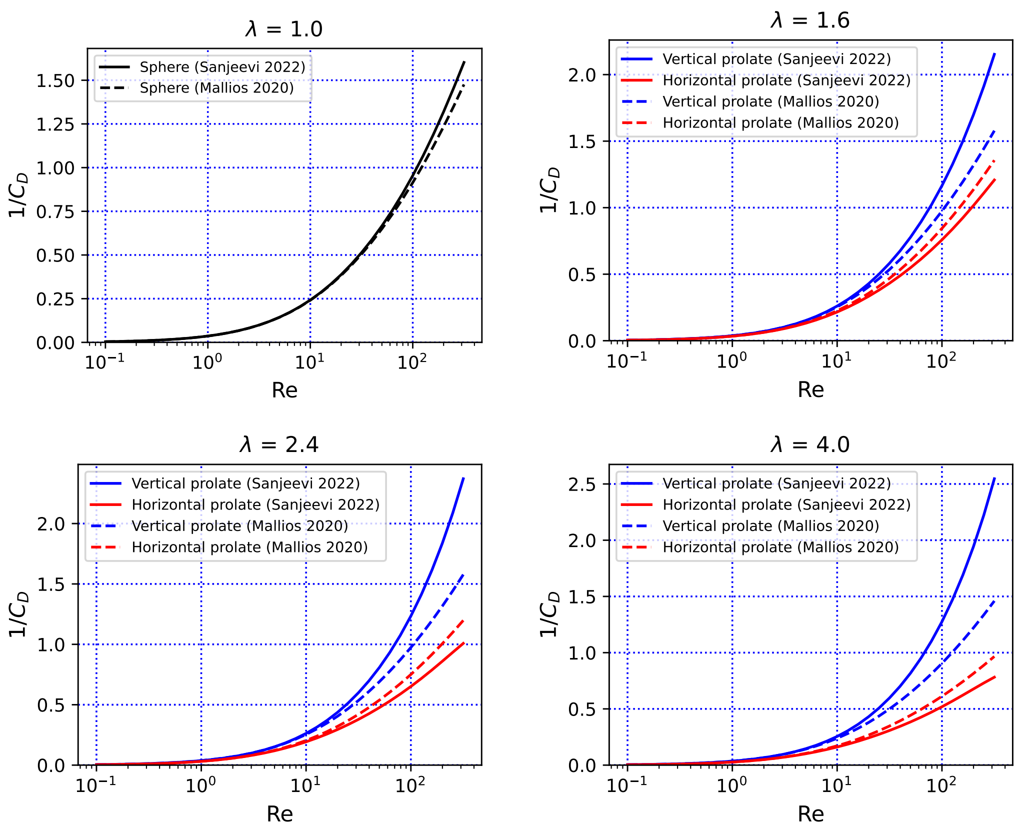

Figure 2: C_D as a function of Re for 4 selected values of λ

D (m)	τ (s)
10^{-5}	8.0×10^{-4}
10^{-4}	8.0×10^{-2}
4.5×10^{-4}	1.6

Table 1: Response time of the speed of particles as a function of their diameter for $\rho_p = 2650\,\mathrm{kg\,m^{-3}}$, in standard atmospheric conditions ($T = 298.15\,\mathrm{K}$, $p = 101325\,\mathrm{Pa}$)

3.1.4 Stokes number

Comment In addition to Re and λ, in the case when $Re > 1$, [the] Stokes number also becomes important. The authors should comment on why they ignored it. They should also include a discussion addressing the time evolution of particle speed in the cases when particles have a finite Re.

