Reviewer 1 : Comments and Answers

In this study, a coupled ocean-sea ice 3Dvar system is extended to include the assimilation of sea ice concentration and sea ice thickness. OSISAF sea ice concentrations and sources of sea ice thickness from CryoSat-2 and SMOS are assimilated in various configurations. The assimilation of OSISAF sic data alongside L4 C2SMOS SIT data with a Desroziers' OE factor of 1 performs best in comparison to both assimilated and independent moorings.

Overall I believe this to be a good paper with strong scientific basis and quality, particularly with strong implementation of a robust data assimilation scheme and good statistical assessment of the results. The plots show the results well. My main concern is that a bit more validation would improve the impact of the paper strongly alongside some discussion of the results.

We thank the Reviewer and we do agree that a second independent validation can improve the quality of the manuscript. Following the recommendation of both Reviewers we enriched the text adding a new validation dataset and discussed the corresponding metrics. The abstract is also reworded and corrected as suggested.

General comments:

In your analysis of the validation against BGEP ULS moorings you also mention the RMSE and BIAS but do not show these results which would be useful to see. I think it would be useful to extend the validation to also look at Operation IceBridge data, which also covers the time period of your experiments, and has a higher spatial coverage than the BGEP ULS moorings do. Although the data only cover March and April it would be useful to show a comparison to them with, for example, BIAS, RMSE or scatter plots of the different runs against the OIB data available during your experimental time period.

Operation IceBridge measurements are now used as a second independent validation dataset and a new section is added called "Validation against Operation IceBridge data ". Specifically we use the IDCSI4 dataset version 1 (http://nsidc.org/data/idcsi4; Kurtz et al., 2015) extracting data from 2011 to 2013 (no data available in 2014). Results are summarized in Figures S1 and S2 below, containing the SIT spatial RMSE and BIAS respectively for different experiments and binned in 2°x2° boxes. Metrics are calculated aggregating statistics from late-March and April (the only months available in the datasets for those years). Such dataset covers several days after the end of the dissemination of satellite data therefore the differences in the SIT distributions among experiments largely depends on the diverse initial conditions from mid-March. Figures S1 and S2 confirm the conclusions discussed with the analysis of BGEP ULS data and extend them to a broader region. Winter assimilation of SIT data (panels d-h) produces a smaller RMSE in March-April statistics compared to SIC-only (panel b-c) or CTRL experiment (panel a). A spatial dipole structure for BIAS (observation minus model) is generally seen in all the experiments with an overestimation of thickness in the Beaufort Gyre and an underestimation in the Atlantic sector. L4DE1 experiment (assimilation of CS2SMOS with Desroziers' error) shows the lowest RMSE and reduces the regional BIAS almost everywhere. SICDE02 (assimilation of SIC with reduced observation error) shows the worst skill in term of regional RMSE and BIAS. However negative/positive BIASES seem to compensate each other leading to a low global BIAS (spatially averaged). This demonstrates that such indicator is not always representative of the actual skill of the model. Subsampling the data (L4SUB, panel h) or increasing the observation errors (L4DE30, panel g) still provide positive feedback in April. The distribution of observation errors as provided by Operation IceBridge is also shown in panel i) for comparison.

