Reviewer 3 : Comments and Answers

I have asked these questions orally on the IICWG workshop, but here is a written version. The paper by Cipollone and colleagues is to my knowledge the first application of the anamorphosis to the assimilation of sea ice variables going through several assimilation cycles. The work carried out is of very high quality and the results are quite encouraging but the paper would deserve a few clarifications before publication.

A Gaussian anamorphosis is a continuous function and is therefore not designed to turn discontinuous probability densities (like the zeroes and 100% of ice concentrations in open water and fully packed ice). In case of accumulation of probability density at a given value ("atoms of probability density"), the piecewise linear mapping to 21 quantiles will diffuse the atoms to neighbouyring values. The authors should explain how both extremities of the distribution are handled.

We thank Dr. Bertino for reading and commenting the Manuscript pointing out the aspects to be explained in more detail. We extended the description the anamorphosis operator (called Mapping below) to clarify how it is employed in the DA system, by including the optimal range of application and possible limitations. The present DA system uses the tangent/adjoint version of the Mapping linearized around the background value $V_{gSIC\text{->}SIC} = [\frac{\partial MAP}{\partial SIC}]_{SIC=SIC_B}$.The derivative is a simple numerical centered first-ordered difference around the background, except for the extreme where it is a backward or forward formulation. The existence of $V_{gSIC\text{->}SIC}$ requires the mapping to be locally continuous and the diffusion towards the neighboring values help in this sense; in the case the derivative does not exist the corresponding increment is zero. We tend to avoid the presence of discontinuous probability densities that reflect an underestimation of model error (under the assumption that the variability of the model reflects its error, i.e., zero standard deviation). To avoid such underestimation, we augment the number of model samples in each point by adding values from neighboring points to construct a transformation that could span all possible physical values. A second possible approach can be the use of values from previous or next month.

Further, the paper does not explain how values out of bounds are treated. If the model forecast produces a sea ice thikness value larger than the largest of the samples, what will be its Gaussian counterpart? In Simon and Bertino 2011, we extrapolated the last piece of the linear mapping of quantile according to an exponential tail (Eq 15). This could be included to avoid trouble.

We added a description concerning the treatment of extreme values. There are two different types of possible extreme values: either in the observations or in the background. Extreme values in the observations, i.e. observations far from the background, can be assimilated with current system. However, the tangent linear approximation is suboptimal because implicitly supposes that final increments do not diverge much from the background, the coefficients in the $V_{gICE\text{->}ICE}$ are valid around the background. This is a common problem of the tangent linear approximation and of variational DA that are not designed for the assimilation of extreme values. The use of background quality checks in the preprocessing serves to remove values that are far from the background and for which the linear approximation does not hold anymore.

In the case that the values out of bounds are in the forecasts, i.e., out of the range of values extracted from the historical simulation, then it is not possible to calculate the derivative of the

Mapping and the operator reduces the increments to zero. We preferred to stay conservative and not correct such extreme events that will be driven only by the model. The idea of extrapolating the distribution can be a solution in case a correction is needed. Probably the best approach would be the use of a hybrid scheme, with a part of the B coming from an ensemble that goes to: i) add the model "error-of-the-day"; ii) update the Mapping with the inclusion of ensemble forecasted values.

We added the following paragraph:

"It is worth to note that the use of tangent/adjoint approximations of the anamorphosis leads the assimilation of extreme values, to be suboptimal (i.e. observations that are far from the background value). Tangent/adjoint approximations of any operator are valid in the proximity of the background value and become less and less accurate in the case of large corrections and highly non-linear operator. This is anyway a limitation that is implicit in any three-dimensional variational scheme. Moreover, the anamorphosis should span all the possible physical values in each grid point. In the case the background is out of the range of values used for the anamorphosis, then it is not possible to calculate the derivative and the corresponding increments are zero. This means that extreme events in the background (not present in the 31 years of simulation) do not receive correction. In Simon and Bertino (2012) they include an exponential tail to the anamorphosis, in order to treat values out of bounds. A further approach could be the use of a hybrid **B** where the ensemble part goes to update the anamorphosis with the inclusion of new model values as well as adding the "error-of-the-day".

Besides these two remarks, some clarifications could be made regarding the algorithm:

- Abstract: "transform sea ice anomalies into Gaussian control variables". Why anomalies and not the full field values?

Following also the comment of the first Reviewer we rephrase the abstract. In the specific case, the sea-ice anomalies that were transformed in the control variables (using the tangent/adjoint operators) refer to $\boldsymbol{\delta x}$ that can be considered anomalies with respect to the background in a statistical sense. The operators used are however the tangent and adjoint version of the Mapping not the full operator. The phrase is changed accordingly:
"The tangent/adjoint versions of an anamorphosis operator are used to transform locally the sea-ice anomalies into Gaussian control variables and back, minimising in the Gaussian one."

