

Review of “Deriving cloud droplet number concentration from surface based remote sensors with an emphasis on lidar measurements” by Gerald Mace

The manuscript presents an original and ingenious approach that uses a combination of common ground-based remote-sensing instruments (ceilometer, cloud radar, and microwave radiometer) for inferring the effective radius (r_e) and number concentration (N_d) of cloud droplets in geometrically thin low-level liquid water clouds over the Southern Ocean. The new method is centred around what’s usually a nuisance to lidar operators: the full attenuation of the laser beam in optically thick clouds. The author expands on earlier work to establish a straightforward physical connection between the distance between cloud base to full attenuation of the lidar signal and N_d (and thus r_e). The new framework requires a few assumptions as well as auxiliary cloud information from measurements with cloud radar and microwave radiometer.

The author finds that the application of the new method is strongly constrained by the range-resolution of common ceilometers and lidars, as this determines the accuracy to which the distance between cloud base to full signal attenuation can be determined. The authors hence developed an optimal estimation retrieval based on the novel framework to incorporate more physically meaningful information from the other remote-sensing instruments into the analysis.

The results are then evaluated through a detailed case study that employs independent airborne in-situ measurements and satellite observations as well as through comparison to N_d and r_e from MODIS observations.

The work is an excellent fit for AMT. However, I believe that major revisions are needed before the work can be published as outlined in my comments below.

I want to disclose that I had some editor-mediated communication with the author to clarify the derivation of Eq. (9). While some of the equations in the manuscript are erroneous, the author has confirmed that their implementation in the analysis routine is correct. I could now follow the presented theory. However, there still seem to be some typos that I will point out below as well.

Major comments:

- **Structure:** I suggest to restructure the manuscript to make it easier for readers to follow the process of development, application, and evaluation. For instance, I suggest to introduce all considered measurements as well as the data comparison strategy before the new method is described. At least for me, knowing what data will or might be used really supports the thought process. It also enables a much more straightforward presentation of the findings later in the manuscript. Here is a potential structure:
 - 1 Introduction
 - 2 Data and methods
 - 2.1 Instruments and comparison approach
 - 2.2 Theory
 - 2.3 An optical estimation algorithm
 - 2.4 OE evaluation
 3. Results and discussion
 - 3.1 Detailed case study
 - 3.2 Long-term comparison
 4. Summary and conclusion.

- **Variables:** I advise the re-evaluate the choice of variable names and to check for their consistency throughout the manuscript.
 - The lidar equation uses both z and r for distance. z might be confused with Z , r might be confused with droplet radius. I find the potential for a mix-up between r_e and r_{\max} (z_{\max} is also used) particularly problematic. Possible solutions include a list of symbols (which might be too extensive for this work) or the use of different signs for distance or r_{\max} .
 - The author switches between different names and signs, particularly between the text and the figures. For instance, liquid water content is both LWC and q . The cloud droplet number concentration is N_d , N_d , or cloud droplet number. The cloud droplet effective radius is r_e , r_e , Re , or effective radius. I suggest to introduce the formula signs and stick to them throughout the manuscript and the figures. There is no need to re-introduce them in the summary.
- **Figures:** Rather than using figure and panel titles, I suggest to provide a full description of a figure in the figure caption. This includes what's shown in the different panels (and in which colour), the time and location of measurements, and the source if measurements are shown.
- **Derivation of Eq. (9):** It would be easier to outline the derivation of Eq. (9) if each of the equations in Eq. (2) and (3) had their own number. I can only arrive at the authors derivation if the following equations are corrected. Eq. (2) should have a denominator of 2:

$$\sigma = \frac{\pi}{2} N_0 D_0^3 \Gamma(\alpha + 3).$$

Eq. (A1), which is a re-arranged version of Eq. (2.4) with Eq. (2.1) substituted for N_0 , should have only the second power of D_0 :

$$\sigma = \frac{\pi}{2} \frac{N_d}{\Gamma(\alpha + 1)} D_0^2 \Gamma(\alpha + 3).$$

The combination of the equation for D_0 and q (Eq. (2.2)), which has no number in the Appendix should feature the term $\Gamma(\alpha + 4)$ from the expression for q :

$$D_0 = \frac{3 q \Gamma(\alpha + 3)}{\rho \sigma \Gamma(\alpha + 4)}.$$

This leads to a corrected equation for B in Eq. (A2):

$$B = \left(\frac{9\pi}{2\rho^2} \frac{\Gamma(\alpha + 3)^3}{\Gamma(\alpha + 4)^2 \Gamma(\alpha + 1)} \right)^{\frac{1}{3}}.$$

Eq. (6) should feature σ rather than r in the second term on the right side:

$$\frac{\partial \ln \beta_{\text{obs}}}{\partial r} = \frac{\partial \ln \sigma}{\partial r} - 2\eta\sigma.$$

The B in Eq. (A4) should not be to the power of 3, while the entire Eqs. (7,8) should be as already stated in the clarification. I might also be wrong so please double-check.

- **Measurement uncertainty:** I am trying to wrap my head around the reasoning in the lidar range-gate spacing and the subsequent discussion of Figure 3. I understand that finer range resolution allows for a better quantification of r_{\max} as it provides to better resolve where exactly the lidar signal becomes saturated. If I have a common range bin of 15 m and my nominal height is at the bin centre, wouldn't it mean that my range uncertainty is 7.5 m rather than the full 15 m? I wonder if measurements capabilities are actually a factor 2 better than currently accounted for?
After the discussion of Figure 3, it is concluded that the r_{\max} as inferable from the lidar measurements is insufficient to retrieve N_d and r_e and that the use of additional information as in the optimal inversion algorithm could compensate for this lack. Later, all presented results are inferred by the OE algorithm. I wonder what the results would look like if the author had used the lidar-derived r_{\max} . Could you please comment or provide an example?
- **Verification:** I suggest to restructure the verification section of the paper. The airborne in-situ measurements allow for a more direct comparison than the MODIS observations. I would therefore discuss those (they would already be introduced in Section 2) first in the detailed case study (currently lines 374 - 386), continue with the addition of MODIS observations during the case study period (currently lines 358 - 373), and conclude with the comparison to the other coinciding MODIS overpasses (currently lines 387 onward).
- **Bigger picture:** I find this to be an excellent paper that could be even better if the author could sort the new method into the bigger picture of cloud remote sensing. You outline what kind of measurements are needed. Maybe you could also provide a check list on ranges of cloud properties for which you would assume the method to be valid? Is it just shallow liquid water clouds that don't precipitate too heavily? Are there other regions or established measurement sites for which the method could be applied? Would such an application require a revision of the OE algorithm? Which knowledge gaps could be closed by a widespread application of the new method?

Specific comments:

- line 174: please give all normalised standard deviations in either percent or without units
- line 252: to wit?
- equation for K_b : second row, third column should be $\partial\sigma/\partial\eta$
- line 275: I suggest to start the section on the evaluation of the OE algorithm here as the text before is still on the functioning of the algorithm
- Tables 2 and 3: Is it possible to combine those table, maybe in a transposed form? I also suggest to include cases 2, 4, and 6 in Table 2 as this is what's stated in the text. Please also check the referencing between the tables if kept separated.
- line 194: not clear how this is shown in Table 2
- Figure 6: Maybe show plots for radar and lidar to 2 km height only?
- Figure 7 and in the text: integers should suffice for N_d

- line 397: please quantify what is meant with good correlation
- Table 4 could be moved to the appendix
- Figure 9: It's hard to be certain but I have the impression that there are more points in the plots than overpasses listed in Table 4...?