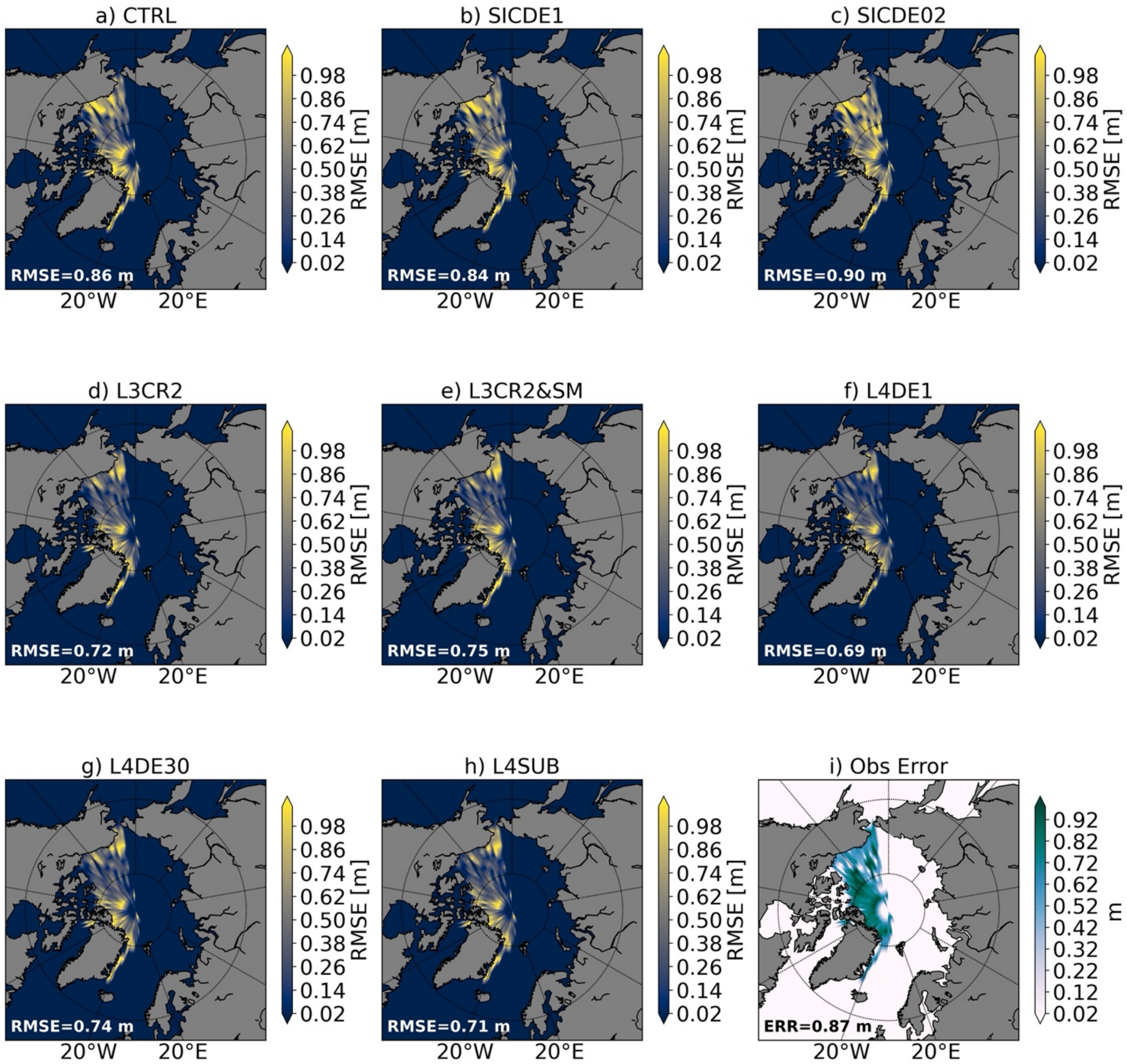

Fig S1. (Panel a-h) Spatial SIT RMSE for different experiments against Operation IceBridge data (available in March-April only) binned with 2°x2° boxes. The spatially averaged RMSE is shown in the picture. Panel i) shows the aggregated IceBridge data together with the spatially averaged observation error associated with the measurements.

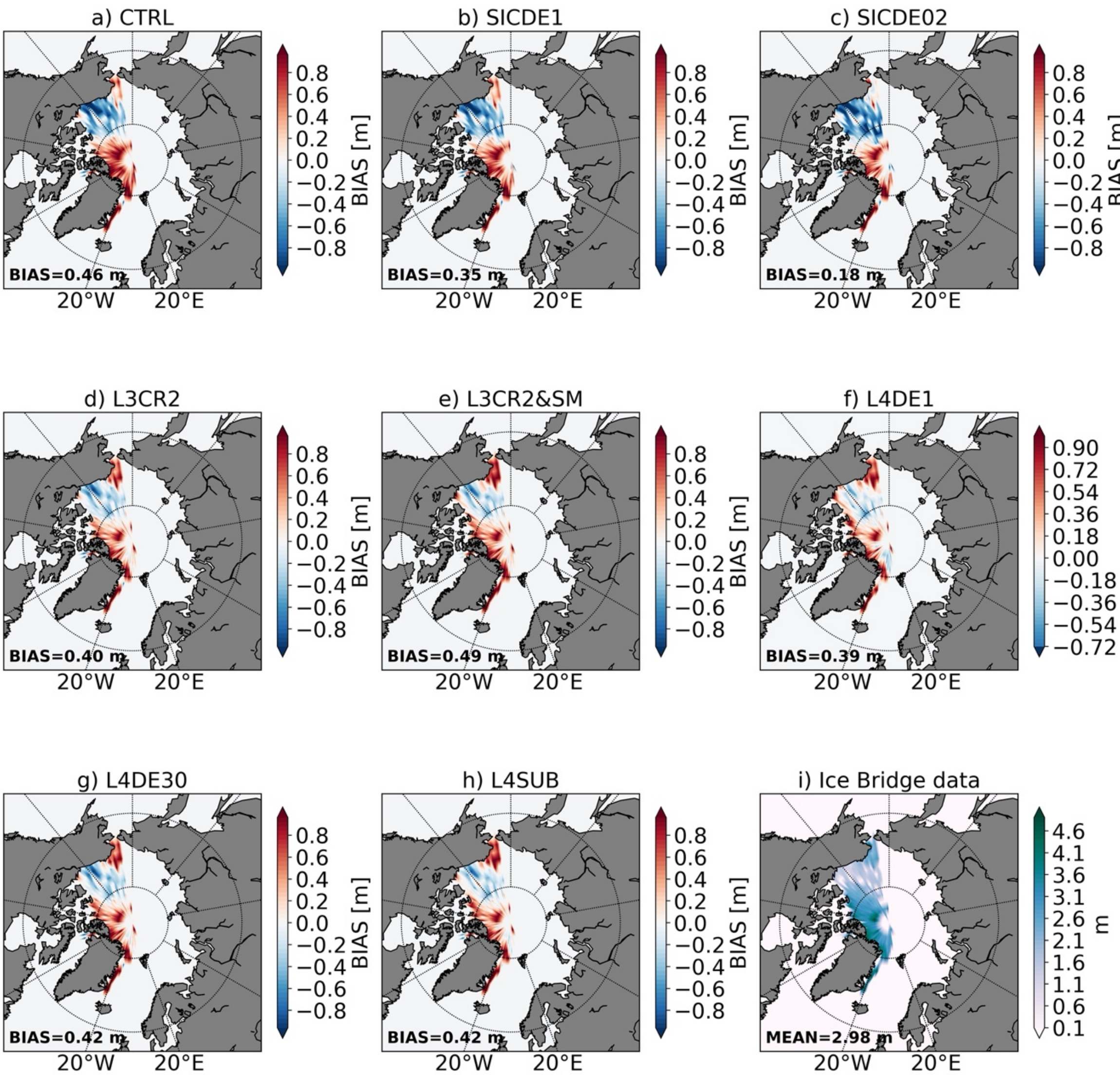

Fig S2. (Panel a-h) Spatial SIT BIAS for different experiments against Operation IceBridge data (available in March-April only) binned with 2°x2° boxes. The BIAS is calculated observation minus model. The spatially averaged BIAS is shown in the picture. Panel i) shows the aggregated IceBridge data together with the spatially averaged mean value associated with the measurements.

