

MPeat2D – A fully coupled mechanical-ecohydrological model of peatland development in two dimensions

Adilan W. Mahdiyasa^{1,2}, David J. Large¹, Matteo Icardi³, Bagus P. Muljadi¹

5 ¹Department of Chemical and Environmental Engineering, University of Nottingham, University Park, Nottingham NG7 2RD, UK

²Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology, Bandung 40132, Indonesia

³School of Mathematical Sciences, University of Nottingham, University Park, Nottingham NG7 2RD, UK

10 *Correspondence to:* Adilan W. Mahdiyasa (adilan.widyawan@itb.ac.id)
David J. Large (david.large@nottingham.ac.uk)

Abstract. Higher dimensional models of peatland development are required to analyse the influence of spatial heterogeneity and complex feedback mechanisms on peatland behaviour. However, the current models exclude the mechanical process that
15 leads to uncertainties in simulating the spatial variability of water table position, vegetation composition, and peat physical properties. Here, we propose MPeat2D, a peatland development model in two dimensions, which considers mechanical, ecological, and hydrological processes together with the essential feedback from spatial interactions. MPeat2D employs poroelasticity theory that couples fluid flow and solid deformation to model the influence of peat volume changes on peatland ecology and hydrology. To validate the poroelasticity formulation, the comparisons between numerical and analytical solutions
20 of Mandel's problems for two-dimensional test cases are conducted. The application of MPeat2D is illustrated by simulating peatland growth over 5000 years above a flat and impermeable substrate with free-draining boundaries at the edges, using constant and variable climate. In both climatic scenarios, MPeat2D produces lateral variability of water table depth, which results in the variation of vegetation composition. Furthermore, the drop of the water table at the margin increases the compaction effect, leading to a higher value of bulk density and a lower value of active porosity and hydraulic conductivity.
25 These spatial variations obtained from MPeat2D are consistent with the field observations, suggesting plausible outputs from the proposed model. By comparing the results of MPeat2D to a one-dimensional model and a two-dimensional model without the mechanical process, we argue that mechanical-ecohydrological feedbacks are important for analysing spatial heterogeneity, shape, carbon accumulation, and resilience of the peatland.

1 Introduction

30 In this paper, we provide a fully coupled mechanical-ecohydrological model of peatland development in two dimensions (2D). The continuum representation of the peatland employed by the proposed model results in the advancement of peatland modelling, particularly if we consider questions relating to the phenomena for which mechanical process and feedback are

essential components. Examples of these phenomena include the analysis of mechanical limits to peatland stability and the relationship between topography, peat physical properties, and carbon accumulation. The purpose of this paper is to explain the formulation of the model, which is developed from an earlier one-dimensional (1D) mechanical-ecohydrological model (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022). Its application is illustrated through the simulations of the long-term peatland growth over millennia under idealised conditions. To consider the consequences of incorporating spatial variability and mechanical process on the peatland behaviour, we compare our results with the existing peatland growth models, including a 1D model without spatial variability MPeat (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022) and a 2D stiff non-continuum model DigiBog (Baird et al., 2012; Morris et al., 2012).

The spatial variability of the peatland is widely evidenced by the changes in the horizontal and vertical directions of peat physical properties, including bulk density, active porosity, and hydraulic conductivity. The horizontal variation of hydraulic conductivity was observed by Lapen et al. (2005), who found that hydraulic conductivity is lower at the margin than at the centre based on the field measurements and analysis of a peatland groundwater flow model. Field observations from Baird et al. (2008) and Lewis et al. (2012), who measured lateral variability of the hydraulic conductivity in a raised and a blanket peatland, respectively, agree with the Lapen et al. (2005) finding. Lewis et al. (2012) also observed the lateral variability in bulk density, which increased from the centre toward the margin. In the vertical direction, deeper peat exhibits a higher value of bulk density and a lower value of active porosity and hydraulic conductivity, with abrupt changes occurring between the unsaturated and saturated zones (Clymo, 2004, 1984; Hoag and Price, 1997; Hoag and Price, 1995; Quinton et al., 2008; Quinton et al., 2000; Fraser et al., 2001). Moreover, the meta-analysis from Morris et al. (2022) with an extensive database of hydraulic conductivity and bulk density also indicates that depth significantly affects these peat physical properties.

The spatial variations of peat physical properties occur because of the fully coupled mechanical-ecohydrological processes. As a porous medium with a low value of Young's modulus (Long, 2005; Mesri and Ajlouni, 2007; Boylan et al., 2008; Dykes and Warburton, 2008), the peat body is susceptible to deformation. The deformation is non-uniform throughout the peatland area due to the spatial variations of water table depth that influence the effective stress (Whittington and Price, 2006; Price, 2003; Price et al., 2005; Waddington et al., 2010). For example, the increase in water table depth at the margin leads to higher bulk density and lower active porosity and hydraulic conductivity, preventing greater water discharge from the deeper peat. Consequently, Lapen et al. (2005) posited that a lower hydraulic conductivity at the margin has a significant influence on maintaining the wet condition at the centre, which in turn affects peat accumulation. Therefore, the spatial variations of peat physical properties potentially provide essential feedback as the peatland develops.

Higher dimensional models of peatland development ignore mechanical feedback (e.g., Ingram, 1982; Winston, 1994; Armstrong, 1995; Korhola et al., 1996; Borren and Bleuten, 2006; Baird et al., 2012; Morris et al., 2012; Swinnen et al., 2019) and assume constant or limited spatial variations of peat physical properties. For example, Borren and Bleuten (2006) proposed a three-dimensional model (3D) of peatland development based on the groundwater flow model (Boussinesq, 1871) and focused on the ecohydrological feedback between water table position with peat production and decomposition following the Clymo (1984) model. The mechanical compaction is assumed to be negligible, and the spatial variations in the bulk density

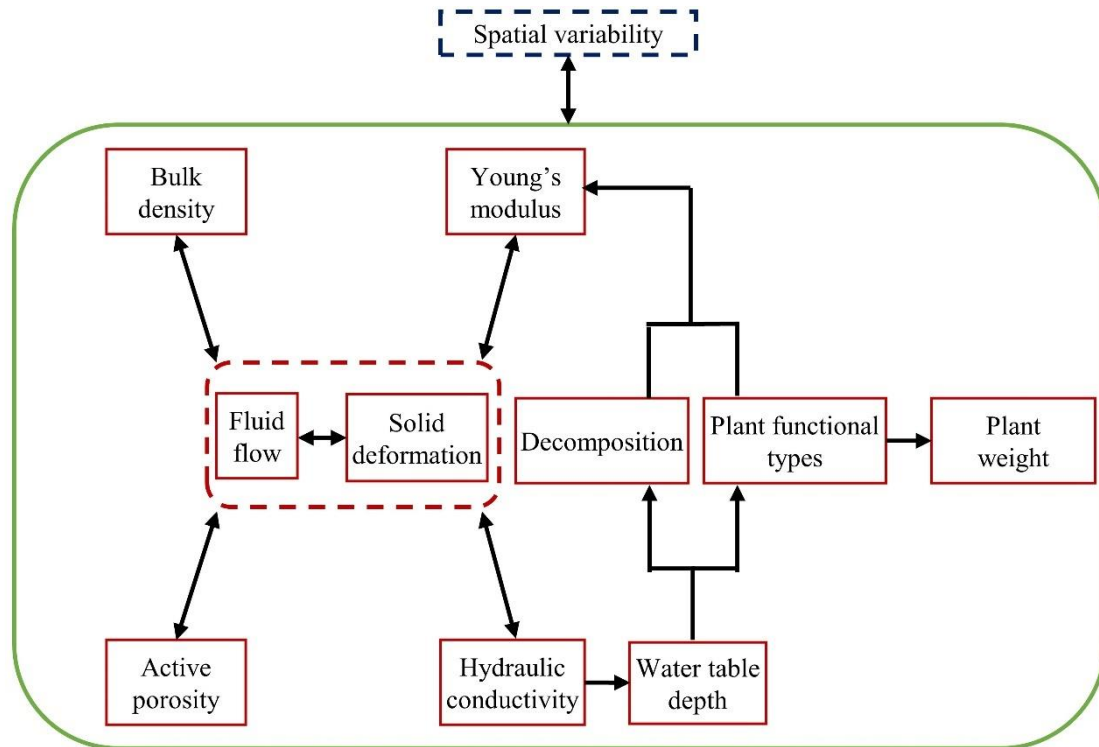
and hydraulic conductivity are obtained based on the empirical relationship between different peatland types, consisting of bog, throughflow fen, and fen. DigiBog (Baird et al., 2012; Morris et al., 2012) is a 1D, 2D, or 3D model of peatland development that accommodates the spatial changes in hydraulic conductivity through the differences in remaining mass that are affected by water table position and decomposition processes (Moore et al., 2005; Quinton et al., 2000). Although DigiBog captures more complex feedback between ecological and hydrological processes than the model from Borren and Bleuten (2006), the omission of mechanical feedback leads to the assumption of constant active porosity and bulk density as the peatland grows. Cobb et al. (2017) developed a 2D tropical peatland growth model to analyse the influence of climate, particularly the rainfall pattern, on carbon storage. This model simulates the dynamics of the water table and peat accumulation through the groundwater flow model (Boussinesq, 1871) and the difference between peat production and decomposition. The carbon storage is estimated from the stable peat surface Laplacian that is affected by the rate of peat production and decomposition. The peat surface Laplacian indicates the curvature of the peat surface, calculated as the sum of second derivatives of surface elevation. Although surface Laplacian provides information related to the peatland morphology, this model ignores the mechanical feedback and assumes a constant value of hydraulic conductivity that becomes the source of uncertainty in estimating the peatland carbon storage.

This paper, therefore, sets out to (1) provide the formulation of a fully coupled mechanical-ecohydrological model of peatland development in 2D called MPeat2D, (2) investigate model outputs in the idealised peatland growth scenario, (3) analyse the potential consequences of mechanical-ecohydrological feedback on the long-term peatland carbon accumulation and resilience by comparison with other peatland development models. The structure of this paper is presented in three main parts. First, we consider the mathematical formulation, consisting of mechanical, ecological, and hydrological submodels, together with the numerical verification of the MPeat2D. Second, we explain how to implement MPeat2D to simulate long-term peatland growth over millennia and provide examples of model outputs. Third, we examine the implications obtained from MPeat2D to understand peatland behaviour and conclude the analysis by addressing the areas in which further development from the model is required. Although MPeat2D is focused on ombrotrophic peatlands with temperate climates, the framework proposed in this paper could be employed to model the other peatland types.