Answer: The Stokes number characterizes a particle advected by a fluid flow. It can be defined as:

$$Stk = \frac{\tau \cdot U}{L}, \tag{1}$$

where τ is a characteristic response time for the particle speed, U a characteristic speed for the flow, and L a characteristic length of the flow. In our system though, it is difficult to define clearly the characteristic length or time for atmospheric flow, and therefore to give a clear expression of the Stokes number. Usually, in atmospheric science, it is considered that, except for the action of gravity, the response time of particles is sufficiently short so that they can be considered to follow passively the trajectories of the air parcels that carry them. Some relevant exceptions to this occur:

1. in the presence of obstacles or vegetation (*e.g.* Pleim et al. (2022)), in which case it is important to determine whether the atmospheric aerosol may be intercepted by vegetation in what is called "dry deposition"
2. in the presence of falling rain drops (*e.g.* Cherrier et al. (2017)), in which case it is important to determine whether the atmospheric aerosol may collide with a raindrop ("below-cloud scavenging").

Other than that, the Stokes number is generally not relevant in the atmospheric science, due to the absence of obstacles in the atmospheric flow.

However, the question of the Reviewer of the Reviewer regarding whether we have to consider the time evolution of particle speed is relevant. For this, in the Stokes case and for a spherical particle, the response time of the particle is given by:

$$\tau = \frac{\rho_p D^2}{18\mu} \tag{2}$$

we can calculate τ as follows for a spherical particle with density $\rho_p = 2650\,\mathrm{kg\,m^{-3}}$ evolving in dry air in standard atmospheric conditions:

Clearly, all particles with diameter $D < 10^{-4}\,\mathrm{m}$ follow the flow with a lag that is very short relative to any relevant timescale for atmospheric motion (except the previously noted cases of interaction with falling raindrops or with vegetation). $D < 10^{-4}\,\mathrm{m}$ is already extremely large for an atmospheric particle, such particles are scarcely observed in the atmosphere. $4.5 \times 10^{-4}\,\mathrm{m}$ in Table 1 as the maximal value of diameter tested has been chosen because this is the size of the largest atmospheric particle collected by van der Does et al. (2018) in their *in situ* observations of giant dust particles. For such a giant particle, the response time of the particle may become non-negligible in some cases of strong turbulent motion. It is also to be noted that the calculation of τ done for Table 1 is done assuming a Stokes regime, but the biggest particles strongly deviate from the Stokes regime: the drag force and its derivative with speed become more reduced. Even taking that into account, the reaction time for these extremely large particles particles to adjust to the motion of the flow is at worst of a few seconds. This is short comparable to the typical time-scales of atmospheric motion, and also short compared to the time steps of GCMs or chemistry-transport models (typically, from one minute to a few minutes).

T (K)	μ (Pa s)
273.15	1.72×10^{-5}
283.15	1.77×10^{-5}
298.15	1.84×10^{-5}

Table 2: Dynamic viscosity of air as a function of temperature

3.1.5 Orientation of particles

Comment: It is well known that prolate particles (spheroidal particles in general see Ardekani et al. (2016)) rotate as they settle under gravity and attain a steady state orientation such that their broadside is horizontal if their R is lower than a critical value. At R above this critical value, they undergo orientation instability such that they do not have a steady a steady state orientation. In such a case, how useful are the assumptions about $\phi = 0$ and $\pi/2$?

Answer: We agree on the observations made by the Reviewer in this comment and the fact that they limit the possible use of our results in their present form. However, results with $\phi = \pi/2$ can be directly used because, as said by the Reviewer and also shown quantitatively in Mallios et al. (2021), the large prolate particles tend to fall with this orientation. This aspect is, in part, already discussed in the manuscript, based on Mallios et al. (2021), which derives a probability distribution function for particle orientation depending on the physical parameters of the problem. For smaller particles, orientation becomes more evenly distributed (and finally random for the smallest ones), as discussed in Mallios et al. (2021).

We are currently working to derive expressions for the settling speed valid for any particular orientation, check them against the Sanjeevi et al. (2022) CFD simulations (which also give data for intermediate orientation), and integrate them over the PDFs of particle orientation given by Mallios et al. (2021). Only this ongoing work will hopefully answer completely and satisfactorily this Reviewer comment.