In the BGEP ULS section, the RMSE and BIAS were computed averaging over a time period. Figures 10 and 11 show the daily timeseries for BGEP ULS data (as downloaded) and compare them to the model values in the three different points. For each day there is a single value for observation and model. Aggregating the three different moorings and calculating the daily RMSE and BIAS timeseries based on three values is not so meaningful, this is why we provide in the text only RMSE and BIAS values averaged on time-period.

Kurtz, N., Studinger, M., Harbeck, J., DePaul Onana, V., and Yi, D.: IceBridge L4 Sea Ice Freeboard, Snow Depth, and Thickness, Version 1, https://doi.org/10.5067/G519SHCKWQV6, 2015.

The authors use a relatively simple 1 category sea ice model but achieve good results with the sea ice assimilation, it would be good to have a discussion of how using a more complex sea ice model might improve or change the results. Some discussion that compares your results to those seen in the other sea ice data assimilation studies you mention in the introduction would also be useful, as well as the reason for any differences or similarities.

Following the Reviewer's suggestion, we modified the Introduction (1) and model description (2):

(1) In the Introduction we extended the discussion about the differences of our configuration with respect to the ones in literature. Simply listing the values of the different metrics proposed by each paper (some of the papers analyse one season only) does not provide any interesting insight in our opinion. On the other hand, discussing in detail the results from different ocean/sea-ice models and scales (regional/global), different DA schemes and different atmospheric forcings would probably require a dedicated intercomparison paper by its own. For example, at a global scale, only univariate approaches are in place to our knowledge. Blockley et al, (2018), Mignac et al. (2022) focused on the assimilation of SIT or SIC and SIT data without cross-correlation terms between them. On the other hand, they use a multi-category sea-ice model and the improvement brought using several categories is difficult to discriminate. The impact of sole assimilation of SIC is present in other systems too, such as Zuo et. al. 2019 but the indirect impact on SIT is not yet shown. Regional systems consider multivariate correlations (Mu et al, 2018, Xie et al. 2018) extracted from ensemble approaches, different mono-category models, and atmospheric forcings/lateral boundaries whose impact on sea-ice evolution are crucial. Recently, new class of sea-ice models with different rheologies are also appearing with multivariate ensemble data assimilation capability such as Cheng et al., (2023) although it still adopts a standalone (uncoupled) sea-ice configuration. The text has changed as follow:

"Nowadays, the sole assimilation of the sea-ice concentration in a univariate fashion, is a well-established approach (Posey et al., 2015; Lemieux et al., 2016; Zuo et al., 2019). Preliminary studies on the addition of a second univariate assimilation scheme for thickness have come out only re- cently at global level. Blockley and Peterson (2018) and Mignac et al. (2022) showed the benefit of using of CryoSat-2 and later CryoSat-2/SMOS data to correct the Arctic thickness distribution, exploiting a variational approach within the FOAM system. They also point out the need for a better estimation of SIT observation errors. At regional scale, multivariate approaches were developed. Xie et al. (2016, 2018) confirm the benefits of the assimilation of CryoSat-2 and SMOS in the TOPAZ regional fore- cast system based on the Ensemble Kalman filter. The main correction comes from the use of CryoSat-2 data, the assimilation of SMOS reduces the error in the thin-ice of about 11 and 22% in March and in November respectively, without degradation in the other variables. Yang et al. (2014) and Mu et al. (2018b) tested the Localized Singular Evolutive Interpolated Kalman filter to integrate thickness data and showed an overall error that is similar to the PIOMAS system (Zhang and Rothrock, 2003) when compared to independent in-situ measurements. Finally, Cheng et al., (2023) has recently showed in a standalone Lagrangian sea-ice model, neXtSIM, interfaced to a deterministic EnKF scheme in a multivariate manner that improvements in SIT estimates indicate the importance of assimilating weekly CS2SMOS SIT while the improvements of SIC and ice extent are moderate but benefit from daily ingestion of the OSI-SAF SIC."

(2) In the model section we discussed the possible improvements brought by a multicategory model: “The use of a multi-category sea-ice model is foreseen in the next future, providing a more complex representation of the sea-ice interaction with the other components of the earth system. The Ice Thickness Distribution scheme (ITD, Thorndike et al. 1975) accounts for the sub-grid (unresolved) physics in a statistical sense: internal/external thermodynamic/mechanic processes can change the total thickness as well as its distribution and therefore can be only partially parametrized by simpler mono-category sea-ice model. On the other hand, the practical discretization of such categories as well as their number should be properly tuned to contain of the computational costs and still provide benefits with respect the mono-category models. In Uotilia et al (2017), the Authors compared a set of simulations performed with a multi and a mono-category sea-ice models: LIM3 and LIM2 respectively. They showed that the decline of Arctic sea-ice extent in the last decade as well as Antarctic seasonal variability are better reproduced with LIM3. However, the impact on the ocean sector is not always positive. Moreover, the discretization has a significant impact on the mean state (Massonnet et al. 2019) and it can be inferred that the optimal configuration is different for Arctic and Antarctic sea-ice. In this context the coupling with a sea-ice DA system can help in reducing the differences between multi/mono category models. A tuned multi-category model can ease the effort of DA and provide a realistic representation of such variables that are not directly corrected by the DA.”