2 Model formulation and verification

MPeat2D is a fully coupled mechanical, ecological, and hydrological model of long-term peatland growth in two dimensions, which takes spatial variability and structure into consideration. MPeat2D is developed based on the continuum concept (Irgens, 2008; Jog, 2015) that assumes peatland constituents, both solid and fluid particles, entirely fill the peatland body. Through this approach, the conservation of mass can be appropriately defined to formulate mechanical processes on the peatland obtained from the coupling between solid deformation and fluid flow or known as poroelasticity that becomes the core of the model (see the mechanical submodel) (Biot, 1941; Detournay and Cheng, 1993; De Boer, 2000; Wang, 2000; Coussy, 2004). The mechanical deformation of peat pores space affects physical properties, including bulk density, active porosity, and hydraulic

conductivity, resulting in different peatland behaviour (Fig. 1). For example, the changes in active porosity and hydraulic conductivity influence the water table position, which in turn determines peat production and decomposition processes (Belyea and Clymo, 2001; Clymo, 1984). Furthermore, the proportion of plant functional types (PFT) and the plant weight are also affected because they are a function of the water table depth (Moore et al., 2002; Munir et al., 2015; Peltoniemi et al., 2016; Kokkonen et al., 2019; Laine et al., 2021). The plant weight at the top surface produces loading that leads to compaction and provides feedback on the peat physical properties. By having fully coupled mechanical, ecological, and hydrological processes, MPeat2D incorporates realistic spatial variability on the peatland and allows for more significant insights into the interplay between these complex feedback mechanisms. As explained below, the formulation of MPeat2D is divided into mechanical, ecological, and hydrological submodels.



110 **Figure 1: Illustrative formulation of MPeat2D that involves mechanical, ecological, and hydrological processes together with the feedback from spatial variability on the peatland under a single mathematical and numerical framework.**

2.1 Mechanical submodel

The mechanical deformation on the peat body is influenced by the stiffness of the peat solid skeleton and the behaviour of the pore fluid. Reeve et al. (2013) found that a higher value of Young's modulus, which represents the stiffness of the material, leads to a lower deformation effect on the peat body. Furthermore, the characteristics of fluid contained in the peat pore space, including gas content and degree of saturation, also significantly affect the deformation due to the presence of pore fluid

pressure (Boylan et al., 2008; Price and Schlotzhauer, 1999; Price, 2003). Therefore, the mechanical submodel is developed based on the poroelasticity theory, which couples solid deformation and fluid flow.

We employ a fully saturated poroelasticity in 2D (Biot, 1941) to model the saturated zone of the peatland below the water table, with the governing equations as follows. The equation of equilibrium can be formulated by considering the stress tensor acting on a small elementary area, as written below

$$\bar{\nabla}^T \boldsymbol{\sigma} + \mathbf{b} = 0, \quad (1)$$

with $\bar{\nabla} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$ and $\mathbf{b} = [0 \quad (\rho_w \phi + \rho(1 - \phi))g]^T$. In this formulation $\boldsymbol{\sigma} = [\sigma_{xx}, \sigma_{yy}, \sigma_{xy}]^T$ is the total stress tensor

(Pa), \mathbf{b} is the body force (N m^{-3}), ρ_w is the water density (kg m^{-3}), ρ is the peat bulk density (kg m^{-3}), ϕ is the active porosity (-) and g is the acceleration of gravity (m s^{-2}). The presentation in terms of matrix form provides convenient notation for the derivation of weak form and numerical calculation (Jha and Juanes, 2014).

The stresses on the peat body are distributed to the solid skeleton and pore fluid, resulting in solid displacement and pore fluid pressure. The stress associated with solid displacement is known as effective stress, and it is defined as

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \alpha \mathbf{m} p, \quad (2)$$

where $\boldsymbol{\sigma}' = [\sigma'_{xx}, \sigma'_{yy}, \sigma'_{xy}]^T$ is the effective stress tensor (Pa), $\boldsymbol{\sigma} = [\sigma_{xx}, \sigma_{yy}, \sigma_{xy}]^T$ is the total stress tensor (Pa), α is the Biot's coefficient, $\mathbf{m} = [1 \quad 1 \quad 0]^T$ is the vector form of Kronecker's delta, and p is the pore water pressure (Pa). The linear constitutive law gives the relation between effective stress tensor and strain tensor through the following equation

$$\boldsymbol{\sigma}' = \mathbf{D} \boldsymbol{\epsilon}, \quad (3)$$

with $\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$. In this formulation $\boldsymbol{\sigma}' = [\sigma'_{xx}, \sigma'_{yy}, \sigma'_{xy}]^T$ is the effective stress tensor (Pa),

E is the Young's modulus (Pa), ν is the Poisson's ratio (-), and $\boldsymbol{\epsilon} = [\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}]^T$ is the strain tensor (-). The relation between strain tensor and displacement is provided by the kinematics relations reads

$$\boldsymbol{\epsilon} = \bar{\nabla} \mathbf{u}, \quad (4)$$

where $\boldsymbol{\epsilon} = [\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}]^T$ is the strain tensor (-) and $\mathbf{u} = [u_x, u_y]^T$ is the displacement (m). Finally, to complete the governing equations of the mechanical submodel, we employ the conservation of mass for solid and fluid constituents. By assuming that water flow in the peat pore space follows Darcy's law and the volumetric strain is the sum of linear strains, we can formulate the relation between solid deformation, pore water pressure, and the water flow in the peat pore space as

$$\alpha \frac{\partial \epsilon}{\partial t} + S_s \frac{\partial p}{\partial t} = \nabla \cdot (\kappa \nabla p), \quad (5)$$

where α is the Biot's coefficient (-), $\epsilon = \epsilon_{xx} + \epsilon_{yy}$ is the volumetric strain (-), S_s is the specific storage (m^{-1}), p is the pore water pressure (Pa), and κ is the hydraulic conductivity (m s^{-1}).

In the unsaturated zone above the water table, we assume that the air pressure is equal to the atmospheric pressure because the water table depth is usually less than 0.5 m in the peatland (Ballard et al., 2011; Swinnen et al., 2019; Mahdiyasa et al., 2022). Consequently, we can expand Eq. (5) to model the unsaturated zone by introducing parameters α_w and M_w that depend on the degree of saturation of water as follows (Cheng, 2020)

$$\alpha_w \frac{\partial \epsilon}{\partial t} + \frac{1}{M_w} \frac{\partial p}{\partial t} = \nabla \cdot (\kappa \nabla p), \quad (6)$$

with $\alpha_w = S_w$ and $M_w = \frac{\gamma_w(1-\lambda)}{\phi\lambda\mu} S_w^{-1/\lambda} (1 - S_w^{1/\lambda})^\lambda$. In this formulation $\epsilon = \epsilon_{xx} + \epsilon_{yy}$ is the volumetric strain (-), p is the pore water pressure (Pa), κ is the hydraulic conductivity (m s^{-1}), S_w is the degree of saturation of water (-), γ_w is the specific weight of water (N m^3), ϕ is the active porosity (-), λ is the first water retention empirical constant (-), and μ is the second water retention empirical constant (m^{-1}).

The discretisation is required in order to solve the partial differential equations from poroelasticity formulation. In 1D, the discretisation is relatively simple because it is conducted over a vertically oriented domain, the length of which represents the height of a peatland (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022). However, in 2D, the discretisation becomes more complex and is carefully done to circumvent numerical instabilities (George, 2000; Edelsbrunner, 2001; Zhu et al., 2006). We implement the Delaunay triangulation, which provides an optimal and non-overlapping connection between the neighbouring triangles from the data sets of points to create a 2D mesh (Shewchuk, 2002). The Delaunay triangulation does not require a predetermined equation for domain descriptions, which is relevant for our model because the internal and external feedback mechanisms influence the shape and domain of the peatland during the development process.

The peat stiffness, represented by Young's modulus, is modelled as a function of decomposition (Zhu et al., 2020) and plant functional types (PFT) (Whittington et al., 2007), following the formulation from Mahdiyasa et al. (2023)

$$E = \chi(1 + \theta_t^\zeta)(b_1c_1 + b_2c_2 + b_3c_3), \quad (7)$$

where E is the Young's modulus (Pa), χ is the first Young's modulus parameter (Pa), ζ is the second Young's modulus parameter (-), θ is the remaining mass (-), b_1, b_2, b_3 are the coefficient to couple PFT with Young's modulus (-), and c_1, c_2, c_3 are the PFT proportions (-) with the indices 1, 2, 3 indicating shrub, sedge, and *Sphagnum*, respectively. The interactions between peat stiffness and the load from plant weight, new layer addition, and body force determine the vertical and horizontal displacement of peat solid particles, which affects the bulk density and active porosity due to the changes in the peat volume. We propose the influence of solid displacement on the peat bulk density and active porosity in 2D as follows

$$170 \quad \rho_t = \frac{\rho_{t-1}}{1 + \beta_\rho \nabla \cdot \mathbf{u}}, \quad (8)$$

$$\phi_t = \frac{\phi_{t-1} + \beta_\phi \nabla \cdot \mathbf{u}}{1 + \nabla \cdot \mathbf{u}}, \quad (9)$$

where ρ is the bulk density (kg m^{-3}), β_ρ is the bulk density parameter ($-$), ϕ is the active porosity ($-$), β_ϕ is the active porosity parameter ($-$), and $\mathbf{u} = [u_x, u_y]^T$ is the displacement (m).

2.2 Ecological submodel

175 We use the formulation from Morris et al. (2015) for the peat production model, written as

$$\psi = 0.001(9.3 + 133z - 0.022(100z)^2)^2(0.1575Temp + 0.0091), \quad \text{for } 0 \leq z \leq 0.668 \quad (10)$$

$$\psi = 0, \quad \text{for } z > 0.668,$$

where ψ is the peat production ($\text{kg m}^{-2} \text{ yr}^{-1}$), z is the water table depth (m), and $Temp$ is the air temperature ($^\circ\text{C}$). Although the peat production model in Eq. (10) has a limitation related to the vegetation composition, this model can couple the ecological and hydrological processes through the dependency between peat production and water table depth. Furthermore, it also includes the effect of air temperature, which leads to a more realistic model. Another approach to model peat production is through the global Thornthwaite Memorial equation (Lieth, 1975) that simulates the primary productivity of the world. However, this model might omit the unique characteristics and the important feedback from the peatland ecosystem.