3.1.6 Stratification of the atmosphere

Comment: It is well known that the earth's atmosphere has density and viscosity stratification (see Magnaudet and Mercier (2020); More and Ardekani (2023)) In such a case how useful are these calculations? Shouldn't one need to include effects of stratification and time dependence then?

Answer: It is true and well-known that the atmosphere is stratified. Density and dynamic viscosity are the key atmospheric variables that affect the settling of particles. Density of air essentially follows an exponential decrease with altitude, with a scale height $H \simeq 8\times10^3$ m. This is an extremely smooth evolution.

Dynamic viscosity of air is a direct function of its temperature:

$$\mu = \frac{\beta T^{\frac{3}{2}}}{T+S}, \tag{3}$$

where $\beta = 1.458 \times 10^{-6}\,\mathrm{kg\,s^{-1}\,m^{-1}\,K^{-\frac{1}{2}}}$ and $S = 110.4\,\mathrm{K}$. The evolution of μ with temperature for selected T values is shown on Table 2, which clearly shows that even for strong variations of temperature (10 K), μ varies only by a couple of percent, so even if we suppose a very sharp stratification of the atmosphere where the temperature changes by 10 K over, say, 100 m, the resulting variation in dynamic viscosity will not be substantial: even in the worst case of an extremely big particle with $D = 4.5 \times 10^{-4}$ m, the response time of the particle speed will be a couple of seconds to adjust its settling speed to the environment (see Tab. 1). During this couple of seconds, with a settling speed $U \simeq 6.4\,\mathrm{m\,s^{-1}}$ obtained with our formulae, the particle will travel over, say, 50 meters, a distance over which the variation of air density is negligible, and that of μ will be, in the worst case, a couple of percent (Tab. 2). Of course, such a giant diameter is a worst-case only. Particles with $D \simeq 10^{-4}$ m typical of giant dust will have a speed $U \simeq 1.5\,\mathrm{m\,s^{-1}}$, and during its response time of about 0.1 s (see Table 1) it will travel a dozen of centimeters, a distance across which atmospheric temperature and density does not have relevant variations.

As a result, the vertical scales of atmospheric stratification leave more than enough time for settling particles to adjust rapidly their vertical speed to its steady-state value, even for the biggest atmospheric particles.

3.1.7 Clearly exclude liquid droplets

Comment: Aerosols can also mean liquid droplets suspended in air. The authors should clearly state that by aerosol they strictly focus on solid particles in air as in the case of liquid droplets, the surface boundary conditions are different and the calculations presented are not valid.

Answer: Many aerosols in the atmosphere are, indeed, liquid. While big hydrometeors can typically be deformed by their interaction with air, liquid aerosol particles tend to be spherical due to surface tension. Liquid particles with a prolate shape have no reason to exist in the atmosphere, and in the discussion of our results we essentially discuss solid particles of mineral dusts. But for the sake of clarity, we will mention clearly that our study is meant only for solid particles.

3.2 Minor

3.2.1 Archimedes number

Comment: Eq 15 should have μ^2 in the denominator. This definition of R is equivalent to something called Archimedes number

Answer: Thank you for having identified this typo. Fortunately, in all the later occurences of this number, the denominator was correctly written with μ^2.

It is correct that the non-dimensional number that we had defined as "the Reynolds number of a sphere having the same volume as the prolate spheroid and obeying the Stokes law" (under the influence of gravity) and named "pseudo-Reynolds number" in Mailler et al. (2023) and in the present manuscript, is none other than the *Archimedes number*. It is always helpful to name things as they should be named and we are extremely grateful to the Reviewer for permitting us to have this non-dimensional number correctly. Consistently, we will use "Archimedes number" throughout the manuscript, and note it Ar instead of R.

3.2.2 Typo line 233

Answer: the typo will be corrected in the revised version.

On behalf of the all the authors,

Sylvain Mailler

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