Cheng, S., Chen, Y., Aydoğdu, A., Bertino, L., Carrassi, A., Rampal, P., and Jones, C. K. R. T.: Arctic Sea ice data assimilation combining an ensemble Kalman filter with a novel Lagrangian sea ice model for the winter 2019–2020, The Cryosphere, 17, 1735–1754, https://doi.org/10.5194/tc-17-1735-2023, 2023

Thorndike, A. S., Rothrock, D. A., Maykut, G. A., and Colony, R. (1975), The thickness distribution of sea ice, *J. Geophys. Res.*, 80(33), 4501– 4513, doi:10.1029/JC080i033p04501.

Uotila, P., Iovino, D., Vancoppenolle, M., Lensu, M., and Rousset, C.: Comparing sea ice, hydrography and circulation between NEMO3.6 LIM3 and LIM2, Geosci. Model Dev., 10, 1009–1031, https://doi.org/10.5194/gmd-10-1009-2017, 2017.

Massonnet, F., Barthélemy, A., Worou, K., Fichefet, T., Vancoppenolle, M., Rousset, C., and Moreno-Chamarro, E.: On the discretization of the ice thickness distribution in the NEMO3.6-LIM3 global ocean–sea ice model, Geosci. Model Dev., 12, 3745–3758, https://doi.org/10.5194/gmd-12-3745-2019, 2019.

Why is SIT RMSE only shown for February, it would be good also to see for maybe sometime earlier in Winter (November?)

Following the suggestion of the Reviewer we calculated the November SIT RMSE and showed it in Figure S3 for different experiments. The analysis of February statistics in the text remains valid for November RMSE as well. It is probably worth to notice that in November the sole assimilation of SIC data (SICDE1 experiment) shows a significant smaller error compared to the CTRL (no assimilation) in large part of the central Arctic and the Beaufort Gyre but degrades the SIT distribution close to the Canadian coastline. We decide to add a comment on the November statistics in the text, without including Fig S3 in the manuscript, being the results similar to February one: “A similar comparison of the November RMSE among experiments extend the validity of the present discussion to the beginning of the freezing period (not shown).” Figure S3 will remain publicly available in this comment/answer section and will be also added in the Supplement.