185 Peat production and the PFT proportion are employed to model the plant weight at the top surface through the following equation (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022; Moore et al., 2002)

$$Y = c_1 \left(10^{\frac{\log_{10}(\psi) + 0.409}{0.985}} \right) (1 + d_1)g + c_2 (10^{\log_{10}(\psi) + 0.001}) (1 + d_2)g + (c_3 0.144) (1 + d_3)g, \quad (11)$$

where Y is the plant weight (Pa), ψ is the peat production ($\text{kg m}^{-2} \text{ yr}^{-1}$), g is the acceleration of gravity (m s^{-2}), c_1, c_2, c_3 are the PFT proportions ($-$), and d_1, d_2, d_3 are the constants for plant wet condition ($-$) with the indices 1, 2, 3 indicating shrub, sedge, and *Sphagnum*, respectively. The proportions of PFT vary depending on the position of the water table, with the shrub becoming the dominant PFT in the low water table condition (Moore et al., 2002; Potvin et al., 2015; Kettridge et al., 2015). Therefore, we use a linear regression model from Mahdiyasa et al. (2023), which was developed from the Moore et al. (2002) data, to model the relationship between PFT proportions with the water table

$$c_1 = 2.23z - 0.28, \quad (12)$$

$$c_2 = -1.42z + 0.63, \quad (13)$$

$$195 \quad c_3 = -0.81z + 0.64, \quad (14)$$

where c_1, c_2, c_3 are the PFT proportions (–) with the indices 1, 2, 3 indicating shrub, sedge, and *Sphagnum*, respectively, and z is the water table depth (m). We assign the minimum value of each PFT proportion equal to zero if the value is negative and normalise the total proportion.

The decomposition processes occur in the saturated and unsaturated zones of the peatland at different rates. In the saturated zone below the water table, the rate of decay is low due to anoxic conditions, while in the unsaturated zone above the water table, the rate of decay is significantly higher as a consequence of oxic conditions that support the decomposition processes. We follow the model from Clymo (1984) to calculate the changes in peat mass due to the decomposition

$$\frac{dm}{dt} = -\eta m, \quad (15)$$

where m is the mass per unit area (kg m^{-2}) and η is the rate of decay (yr^{-1}). We do not include the influence of temperature and recalcitrance in the decomposition model because they will increase the number of empirical parameters and assumptions, which might lead to a higher uncertainty of the model. The effect of temperature on the decomposition process could be employed through Q_{10} parameter (Morris et al., 2015). However, this parameter has a high range of values between 1 and 10, which depends on the peatland types and temperature characteristics (Xiang and Freeman, 2009; Hardie et al., 2011). Moreover, the inclusion of the recalcitrance effect requires additional assumptions related to the changes in the rate of decay that could decline linearly or quadratically, as shown by Clymo et al. (1998).

The effect of decomposition is represented as the remaining mass, which is defined as the ratio between mass at time t , which has experienced decay, and the initial mass (Mahdiyasa et al., 2022; Baird et al., 2012; Morris et al., 2012; Morris et al., 2015)

$$\theta_t = \frac{m_t}{m_0}, \quad (16)$$

where θ is the remaining mass (–), m_t is the mass per unit area at time t (kg m^{-2}), and m_0 is the initial mass per unit area (kg m^{-2}).

2.3 Hydrological submodel

We model the peatland groundwater flows in 2D using the Boussinesq equation (Bartlett and Porporato, 2018) subject to net rainfall that acts as a source term (Cobb et al., 2017; Baird et al., 2012; Morris et al., 2012)

$$S_y \frac{\partial W}{\partial t} = \nabla \cdot (T \nabla W) + r, \quad (17)$$

where W is the water table height (m), S_y is the specific yield (–), T is the transmissivity ($\text{m}^2 \text{yr}^{-1}$), and r is the net rainfall (m yr^{-1}) that is defined as precipitation minus evapotranspiration. The Boussinesq equation is based on the Dupuit and Forchheimer (D-F) assumption (Bartlett and Porporato, 2018), which states that groundwater flows horizontally in unconfined aquifers. The D-F assumption is appropriate to model peatland groundwater flow because the peatland lateral distance is much wider than the thickness, which leads to the dominant horizontal flow. We assume that the height of the water table cannot

225 surpass the height of the peatland because the water will flow as surface water (Mahdiyasa et al., 2022; Morris et al., 2011). This would appear to be a realistic assumption because we do not simulate patterned peatlands. Consequently, the water table depth is obtained from the difference between peatland height and water table height as follows

$$z = h - W, \quad (18)$$

where z is the water table depth (m), h is the peatland height (m), and W is the water table height (m).

230 The mechanical deformation changes the peat pore structure, leading to variations in the active porosity (Eq. (9)) and influencing water flow through the pore space. Therefore, we implement the hydraulic conductivity model from Mahdiyasa et al. (2022), who formulate the changes in hydraulic conductivity as a function of active porosity

$$\kappa_t = \kappa_0 \left(\frac{\phi_t}{\phi_0} \right)^\xi, \quad (19)$$

235 where κ is the hydraulic conductivity (m s^{-1}), κ_0 is the initial value of hydraulic conductivity (m s^{-1}), ϕ is the active porosity ($-$), ϕ_0 is the initial value of active porosity ($-$), and ξ is the hydraulic conductivity parameter ($-$). Compared to the hydraulic conductivity model developed by Morris et al. (2022) from the meta-analysis of northern peat samples, Eq. 19 provides a more straightforward approach to analyse the influence of mechanical deformation on peatland hydrology.

2.4 Numerical verification

The verification is focused on the mechanical submodel, particularly the poroelasticity formulation, by comparing numerical 240 calculations with analytical solutions from Mandel's problem (Mandel, 1953). Uniform vertical load $2F$ is applied to a rectangular sample through a rigid and frictionless plate of width $2a$ and height $2H$, with drainage to the two sides in lateral condition as shown in Fig. 2. The deformation of the sample is forced to be in-plane strain condition by preventing all deformation in the direction perpendicular to the plane. The pore water pressure distribution will be homogeneous at the instant loading, but when drainage starts, the pore water pressure at two sides, $x = -a$ and $x = a$, are reduced to zero and followed 245 by the pore water pressure in the interior. Because the discharge has only a horizontal component, the pore water pressure, stress, and strain are independent of the y -coordinate. Furthermore, $\sigma_{xx} = 0$, $\sigma_{xy} = 0$, u_x is independent of y , and u_y is independent of x . Since the problem is symmetric, we solve only the upper right quadrant of the xy plane. We use 441 nodes and 800 elements to generate the simulations. The data for analytical and numerical solutions of this problem are stated in Table 1.

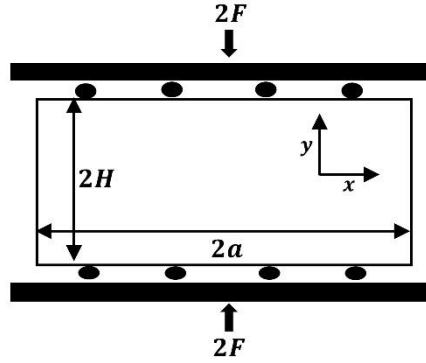
250 The analytical solutions to Mandel's problem for the pore water pressure, horizontal, and vertical displacement are (Cheng and Detournay, 1988; Abousleiman et al., 1996; Phillips and Wheeler, 2007)

$$p = \frac{2FB(1+\nu_u)}{3a} \sum_{i=1}^{\infty} \frac{\sin \omega_i}{\omega_i - \sin \omega_i \cos \omega_i} \left(\cos \frac{\omega_i x}{a} - \cos \omega_i \right) \exp \left(-\frac{\omega_i^2 c_p t}{a^2} \right), \quad (20)$$

$$u_x = \left[\frac{F\nu}{2Ga} - \frac{F\nu_u}{Ga} \sum_{i=1}^{\infty} \frac{\sin \omega_i \cos \omega_i}{\omega_i - \sin \omega_i \cos \omega_i} \exp\left(-\frac{\omega_i^2 c_v t}{a^2}\right) \right] x + \frac{F}{G} \sum_{i=1}^{\infty} \frac{\cos \omega_i}{\omega_i - \sin \omega_i \cos \omega_i} \sin \frac{\omega_i x}{a} \exp\left(-\frac{\omega_i^2 c_v t}{a^2}\right), \quad (21)$$

$$u_y = \left[-\frac{F(1-\nu)}{2Ga} + \frac{F(1-\nu_u)}{Ga} \sum_{i=1}^{\infty} \frac{\sin \omega_i \cos \omega_i}{\omega_i - \sin \omega_i \cos \omega_i} \exp\left(-\frac{\omega_i^2 c_v t}{a^2}\right) \right] y, \quad (22)$$

255 with $\tan \omega_i = \frac{1-\nu}{\nu_u - \nu} \omega_i$. In this analytical solution p is the pore water pressure (Pa), u_x is the horizontal displacement (m), u_y is the vertical displacement (m), F is the force (N), B is the Skempton's coefficient (-), G is the shear modulus (Pa), ν is the Poisson's ratio (-), and ν_u is the undrained Poisson's ratio (-).



260 **Figure 2: The illustration of Mandel's problem for the two-dimensional poroelasticity verification.**

Table 1: Input data for numerical and analytical solutions of Mandel's problem.