- l40: the result of the anamoprhosis is not strictly Gaussian. It would be fair to write "more Gaussian" or "closer to a Gaussian".

We change the sentence in "The operator transforms the probability density functions of SIC/SIT anomalies towards Gaussian-like ones performing the minimization in this space".

- Section 3: linearizing the anamorphosis operator seems to defeat the purpose of the anamorphosis. Can you clarify why and what is done there in practise with a piecewise linear quantile mapping?

The linearization consists of a numerical derivative of the quantile mapping in each grid point around the background value. This is the classical approach where the full Mapping is replaced locally with the first two terms of the Taylor expansion: the approximation holds if the increments are not far from the background value. Using a local Gaussian space in each point of the grid is optimal for a correct application of the rest of the Control Variable Transformation, i.e., the cross-correlation and horizontal diffusion. The latter mimics the Gaussian spread of information in the surrounding points based on a reference-length using three iterations of a first-order recursive filter. The benefit turns to be significative for example close to sea-ice edge. In the physical space, correlating two points that have opposite distributions (say being close to 100% of SIC in one point and close to 0% in the other) can populate the surrounding points with values that do not fall in the range of their distributions. The use of a gaussian space re-center the increments in the range of physical values. In this sense this local mapping easily allows the use of diverse correlation length for each grid point, as the maps provided for example by CS2SMOS. Moreover, such operator helps the future coupling with Gaussian-like ocean variables such as temperature and salinity. Without such Mapping, the isotropic spread of temperature or salinity increments on the edge of sea-ice, would lead to a corresponding spread in SIC and SIT, potentially destroy, or smooth the edge of sea ice. We rephrase the corresponding paragraph to be clearer:

"The use of local Gaussian space in each point of the grid turns out to be crucial for a correct application of the horizontal correlation operator especially close to sea-ice edge. Figure 2 shows the sea-ice increments in a test case, says the third week of February 2015, generated with and without the application of $V_{gICE \rightarrow ICE}$ with a large fixed correlation length of 150km and three iterations of a first order recursive filter. Green solid line corresponds to the mean sea-ice edge in that week, SIC and SIT are jointly assimilated close to the sea-ice edge. In the physical space an isotropic spread of information towards the ice-free areas is seen (Figures 2.c,d). The use of $V_{gICE \rightarrow ICE}$ "re-center" the increments (in the Gaussian space) within the range of physical values, reducing the wrong isotropic diffusion (Figures 2.a,b) and following the variability of the specific region"

- If V gICE->ICE and V^T ICE->gICE are linearized anamorphosis functions, where are the nonlinear anamorphosis functions used? If they are both nonlinear they must be the forward and backward anamorphosis functions, please clarify.

Being the mapping empirical, the linearization is numerical around the background value. The code reads the full mapping, reads the background state value, and compute the derivative with a simple first-order central difference around the background value.

- l.87: isn't it conditioned to the model background rather than analysis?

Thank you for the correction, the text is changed accordingly.

- l136: the word "correctly" implies that there is a correct reference SIT, but here I think that you mean "similar" with and without anamorphosis. Otherwise, mention which reference is used.

Agreed with the comment, now it reads "the spatial structure is similar in the two cases, while the magnitude slightly differs "

- Figure 9: the colour shade for 1 and open water look the same to me (maybe because I am colour blind). Can you pick a red share instead?

Thank you for pointing out, the color palettes will be changed in all the figures to easy the Readers.

- Figures 8 and 9, the label CRYO2SMOS does not correspond to the name of the experiment in the text.

Thank you, corrected.

Typos:

- l1: Cryosphere

- l65: context

We corrected both, thank you.

- l79: coupling among or coupling between?

“Coupling between“ is used

- C-GLORS is sometimes spelled CGLORS without the dash.

This is something we should have known, we correct to C-GLORS, thank you

- l114: Define the acronym CVT.

Thank you, corrected

- Use a capital letter for Gaussian since it comes from a person's name.

We now use the capital letter everywhere. Formally, being Gauss a person’s name, the correct form is the possessive, i.e. Gauss’. The use of the adjective forms probably followed the same evolution discussed by Wright for Green function,

Wright, M. “Green function or green's function?”, *Nature Phys* **2**, 646 (2006). https://doi.org/10.1038/nphys411

Reference:

Simon, E., & Bertino, L. (2012). Gaussian anamorphosis extension of the DEnKF for combined state parameter estimation: Application to a 1D ocean ecosystem model. Journal of Marine Systems, 89(1), 1–18. https://doi.org/doi:10.1016/j.jmarsys.2011.07.007
Citation: https://doi.org/10.5194/egusphere-2023-254-CC1