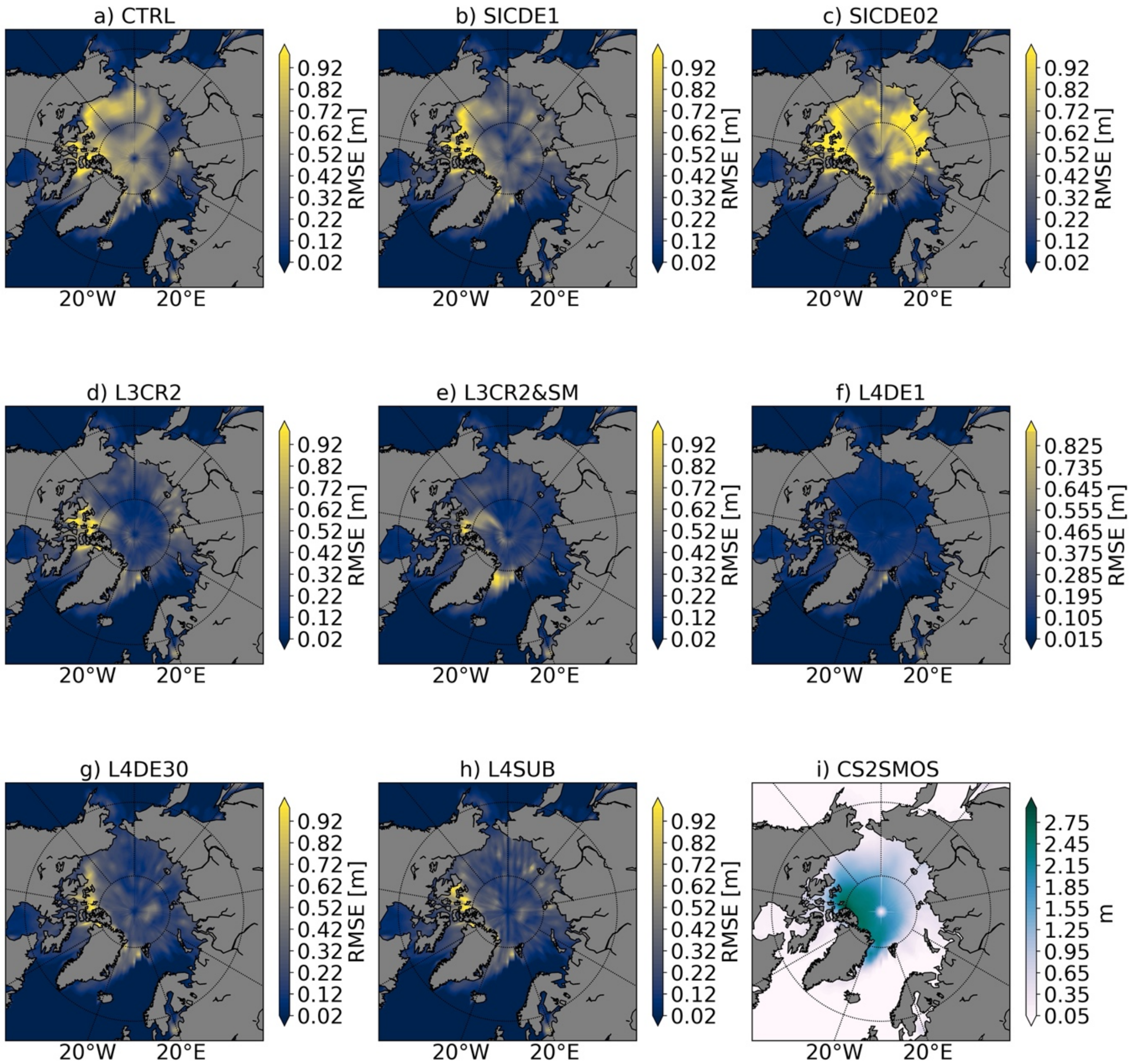

Fig S3. (Panel a-h) Spatial SIT RMSE for different experiments against CS2SMOS data aggregating November statistics and binned in 2°x2° boxes. Panel i) shows the November CS2SMOS spatial mean.

Specific comments:

Throughout the paper: You use ingestion or ingesting many times when assimilation or assimilating sounds fine, ingestion sounds quite strange and not correct.

We limited the use of “ingestion/ingesting” and used synonyms throughout the text.

In quite a few places you have used Cryosat-2 when you should use CryoSat-2

Thank you, corrected.

The abstract feels a little clumsy in presentation and wording and could be rewritten in a clearer way, there are also many grammatical/spelling errors in the abstract highlighted below

We thank the Reviewer, the abstract has been rephrased following comments and apply also the corrections suggested below in the text.

"In the last decade, various satellite missions have been monitoring the status of the Cryosphere and its evolution. Beside sea-ice concentration data, available since the 80s, sea-ice thickness retrievals are now ready to be used in global operational prediction and reanalysis systems. Nevertheless, while univariate algorithms are commonly used to constrain sea-ice area or volume at global level, multivariate approaches are yet to be employed due to highly non-Gaussian distribution of sea-ice variables together with the low accuracy of thickness observations. This study extends a 3Dvar system, called OceanVar and routinely employed in the production of global/regional operational/reanalysis products, to process sea-ice variables. The tangent/adjoint versions of an anamorphosis operator are used to transform back and forward the sea-ice anomalies into local Gaussian control variables, minimizing in the latter space. The benefit brought by such transformation is described. Several sensitivity experiments are carried out using a suite of diverse datasets. The sole assimilation of the CryoSat-2 provides a good spatial representation of thickness distribution but overestimates the total volume that requires the inclusion of SMOS data to converge towards the observation estimates. The intermittent availability of thickness data can lead to potential jumps in the evolution of the volume and requires a dedicated tuning. The use of the merged L4 product CS2SMOS shows the best skill score when validated against independent measurements during the melting season when satellite data are not available. This new sea-ice module is meant to simplify the future coupling with ocean variables."

Line 1: cryoshopere -> the cryosphere, evolution over time -> evolution

Line 6: those variables are treated -> these variables are treated

Line 9: the assimilation of the sole Cryosat-2 -> The sole assimilation of Cryosat-2 sea ice thickness

Line 10: along the year -> throughout the year

Line 11: The use of merged L4 product -> The use of the merged L4 product

Line 15: have been offering -> have offered

Line 16: Thickness extimates were firstly derived -> Thickness estimates were first derived

Line 23: general agreement in the extension -> I assume you mean general agreement in the sea ice extent?

Line 28: while the assimilation of the sole concentration -> while the sole assimilation of sea ice concentration

Line 35: routinely -> routine

Line 38: gaussianity -> gaussian

Line 54: Change sentence to begin with "In the past few decades" instead of using "in the last decades" in middle of sentence.

Line 56-57: Laxon et al., 2013 should be referenced here also in terms of the CryoSat-2 SIT retrieval

Line 66: year-round product that guesses -> year round product that estimates

Line 122: The number of sampling -> The sample size

The above corrections are now included, thank you

Line 142: The sentence is difficult to understand, I am not sure what

We rephrased the sentence in :

"The use of local Gaussian space in each point of the grid turns out to be crucial for a correct application of the horizontal correlation operator especially close to sea-ice edge. "

Line 152: will be possibly investigated -> may be investigated

Figure 3 title: Diagnosys -> Diagnosis

We corrected both, thank you

Figure 4 and 5: very difficult to see SICDE1 in plot due to colour scheme chosen, suggest choose a different scheme for this experiment. Could also change x axis labels in RHS plots from numbers to month initials (i.e. 2, 4, 6 etc to J, F, M, A, M…).

Thank you for pointing out, the color scheme is changed to easy the reading from all Users, we also changed the numbers to month initials.

Line 209: significative -> significant

Corrected, thank you

Line 214: "While THE L4DE1 provides the best skill score, the other two experiments show similar spatial RMSE and BIAS" – It looks to me like the spatial pattern is different with L4DE1 having higher RMSE further from the east coast of Greenland, whereas in the other two experiments it is closer, and they also have high RMSE in Beaufort/CAA, whereas L4DE1 does not have this spatial pattern.

We rephrase and discuss the comparison. "L4DE1 shows a rather small and spatially uniform RMSE and BIAS across the basin except for the Greenland coastline where the RMSE peak up to 0.9m at the interface between open sea and sea-ice edge. The other two experiments (L4DESUB and L4DE30) have similar skill among themselves, with larger RMSE and BIAS compared to L4DE1 close to Canadian/Greenland coastlines and in the Beaufort Gyre."