Name	Symbol	Value	Unit
Horizontal and vertical distance	a	1	m
Consolidation coefficient	c_v	0.17	$\text{m}^2 \text{s}^{-1}$
Force	F	2×10^4	N
Initial value of pore water pressure	p_0	1×10^4	Pa
Porosity	ϕ	0.375	-
Poisson's ratio	ν	0.2	-
Undrained Poisson's ratio	ν_u	0.5	-
Hydraulic conductivity	κ	1×10^{-5}	m s^{-1}
Specific storage	S_s	3.5×10^{-10}	m^{-1}
Bulk modulus	K	1.2×10^8	Pa
Skempton's coefficient	B	0.95	-
Shear modulus	G	4×10^7	Pa

The comparison between numerical and analytical solutions for Mandel's problem for normalised pore water pressure, normalised horizontal displacement, and normalised vertical displacement are shown in Fig. 3 at various dimensionless times $t^* = c_v t/a^2$. The mean absolute errors for normalised pore water pressure and displacement are small. The first variable,

normalised pore water pressure, has a mean absolute error of around 3.8×10^{-3} , 3.7×10^{-4} , and 5.2×10^{-6} at dimensionless time equal to 0.01, 0.1, and 0.5, respectively. For the second variable, normalised horizontal displacement, has a mean absolute error of around 2.8×10^{-3} and 1.4×10^{-6} at dimensionless time equal to 0.1 and 0.5, respectively. Finally, the mean absolute error of normalised vertical displacement is about 1.1×10^{-3} at dimensionless time equal to 0.1 and 5.8×10^{-7} at dimensionless time equal to 0.5.

Mandel's problem has an interesting characteristic related to the behaviour of pore water pressure. In the centre of the sample, the pore water pressure will be higher than the initial pressure for a small time interval. The value of normalised pore water pressure is greater than one at $t^* = 0.01$ and $t^* = 0.1$ (Fig. 3a). This phenomenon is denoted as the Mandel-Cryer effect, and it occurs due to the deformation and rigid plate conditions producing an additional source term for the pore water pressure distribution (Phillips and Wheeler, 2007; Duijn and Mikelic, 2021).

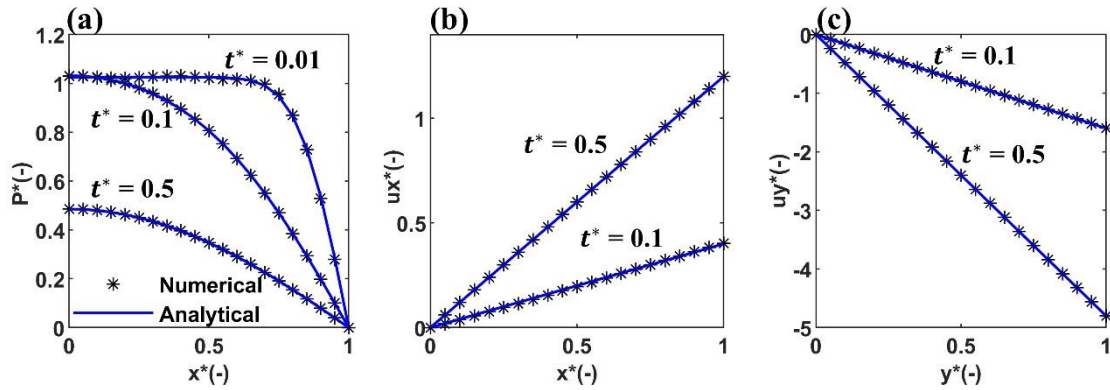


Figure 3: Numerical and analytical solutions of Mandel's problem at various dimensionless time $t^* = c_v t/a^2$ for (a) normalised pore water pressure $P^* = \frac{p}{p_0}$, (b) normalised horizontal displacement $u_x^* = \frac{u_x}{u_{x0}}$, and (c) normalised vertical displacement $u_y^* = \frac{u_y}{u_{y0}}$. The initial displacements in the horizontal and vertical directions are obtained from $u_{x0} = \frac{Fv_u x}{2Ga}$ and $u_{y0} = \frac{F(1-v_u)y}{2Ga}$, respectively. In this verification, $x^* = \frac{x}{a}$ is the normalised horizontal distance and $y^* = \frac{y}{a}$ is the normalised vertical distance.

3 Model implementation

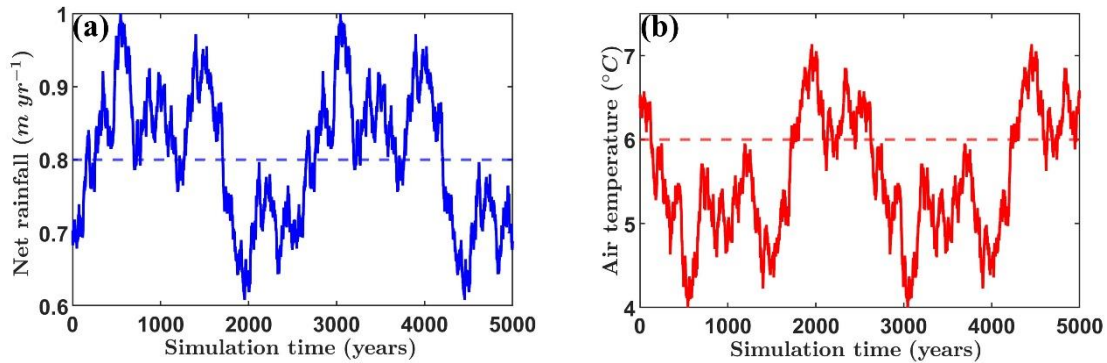
We simulate long-term peatland development over 5000 years with flat, impermeable, and rigid substrates, constrained by the parallel rivers at the edges (Ingram, 1982), with the parameter values summarised in Table 2. We assume the rivers do not incise, which could affect the water discharge (Glaser et al., 2004). Therefore, we implement no displacement and no flux boundary conditions at the bottom and zero pore water pressure at the edges. To reduce the computational time, we model half of the peatland domain from the central vertical axis to the one river with a distance of 500 m due to the symmetric growth assumption of the peatland (Baird et al., 2012; Morris et al., 2012). The boundary conditions for the central axis are impermeable without experiencing horizontal displacement.

The total load on this system is associated with the surficial peat addition (Eq. (10)), plant weight (Eq. (11)), and body force (Eq. (1)). The surficial peat addition and plant weight are applied at the surface, while the body force acts throughout the peatland area. The surface loadings are influenced by the peat production and vegetation composition consisting of shrub, sedge, and *Sphagnum*. Different from the surface loadings that are controlled by external sources, the body force is obtained from peatland self-weight, which is determined by the peat bulk density, water density, and active porosity.

To illustrate how MPeat2D works, we run two groups of simulations with different climate inputs. In the first group, we employ constant net rainfall (0.8 m yr^{-1}) and air temperature ($6 \text{ }^\circ\text{C}$) to provide the basic simulation related to the influence of mechanical-ecohydrological feedback and spatial heterogeneity of peat physical properties, water table depth, PFT proportion, and plant weight on peatland behaviour (Fig. 4). In the second group, we use a non-constant annual time series of net rainfall and air temperature generated from sinusoidal function with some noise (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022), with the range value of $0.6 - 1 \text{ m yr}^{-1}$ and $4 - 7^\circ\text{C}$, respectively (Morris et al., 2015; Young et al., 2021; Young et al., 2019). Through this approach, we can capture the wet and dry climatic influence on the long-term development of peatlands and maintain the simplicity of our climate reconstruction as the input variable.

Table 2: Parameter default values for the simulations.

Name	Symbol	Value	Unit	Reference
Unsaturated zone decay rate	η_{un}	5×10^{-2}	yr^{-1}	Clymo (1984)
Saturated zone decay rate	η_{sa}	8×10^{-5}	yr^{-1}	Clymo (1984)
Biot's coefficient	α	1	—	Terzaghi (1943)
Poisson's ratio	ν	0.2	—	Present study
Bulk density initial value	ρ_0	50	kg m^{-3}	Lewis et al. (2012)
Bulk density parameter	β_ρ	3	—	Present study
Active porosity initial value	ϕ_0	0.8	—	Quinton et al. (2000)
Active porosity parameter	β_ϕ	2	—	Present study
Hydraulic conductivity initial value	κ_0	1×10^{-2}	m s^{-1}	Hoag and Price (1995)
Hydraulic conductivity parameter	ξ	15	—	Mahdiyasa et al. (2022)
Specific yield	S_y	1.4×10^{-2}	—	Bourgault et al. (2017)
Degree of saturation of water	S_w	0.4	—	Mahdiyasa et al. (2022)
Water retention empirical constant 1	λ	0.5	—	Mahdiyasa et al. (2022)
Water retention empirical constant 2	μ	0.4	m^{-1}	Mahdiyasa et al. (2022)
Specific storage	S_s	1.4×10^{-2}	m^{-1}	Hogan et al. (2006)
Young's modulus parameter 1	χ	4×10^5	Pa	Present study
Young's modulus parameter 2	ζ	0.1	—	Mahdiyasa et al. (2022)
Shrub-Young's modulus parameter	b_1	1.25	—	Mahdiyasa et al. (2023)
Sedge-Young's modulus parameter	b_2	1	—	Mahdiyasa et al. (2023)
<i>Sphagnum</i> -Young's modulus parameter	b_3	0.75	—	Mahdiyasa et al. (2023)
Shrub constant	d_1	0.4	—	Mahdiyasa et al. (2022)
Sedge constant	d_2	0.4	—	Mahdiyasa et al. (2022)
<i>Sphagnum</i> constant	d_3	20	—	Mcneil and Waddington (2003)



310 **Figure 4: The climate profile for (a) net rainfall and (b) air temperature over 5000 years under constant and non-constant conditions. The values of constant climate are 0.8 m yr^{-1} and $6 \text{ }^\circ\text{C}$ indicated by dashed lines, while the non-constant climate specified by continuous lines fluctuating between 0.6 and 1 m yr^{-1} for net rainfall and between 4 and $7 \text{ }^\circ\text{C}$ for air temperature.**

We compare the results obtained from MPeat2D with the previously developed mechanical-ecohydrological model of peatland development in 1D called MPeat (Mahdiyasa et al., 2022; Mahdiyasa et al., 2023) and with the 2D ecohydrological model of peatland growth that ignores the mechanical feedback DigiBog (Morris et al., 2012; Baird et al., 2012). The comparison with

315 MPeat is conducted for the water table depth, peatland height, and cumulative carbon from the centre area using the same parameters and climatic influence summarised in Table 2 and Fig. 4, respectively. In both models, cumulative carbon is obtained from cumulative mass multiplied by 47% of the carbon content (Loisel et al., 2014). The comparison with DigiBog employs the model version provided by Baird et al. (2012) and Morris et al. (2012), which is run under a constant climate with the value of bulk density equal to 100 kg m^{-3} and the hydraulic conductivity parameters a and b equal to $1 \times 10^{-5} \text{ m s}^{-1}$ and

320 8, respectively. We chose this model version of DigiBog because it has similar characteristics to MPeat2D, including the flat and impermeable substrate with the symmetric assumption of peatland growth. Moreover, both models also assume that water ponding was lost immediately to the margin. In contrast, the more recent DigiBog versions (e.g., Young et al., 2017; Young et al., 2019; Young et al., 2021) produce incomparable simulation results to the MPeat2D because of the different and complex parametrisation and setup. The more recent DigiBog versions employ a layer lumping system after some specific time and

325 thickness. This approach results in faster simulation because it reduces the number of layers that become the domain of calculation. However, the different parametrisation of the layer lumping appears to change the results and stability of the DigiBog. Furthermore, the more recent version includes the parameter of mineral soil and water ponding thickness, which also influences the model outputs. These additional features and parameters lead to incomparable conditions with MPeat2D. The

output variables from the comparison between MPeat2D and DigiBog are peatland shape in cross section, thickness, and water
330 table depth.