Line 219: fairy well -> fairly well

Line 227: reanalysis purpose -> the purpose of reanalysis

We corrected both, thank you

Figure 6: The colour scheme uses white for both lowest and highest RMSE, therefore within the ice pack I am not sure if the white colour is indicating highest or lowest RMSE values?

The color scheme is changed to ease the reading from all Users.

Line 251: This sharp jump -> This sharp discontinuity or increment

Corrected, thank you

Figure 8: Very difficult to see L4DE30 in plot due to colour chosen for this line.

The color scheme is changed to ease the reading from all Users.

Line 276: extimates -> estimates

Line 281: jumps -> increments/discontinuities

We corrected both, thank you

Sentence Line 284-285 “The reasons can be sought in the peculiar aspects of sea-ice variables that prevent a smooth ingestion in global analysis/reanalysis systems already in place.” This sentence sounds very strange and not correct, needs rewording.

We decide to drop the sentence that is too general and can generate confusion. The sentence was meant to introduce the forthcoming discussion about non-Gaussianity of sea-ice distribution and intermittent availability of satellite thickness data along the year. However, we agreed there is no need of it.

Line 292: routinely production -> routine production

Line 293: “to cope with” -> “to benefit from”

We corrected both, thank you

Laxon, S.W., Giles, K.A., Ridout, A.L., Wingham, D.J., Willatt, R., Cullen, R., Kwok, R., Schweiger, A., Zhang, J., Haas, C. and Hendricks, S., 2013. CryoSat-2 estimates of Arctic sea ice thickness and volume. Geophysical Research Letters 40, no. 4 (2013): 732-737

L. Bertino : Comments and Answers

I have asked these questions orally on the IICWG workshop, but here is a written version. The paper by Cipollone and colleagues is to my knowledge the first application of the anamorphosis to the assimilation of sea ice variables going through several assimilation cycles. The work carried out is of very high quality and the results are quite encouraging but the paper would deserve a few clarifications before publication.

A Gaussian anamorphosis is a continuous function and is therefore not designed to turn discontinuous probability densities (like the zeroes and 100% of ice concentrations in open water and fully packed ice). In case of accumulation of probability density at a given value ("atoms of probability density"), the piecewise linear mapping to 21 quantiles will diffuse the atoms to neighbouyring values. The authors should explain how both extremities of the distribution are handled.

We thank Dr. Bertino for reading and commenting the Manuscript pointing out the aspects to be explained in more detail. We extended the description the anamorphosis operator (called Mapping below) to clarify how it is employed in the DA system, by including the optimal range of application and possible limitations. The present DA system uses the tangent/adjoint version of the Mapping linearized around the background value $V_{gSIC\text{->}SIC} = [\frac{\partial MAP}{\partial SIC}]_{SIC=SIC_B}$.The derivative is a simple numerical centered first-ordered difference around the background, except for the extreme where it is a backward or forward formulation. The existence of $V_{gSIC\text{->}SIC}$ requires the mapping to be locally continuous and the diffusion towards the neighboring values help in this sense; in the case the derivative does not exist the corresponding increment is zero. We tend to avoid the presence of discontinuous probability densities that reflect an underestimation of model error (under the assumption that the variability of the model reflects its error, i.e., zero standard deviation). To avoid such underestimation, we augment the number of model samples in each point by adding values from neighboring points to construct a transformation that could span all possible physical values. A second possible approach can be the use of values from previous or next month.

"Such operator [the $V_{gSIC\text{->}SIC}$] requires the transformation to be locally continuous, in the case the numerical derivative does not exist the corresponding increment is zero and no correction is generated. To avoid the presence of discontinuous probability densities, that reflects an underestimation of model error, we enrich the sample size for each point …"

Further, the paper does not explain how values out of bounds are treated. If the model forecast produces a sea ice thikness value larger than the largest of the samples, what will be its Gaussian counterpart? In Simon and Bertino 2011, we extrapolated the last piece of the linear mapping of quantile according to an exponential tail (Eq 15). This could be included to avoid trouble.

We added a description concerning the treatment of extreme values. There are two different types of possible extreme values: either in the observations or in the background. Extreme values in the observations, i.e. observations far from the background, can be assimilated with current system. However, the tangent linear approximation is suboptimal because implicitly supposes that final increments do not diverge much from the background, the coefficients in the $V_{gICE\text{->}ICE}$ are valid around the background. This is a common problem of the tangent linear approximation and of variational DA that are not designed for the assimilation of extreme values. The use of background quality checks in the preprocessing serves to remove values that are far from the background and for which the linear approximation does not hold anymore.

In the case that the values out of bounds are in the forecasts, i.e., out of the range of values extracted from the historical simulation, then it is not possible to calculate the derivative of the Mapping and the operator reduces the increments to zero. We preferred to stay conservative and not correct such extreme events that will be driven only by the model. The idea of extrapolating the distribution can be a solution in case a correction is needed. Probably the best approach would be the use of a hybrid scheme, with a part of the B coming from an ensemble that goes to: i) add the model "error-of-the-day"; ii) update the Mapping with the inclusion of ensemble forecasted values.