The sensitivity analysis of MPeat2D is conducted by changing the first Young's modulus parameter χ to 3.5×10^5 Pa and
 4.5×10^5 Pa. We performed one at a time sensitivity analysis by focusing on the variation of one parameter and set all other
parameters to remain the same as the baseline value (Table 2). The first Young's modulus parameter χ is chosen because it
determines the peat stiffness, which in turn influences the mechanical deformation of the peat body. Output variables examined
335 from the sensitivity analysis include the spatial variations of bulk density, active porosity, and hydraulic conductivity.

4 Simulation results

4.1 First group: Constant climate

In the initial stage of development, the peatland shape is relatively flat, with a low value of bulk density and a high value of
active porosity and hydraulic conductivity. By 5000 years, a dome-shaped peatland is produced with the maximum thickness
340 obtained at the centre and decreases toward the margin. The increasing thickness leads to higher loading and a more significant
deformation effect on the peat pore structure, which affects the peat physical properties. The changes in peat physical properties
during the development process exhibit spatial variabilities in the vertical and horizontal directions with the range value
between 50 and 100 kg m^3 , 0.49 and 0.8, and 5.5×10^{-6} and $1 \times 10^{-2} \text{ m s}^{-1}$ for bulk density, active porosity, and
hydraulic conductivity, respectively (Fig. 5).

345

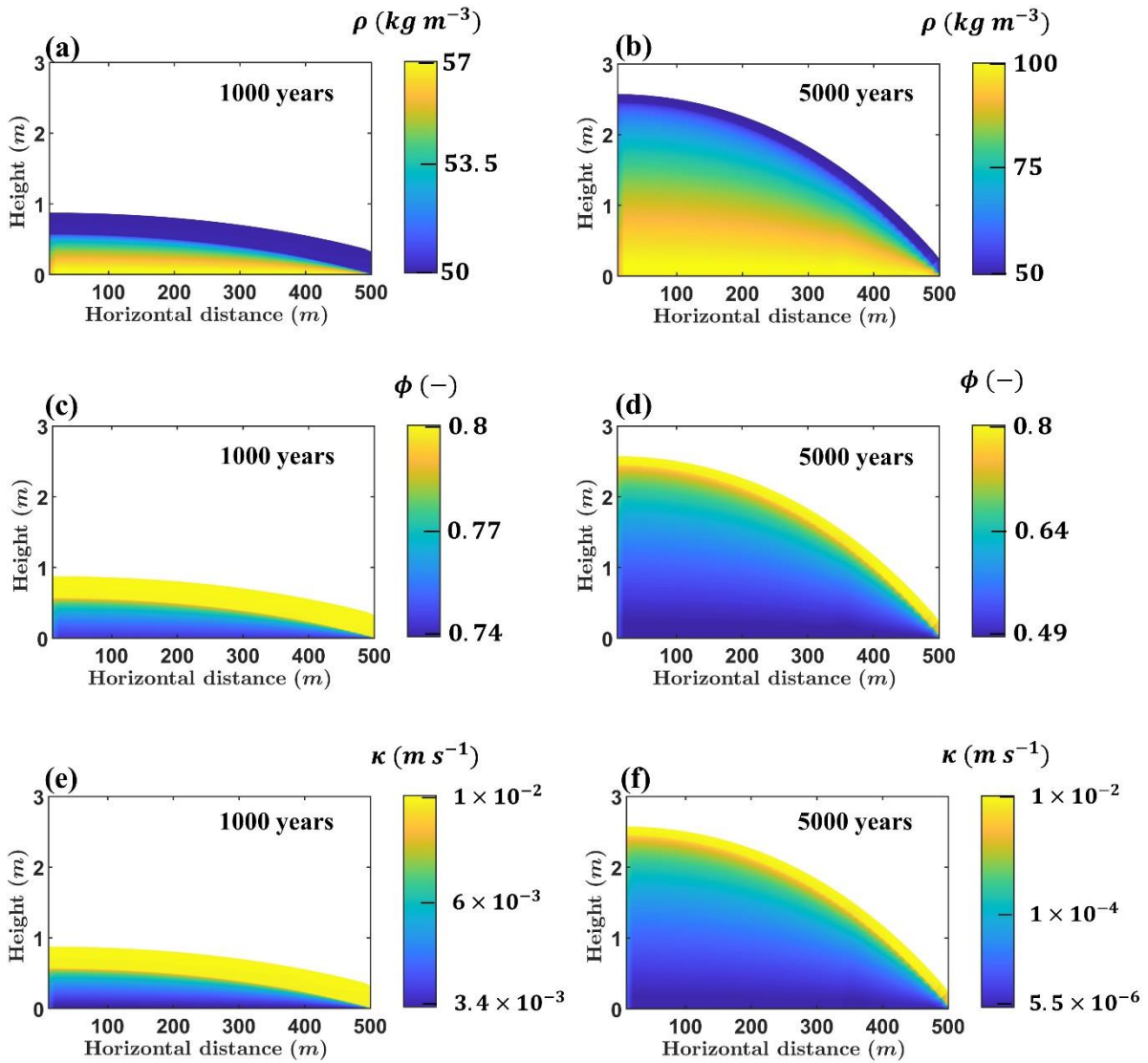


Figure 5: The profiles of (a and b) bulk density, (c and d) active porosity, and (e and f) hydraulic conductivity with spatial variability in the vertical and horizontal directions under constant climate.

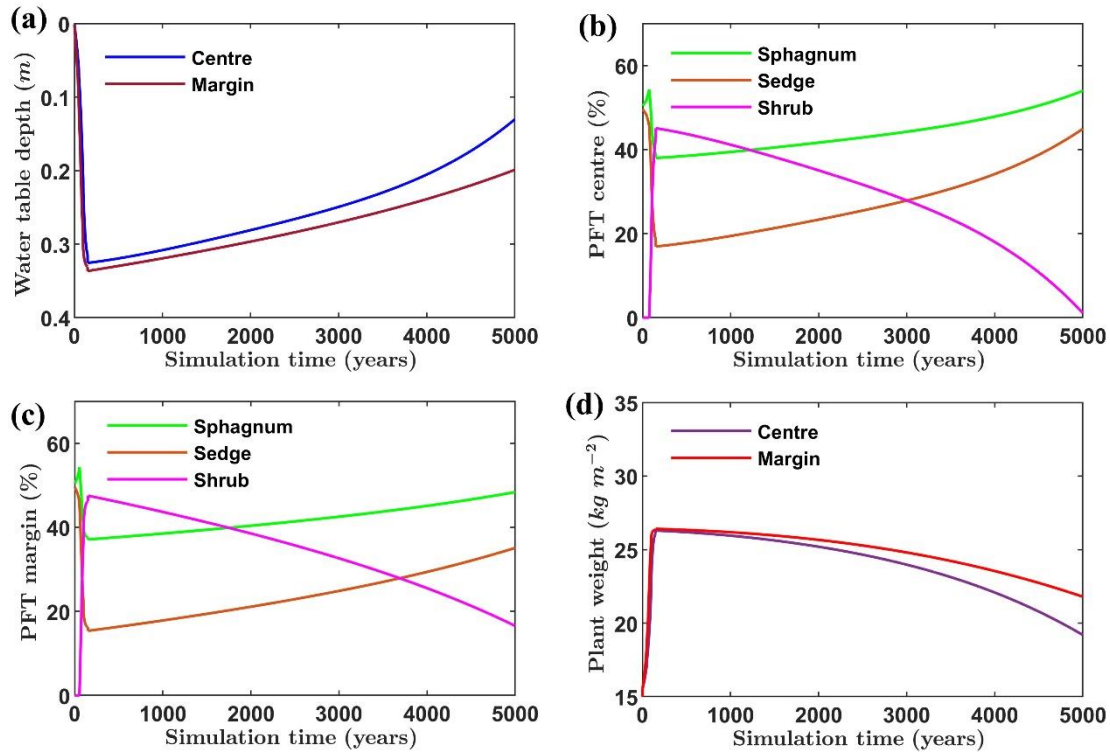


Figure 6: The variations of (a) water table depth, (b) plant functional types (PFT) proportion at the centre, (c) PFT proportion at the margin, and (d) plant weight over 5000 years under constant climate. The centre is defined at the central vertical axis, while the margin is located at a horizontal distance of 500 m from the centre.

355 Over 5000 years of development, water table depth decreases, resulting in the wetter condition of the peatland (Fig. 6). This condition occurs because the loading from peat accumulation increases as the peatland grows, which provides internal feedback mechanisms on the water balance through the deformation of peat pore space. The difference in the final simulation year between the water table depth at the centre (0.13 m) and at the margin (0.20 m), which is separated by a horizontal distance of 500 m, leads to the variation in vegetation composition and plant weight. The proportion of shrub is lower at the centre compared to the margin, with the value of about 1% and 17%, respectively. In contrast, the sedge and *Sphagnum* proportions reduce from around 45% and 54% at the centre to 35% and 48% at the margin, respectively. The variations in the vegetation composition affect the distribution of plant weight, with the centre (19 kg m⁻²) providing a lower value of loading than the margin (22 kg m⁻²).

360

365 4.2 Second group: Non-constant climate

Under a non-constant climate, the profiles of peat physical properties are similar to those in the constant climate case. The value of bulk density increases while active porosity and hydraulic conductivity decrease from the centre toward the margin (Fig. 7). The main difference is that the range values of bulk density (50 – 104 kg m³), active porosity (0.47 – 0.8), and

hydraulic conductivity ($3.7 \times 10^{-6} - 1 \times 10^{-2} \text{ m s}^{-1}$) are higher in the second group after 5000 years. This condition
 370 indicates a more significant effect of mechanical deformation on the peat pore space due to the changing climate.

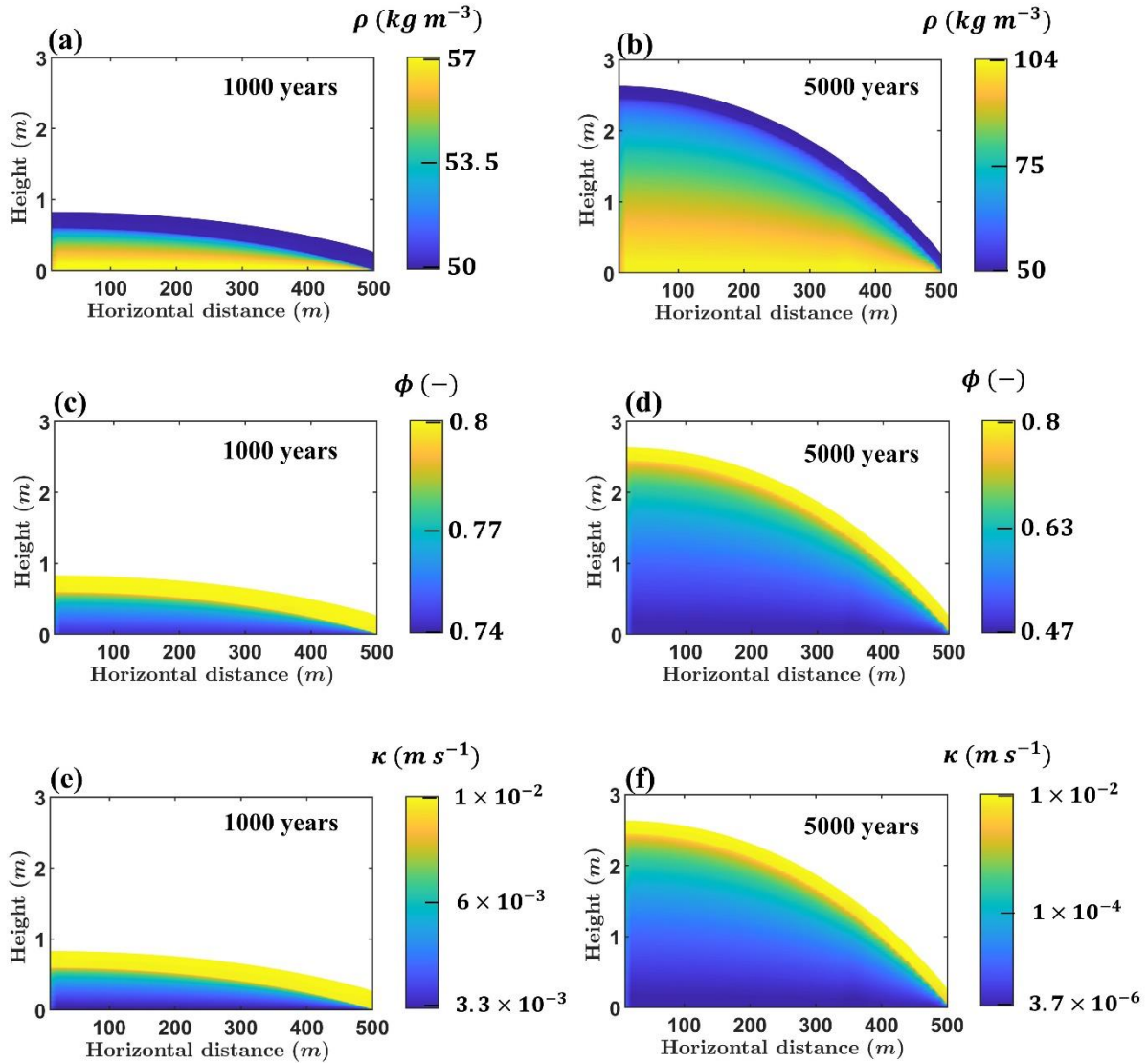
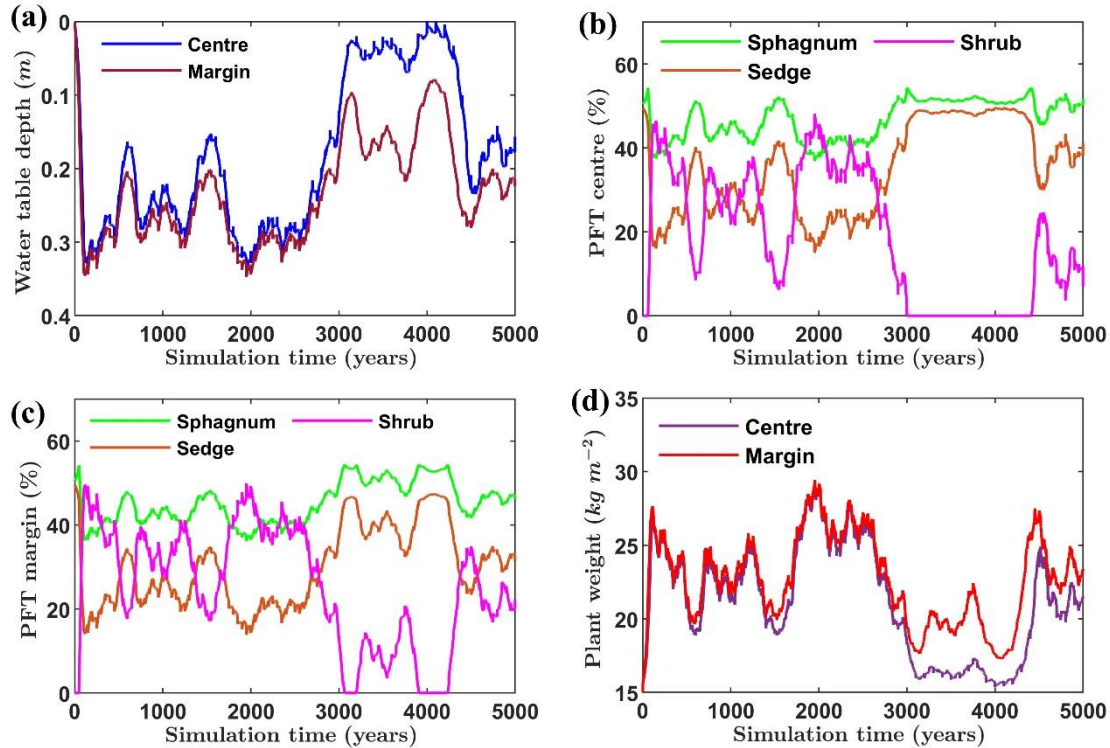


Figure 7: The profiles of (a and b) bulk density, (c and d) active porosity, and (e and f) hydraulic conductivity with spatial variability in the vertical and horizontal directions under non-constant climate.

After the unsaturated zone is developed, around 150 years since peatland initiation, water table depth experiences fluctuations
 375 and exhibits lateral variability. The margin, which is located at a horizontal distance of 500 m from the centre, experiences drier conditions indicated by a higher water table depth compared to the centre (Fig. 8). Furthermore, the spatial variability of water table depth results in lateral changes in PFT proportions and plant weight. For example, around the year 3750, the water table depth is about 0.07 m at the centre, while at the margin, the water table is located about 0.22 m below the surface.

Consequently, the shrub proportion increases from 0% at the centre to 21% at the margin, while sedge and *Sphagnum* decrease from 48% and 52% to 32% and 47% from the centre to the margin, respectively. This condition produces a spatial variation in plant weight between the centre and the margin with values of about 17 kg m⁻² and 22 kg m⁻².



385 **Figure 8: The variations of (a) water table depth, (b) plant functional types (PFT) proportion at the centre, (c) PFT proportion at the margin, and (d) plant weight over 5000 years under non-constant climate. The centre is defined at the central vertical axis, while the margin is located at a horizontal distance of 500 m from the centre.**

4.3 Comparison between MPeat2D with MPeat and DigiBog

390 The comparison between MPeat2D and MPeat is conducted based on the simulation at the centre of the peatland. Under constant climate, the emergence of the unsaturated zone, represented by the non-zero values of water table depth, is faster in the MPeat2D than in the MPeat, with a difference of about 360 years (Fig. 9a). Moreover, the water table depth obtained from MPeat2D (0.13 m) is lower than MPeat (0.3 m) in the final simulation year. Although MPeat2D and MPeat estimate similar peatland height with values of about 2.57 m and 2.52 m (Fig. 9b), respectively, the cumulative carbon obtained from
 395 MPeat2D (89 kg C m⁻²) is higher compared to the MPeat (84 kg C m⁻²) over 5000 years (Fig. 9c).