We added the following paragraph:

"It is worth to note that the use of tangent/adjoint approximations of the anamorphosis leads the assimilation of extreme values, to be suboptimal (i.e. observations that are far from the background value). Tangent/adjoint approximations of any operator are valid in the proximity of the background value and become less and less accurate in the case of large corrections and highly non-linear operator. This is anyway a limitation that is implicit in any three-dimensional variational scheme. Moreover, the anamorphosis should span all the possible physical values in each grid point. In the case the background is out of the range of values used for the anamorphosis, then it is not possible to calculate the derivative and the corresponding increments are zero. This means that extreme events in the background (not present in the 31 years of simulation) do not receive correction. In Simon and Bertino (2012) they include an exponential tail to the anamorphosis, in order to treat values out of bounds. A further approach could be the use of a hybrid **B** where the ensemble part goes to update the anamorphosis with the inclusion of new model values as well as adding the "error-of-the-day".

Besides these two remarks, some clarifications could be made regarding the algorithm:

- Abstract: "transform sea ice anomalies into Gaussian control variables". Why anomalies and not the full field values?

Following also the comment of the first Reviewer we rephrase the abstract. In the specific case, the sea-ice anomalies that were transformed in the control variables (using the tangent/adjoint operators) refer to $\boldsymbol{\delta x}$ that can be considered anomalies with respect to the background in a statistical sense. The operators used are however the tangent and adjoint version of the Mapping not the full operator. The phrase is changed accordingly:
"The tangent/adjoint versions of an anamorphosis operator are used to transform locally the sea-ice anomalies into Gaussian control variables and back, minimising in the Gaussian one."

- l40: the result of the anamoprhosis is not strictly Gaussian. It would be fair to write "more Gaussian" or "closer to a Gaussian".

We change the sentence in "The operator transforms the probability density functions of SIC/SIT anomalies towards Gaussian-like ones performing the minimization in this space".

- Section 3: linearizing the anamorphosis operator seems to defeat the purpose of the anamorphosis. Can you clarify why and what is done there in practise with a piecewise linear quantile mapping?

The linearization consists of a numerical derivative of the quantile mapping in each grid point around the background value. This is the classical approach where the full Mapping is replaced locally with the first two terms of the Taylor expansion: the approximation holds if the increments are not far from the background value. Using a local Gaussian space in each point of the grid is optimal for a correct application of the rest of the Control Variable Transformation, i.e., the cross-correlation and horizontal diffusion. The latter mimics the Gaussian spread of information in the surrounding points based on a reference-length using three iterations of a first-order recursive filter. The benefit turns to be significative for example close to sea-ice edge. In the physical space, correlating two points that have opposite distributions (say being close to 100% of SIC in one point and close to 0% in the other) can populate the surrounding points with values that do not fall in the range of their distributions. The use of a gaussian space re-center the increments in the range of physical values. In this sense this local mapping easily allows the use of diverse correlation length for each grid point, as the maps provided for example by CS2SMOS. Moreover, such operator helps the future coupling with Gaussian-like ocean variables such as temperature and salinity. Without such Mapping, the isotropic spread of temperature or salinity increments on the edge of sea-ice, would lead to a corresponding spread in SIC and SIT, potentially destroy, or smooth the edge of sea ice. We rephrase the corresponding paragraph to be clearer:

“The use of local Gaussian space in each point of the grid turns out to be crucial for a correct application of the horizontal correlation operator especially close to sea-ice edge. Figure 2 shows the sea-ice increments in a test case, says the third week of February 2015, generated with and without the application of $V_{gICE\rightarrow ICE}$ with a large fixed correlation length of 150km and three iterations of a first order recursive filter. Green solid line corresponds to the mean sea-ice edge in that week, SIC and SIT are jointly assimilated close to the sea-ice edge. In the physical space an isotropic spread of information towards the ice-free areas is seen (Figures 2.c,d). The use of $V_{gICE\rightarrow ICE}$ "re-center" the increments (in the Gaussian space) within the range of physical values, reducing the wrong isotropic diffusion (Figures 2.a,b) and following the variability of the specific region”

- If V gICE->ICE and V^T ICE->gICE are linearized anamorphosis functions, where are the nonlinear anamorphosis functions used? If they are both nonlinear they must be the forward and backward anamorphosis functions, please clarify.

Being the mapping empirical, the linearization is numerical around the background value. The code reads the full mapping, reads the background state value, and compute the derivative with a simple first-order central difference around the background value.

- l.87: isn't it conditioned to the model background rather than analysis?

Thank you for the correction, the text is changed accordingly.

- l136: the word "correctly" implies that there is a correct reference SIT, but here I think that you mean "similar" with and without anamorphosis. Otherwise, mention which reference is used.

Agreed with the comment, now it reads “the spatial structure is similar in the two cases, while the magnitude slightly differs “

- Figure 9: the colour shade for 1 and open water look the same to me (maybe because I am colour blind). Can you pick a red share instead?

Thank you for pointing out, the color palettes will be changed in all the figures to easy the Readers.

- Figures 8 and 9, the label CRYO2SMOS does not correspond to the name of the experiment in the text.

Thank you, corrected.

Typos:

- l1: Cryosphere

- l65: context

We corrected both, thank you.

- l79: coupling among or coupling between?

“Coupling between“ is used

- C-GLORS is sometimes spelled CGLORS without the dash.

This is something we should have known, we correct to C-GLORS, thank you

- l114: Define the acronym CVT.

Thank you, corrected

- Use a capital letter for Gaussian since it comes from a person's name.