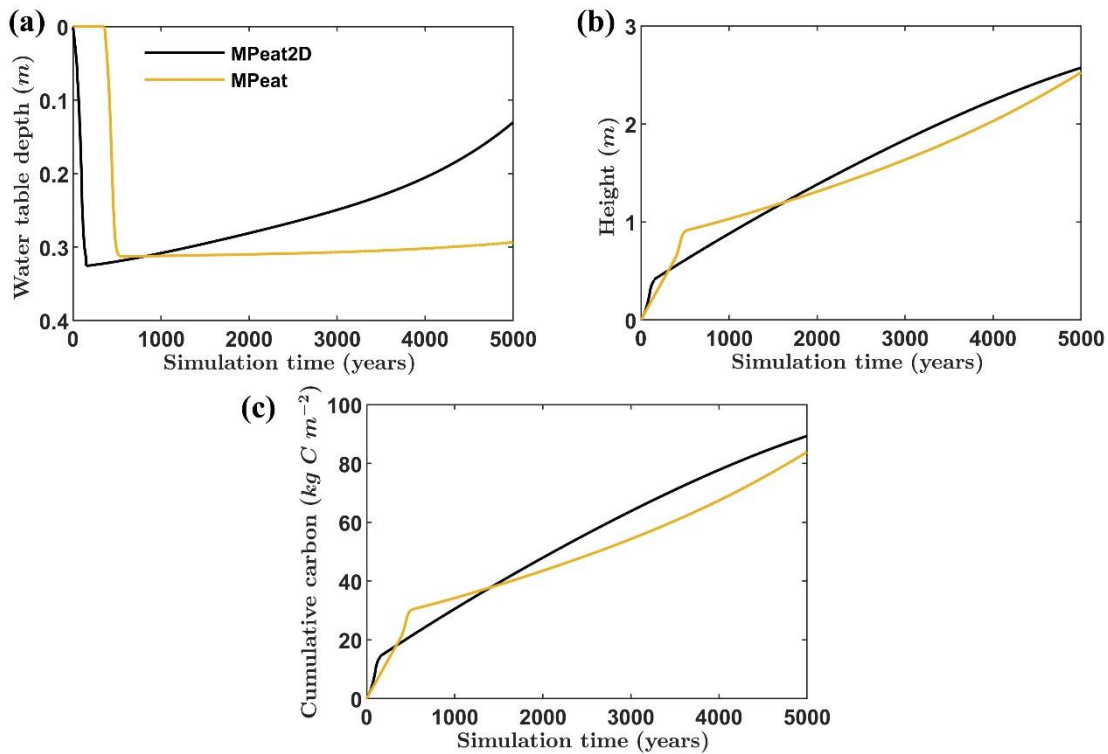
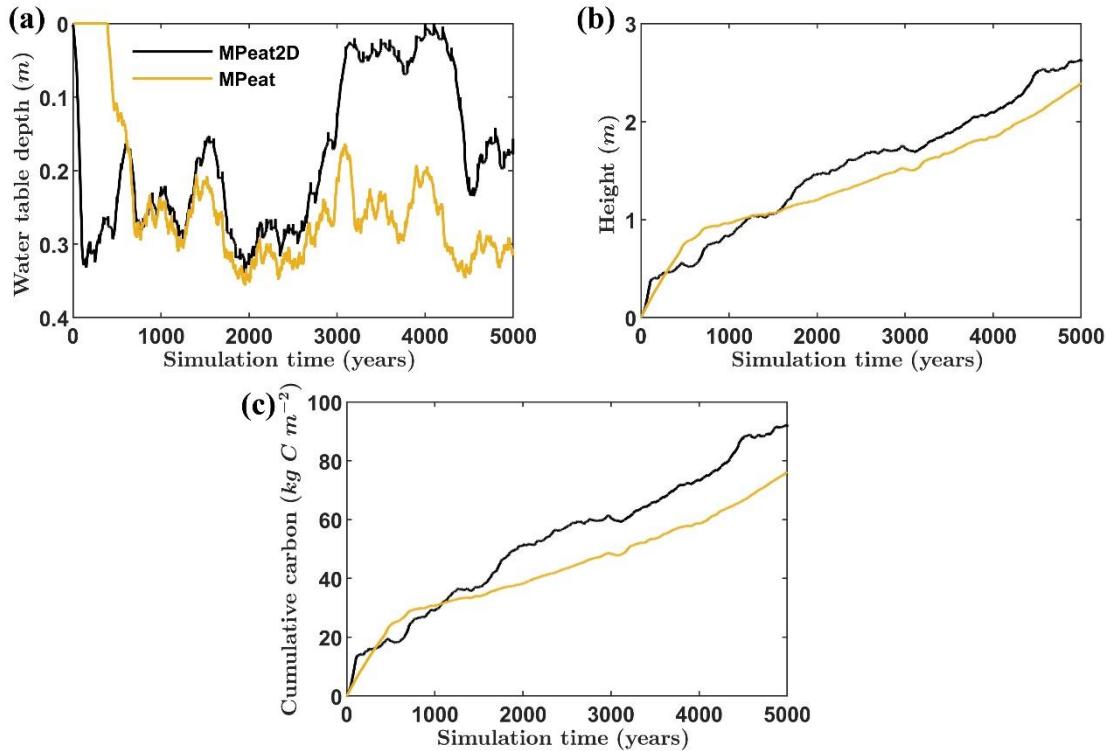


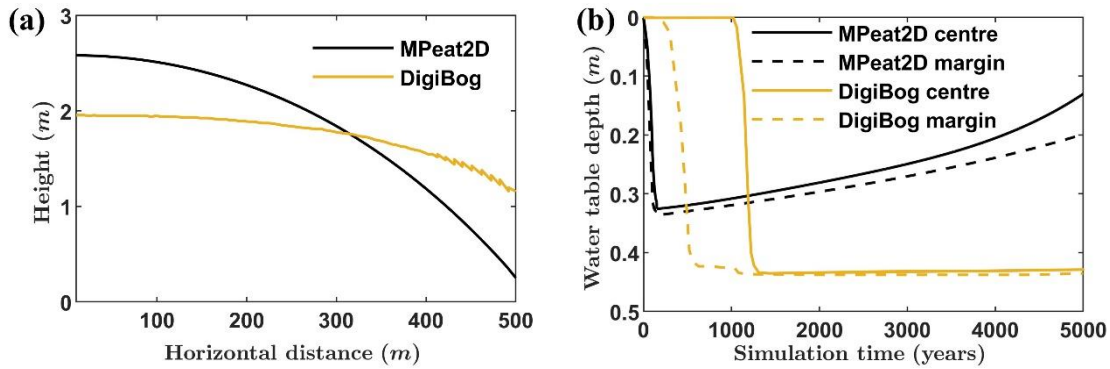
Figure 9: The comparison between MPeat2D and MPeat at the centre for (a) water table depth, (b) peatland height, and (c) cumulative carbon over 5000 years under constant climate.

400 The comparisons between MPeat2D and MPeat for water table depth, peatland height, and cumulative carbon at the centre
 area under non-constant climate are shown in Fig. 10. The appearance of the unsaturated zone is around 400 years earlier in
 the MPeat2D than MPeat based on the non-zero values of the water table depth since peatland initiation. After the unsaturated
 zone is developed in both models, MPeat2D predicts a lower water table depth compared to the MPeat with the range value of
 0 – 0.34 m and 0.16 – 0.36 m, respectively. Peatland height and cumulative carbon obtained from MPeat2D around 2.61 m
 405 and 92 kg C m^{-2} are more significant than the MPeat estimated values of about 2.39 m and 76 kg C m^{-2} , after 5000 years.



410 **Figure 10: The comparison between MPeat2D and MPeat at the centre for (a) water table depth, (b) peatland height, and (c) cumulative carbon over 5000 years under a non-constant climate.**

The comparison between MPeat2D and DigiBog is run only under a constant climate due to the limitations of the chosen DigiBog version (Morris et al., 2012; Baird et al., 2012). Over 5000 years, the peatland height at the centre obtained from MPeat2D is higher compared to DigiBog, with a difference of about 0.6 m (Fig. 11a). Furthermore, both models produce
 415 peatland shapes that experience the formation of a cliff at the margin. The height of the cliffs obtained from MPeat2D and DigiBog are about 0.27 m and 1.15 m, respectively. The spatial variations of water table depth between the centre and the margin are more significant in the MPeat2D compared to DigiBog (Fig. 11b). In the final simulation year, MPeat2D simulates water table depth with values of about 0.13 m at the centre and 0.2 m at the margin, while DigiBog produces water table depth of around 0.43 m and 0.44 m at the centre and margin, respectively.

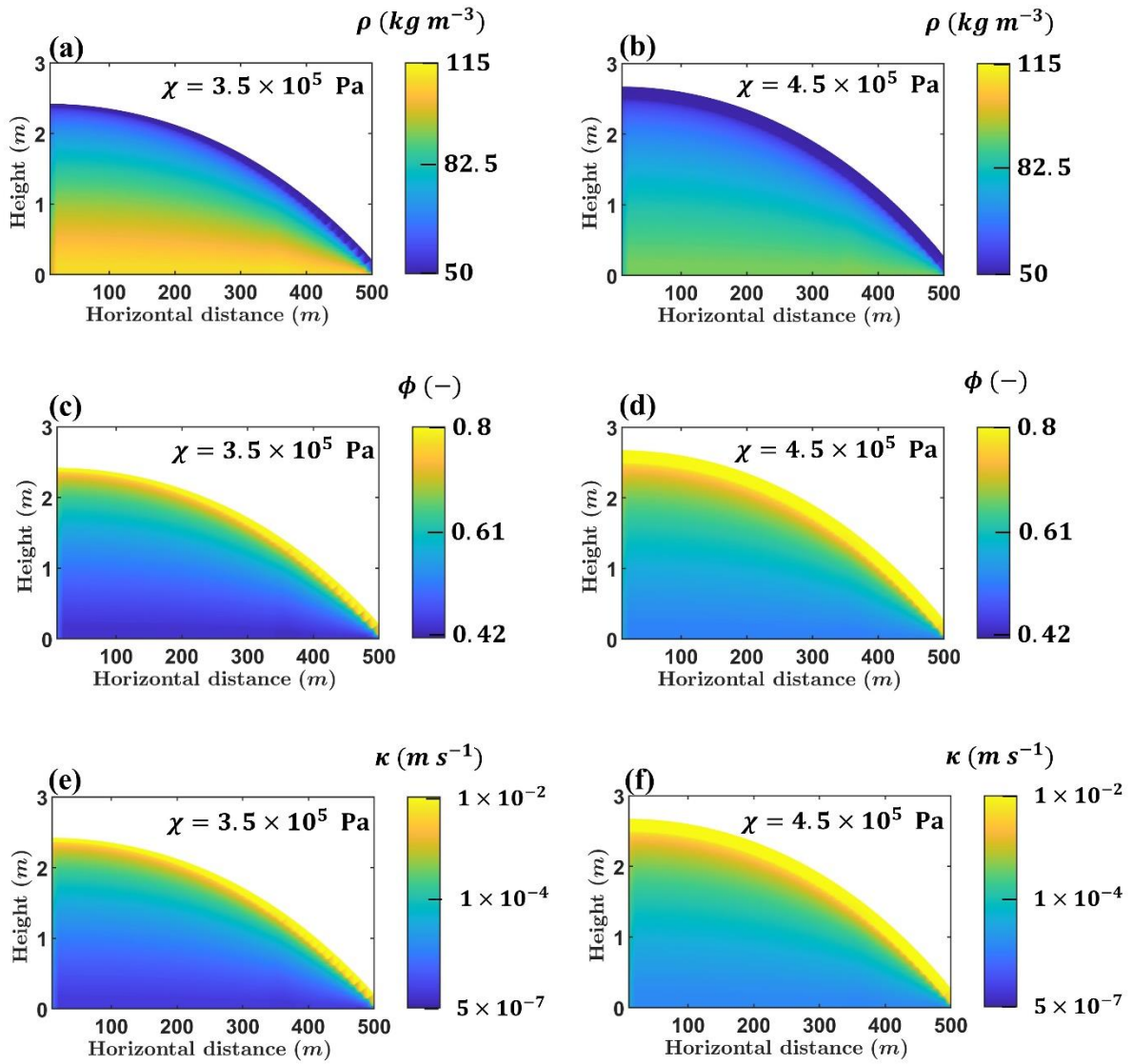


420 **Figure 11: The comparison between MPeat2D and DigiBog Bog 2 (Morris et al., 2012) for (a) peatland shape and (b) water table depth over 5000 years. Both models assume that the peatland develops above the flat and impermeable substrate with constant climate.**