We now use the capital letter everywhere. Formally, being Gauss a person’s name, the correct form is the possessive, i.e. Gauss’. The use of the adjective forms probably followed the same evolution discussed by Wright for Green function,

Wright, M. “Green function or green's function?”, *Nature Phys* **2**, 646 (2006). https://doi.org/10.1038/nphys411

Reference:

Simon, E., & Bertino, L. (2012). Gaussian anamorphosis extension of the DEnKF for combined state parameter estimation: Application to a 1D ocean ecosystem model. Journal of Marine Systems, 89(1), 1–18. https://doi.org/doi:10.1016/j.jmarsys.2011.07.007

Reviewer 2 : Comments and Answers

The paper presents experiments where sea ice concentration and thickness observations are assimilated in a coupled ice-ocean model. The correlation between sea ice concentration and thickness observations is taken into account. The method of Desroziers (2005) is used to better characterize the observation-error statistics. Also the Degrees of Freedom for Signal (DFS) developed by Cardinali et al. (2004) is used to measure the influence of the observations on the analysis.

The results of the experiments are first compared to one of the same sea ice observations dataset that were assimilated. Since only the analyses and not the forecasts are verified, the exercise is more to verify the assimilation methodology than the accuracy of the analyses. Fortunately, the experiments are also verified against independent observations of ice thickness in the Beaufort Sea.

We thank the Reviewer, and we agree that the quality of the Manuscript can increase expanding the validation section with a paragraph "Validation against Operation IceBridge data ". Following a similar comment from the Reviewer 1, we add a second independent comparison against measurements from NASA Operation IceBridge project (http://nsidc.org/data/idcsi4 ; Kurtz et al., 2015) that cover several regions of the Western Arctic for several days in March and April between 2011 and 2013. The results are summarized in the initial answer to the first Reviewer (please see figure S1 and S2) and confirm the conclusions coming from the comparison against BGEP ULS moorings. Winter assimilation of SIT data produces a smaller RMSE in March-April statistics compared to SIC-only or CTRL experiment (no assimilation). Among the different experiments with SIT assimilation, the L4DE1 run (assimilation of CS2SMOS with Desroziers' error) shows the lowest RMSE and reduces the regional BIAS almost everywhere.
Kurtz, N., Studinger, M., Harbeck, J., DePaul Onana, V., and Yi, D.: IceBridge L4 Sea Ice Freeboard, Snow Depth, and Thickness, Version 1, https://doi.org/10.5067/G519SHCKWQV6, 2015.

In the section describing the assimilation method, we see gICE and gSIC that are used interchangeably. I think it would be better to have just one expression throughout, if these point to the same thing.

The difference between gICE and gSIC was not explained, we thank the Reviewer and clarify in the text. While gSIC and gSIT refer specifically to the corresponding gaussian counterpart of SIC and SIT, gICE is used in a more general way, $V_{\mathbf{gICE->ICE}}$ points to a generic mapping of the sea-ice variable from Gaussian space to the physical one and can be associated separately to both SIC and SIT.

"…. gICE refers to both the operators applied independently to gSIC and gSIT. "

Figures 10 and 11 seem to be identical. Please remove figure 11 and only refer to figure 10 in the text. Also please revise the caption for this figure. Currently the caption for figure 10 and 11 are slightly different but I am not sure which one is more correct.

Figures 10 and 11 gather results from two different subsets of experiments from Table 1 to highlight different aspects. Fig 10 compares the sole assimilation of SIC with the experiments where both SIC and SIT are used, in particular, L4DE1 experiment (assimilation of the merged L4 product) and its subsampled version (L4DESUB). Fig 11 groups experiments that uses different thickness datasets where the impact of the sole assimilation of the CryoSat-2 and the impact of the independent assimilation of CryoSat-2 and SMOS data are compared to the L4DE1 experiments. Anyway, we agree that the color scheme can confuse the Reader, therefore, we changed the figures to avoid the use of the same color for different experiments.

Specific comments:

Line 20: What about the first CryoSat mission ?

The very first satellite associated with the CryoSat mission was lost due to a failure during its launch in 2008. The second launch was successful, and it is common to refer to the second mission of CryoSat as CryoSat-2.

Line 65: "contest" or "context" ?

Context is the correct word, thank you.

Line 88: 'x' should be called the state vector, and 'x_a' is the final analysis state (the value of 'x' that minimize the cost function).

We rephrase the line, "δx label the increments that correspond to the difference between the final analysis state x_a and the initial ocean state x_b, in the minimum of the cost function." The definition of x as the state vector comes a few lines below.

Line 113: Please define "CVT".

The acronym was indeed not defined, now we added "Control Vector Transformation or CVT", thanks

Line 146: Should "$V_{ICE \rightarrow}$gICE" be "$V^T_{ICE \rightarrow}$gICE" ?

Thank you, gICE was misplaced, we correct the initial "$V_{ICE\text{->}gICE}$" to "$V_{gICE \rightarrow ICE}$"

Line 151: I think that "that it has been shown" should be "that has been shown".

Corrected, Thanks

Technical corrections

Line 122: "31-year-long" should be "31-year long" for consistency with the caption of figure 1.

We uniform text and caption to "31 year-long" that means a single simulation lasting 31 years.

Figure 1: In the caption, "Panel g and f show the same as e,f for September" should be "Panels g and h show the same as e,f for September".

Figure 4: In the caption, "Panels" should be "panels".

Line 203: "Beaufourt" should be "Beaufort".

Line 219: "0.5 * 10^{13}km" should be "0.5 * 10^{13} m^3"

Line 253: "is usual higher" should be "is usually higher".

Line 255: "…contribute to the modify the model…" should be "…contribute to modify the model…".

Line 275: "indestion" should be "ingestion".

We thank you the Reviewer for the corrections. We applied all in the revised version of the manuscript.