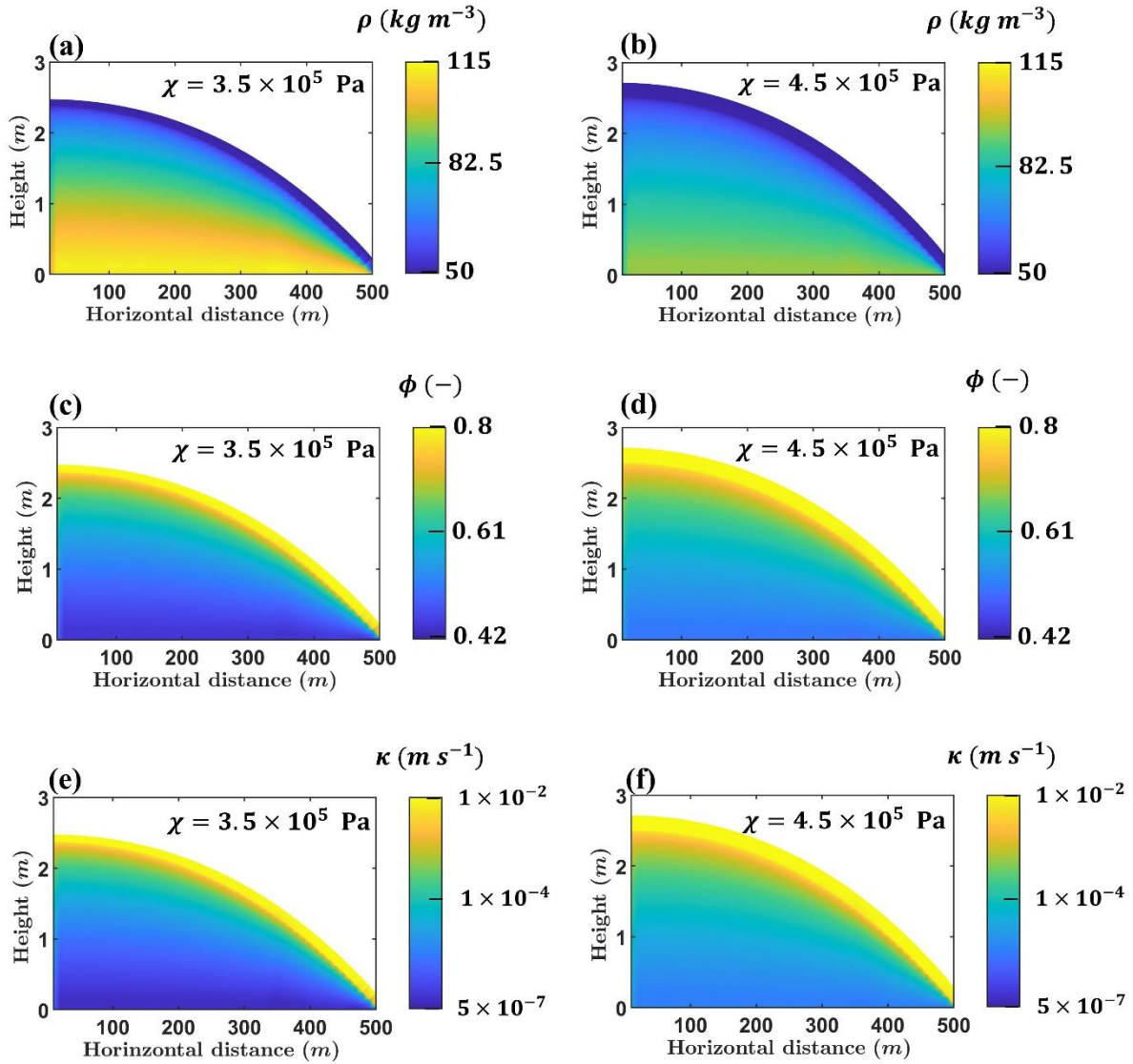
425 4.4 Sensitivity analysis

We changed the first peat Young's modulus parameter χ , which is an essential variable in the MPeat2D because it determines the peat stiffness and the compaction effect on the peat pore structure. Under constant climate (Fig. 12), reducing the first peat Young's modulus parameter χ to 3.5×10^5 Pa led to a higher value of bulk density ($50 - 111 \text{ kg m}^3$) and lower values of active porosity ($0.44 - 0.8$) and hydraulic conductivity ($1.2 \times 10^{-6} - 1 \times 10^{-2} \text{ m s}^{-1}$). The more significant effect of compaction due to a lower first peat Young's modulus parameter χ results in a decreasing peatland height at the centre by about 6% compared to the baseline value after 5000 years. Contrastingly, increasing the first peat Young's modulus parameter χ to 4.5×10^5 Pa produced a lower bulk density ($50 - 94 \text{ kg m}^3$) and higher active porosity ($0.52 - 0.8$) and hydraulic conductivity ($1.6 \times 10^{-5} - 1 \times 10^{-2} \text{ m s}^{-1}$). These conditions increased the peatland height at the centre by about 4% compared to the baseline value in the final simulation year.

435 Under non-constant climate (Fig. 13), the influence of parameter χ on the peat physical properties and thickness is similar to that observed in the constant climate scenario. Reducing χ to 3.5×10^5 Pa resulted in a higher bulk density ($50 - 114 \text{ kg m}^3$) and lower active porosity ($0.42 - 0.8$) and hydraulic conductivity ($6.9 \times 10^{-7} - 1 \times 10^{-2} \text{ m s}^{-1}$). As a consequence, the peatland height at the centre was reduced by about 5% compared to the baseline value after 5000 years. In contrast, increasing χ to 4.5×10^5 Pa led to a lower bulk density ($50 - 96 \text{ kg m}^3$) and higher active porosity ($0.51 - 0.8$) and hydraulic conductivity ($1.2 \times 10^{-5} - 1 \times 10^{-2} \text{ m s}^{-1}$), which in turn produced a more significant peatland thickness at the centre by about 4% compared to the baseline value over 5000 years of simulations.



445 **Figure 12: The sensitivity analysis of MPeat2D by changing the first Young's modulus parameter χ with the output variables are (a and b) bulk density, (c and d) active porosity, and (e and f) hydraulic conductivity under constant climate over 5000 years. In the base run χ is equal to $4 \times 10^5\ Pa$.**



450 **Figure 13: The sensitivity analysis of MPeat2D by changing the first Young's modulus parameter χ with the output variables are (a and b) bulk density, (c and d) active porosity, and (e and f) hydraulic conductivity under non-constant climate over 5000 years. In the base run χ is equal to $4 \times 10^5 \text{ Pa}$.**

5 Discussion

The most important result from MPeat2D is the ability to model the influence of spatial variability on long-term peatland behaviour. The addition of the second dimension provides significant impacts on the analysis of peat physical properties because it allows the bulk density, active porosity, and hydraulic conductivity to change in the horizontal and vertical directions. We found that the bulk density increases systematically from the centre to the margin, while the active porosity and

455

hydraulic conductivity experience an opposite pattern with decreasing values from the peatland interior to the edges (Figs. 5 and 7). The horizontal variability of the peat physical properties is not only caused by different effects of decomposition (Morris et al., 2012) but also by different effects of compaction between the margin and the centre. The steeper hydraulic gradient at the margin promotes water release and reduces the position of the water table (Figs. 6a and 8a) (Reeve et al., 2006; 460 Lewis et al., 2012; Kværner and Snilsberg, 2011; Regan et al., 2019), which results in higher loading from plant weight (Figs. 6d and 8d) and effective stress. In contrast, peatland topography at the centre is mainly flat, leading to the shallow water table position that limits the deformation of the peat pore space.

At smaller scales of a few meters, another possible factor affecting the horizontal variance of peat physical properties is the peatland microform. The measurement from Whittington and Price (2006) indicated that bulk density and hydraulic 465 conductivity differ substantially in the lateral direction over distances of a few metres between hummocks, lawns, and hollows. Moreover, Baird et al. (2016) showed that the difference in the hydraulic conductivity between contiguous microforms could vary by more than an order of magnitude. The variation in the water table position and plant functional types between peatland microforms (Eppinga et al., 2008; Malhotra et al., 2016; Moore et al., 2019), which significantly affect the loading, effective stress, and compaction on the peat pore space, might become a reasonable explanation for this behaviour. However, Baird et 470 al. (2016) found that the change in hydraulic conductivity is less evident at a deeper location between adjacent hummocks and hollows, which suggests that the lateral variability of hydraulic conductivity at the small scale between the microhabitat types beyond the uppermost peat is less clear.

The changes of peat physical properties in the vertical direction, from the top surface to the bottom layer, obtained from MPeat2D show an increasing value of bulk density and a decreasing value of active porosity and hydraulic conductivity 475 (Clymo, 1984; Hoag and Price, 1995; Hoag and Price, 1997; Quinton et al., 2008; Quinton et al., 2000; Fraser et al., 2001). The rapid changes occur at the transition between the unsaturated and saturated zones, indicating significant compaction due to the substantial increase of effective stress (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022). The fluctuations of peat physical properties become gradual in the saturated zone because pore water pressure reduces the effective stress and limits the deformation of the peat solid skeleton. Price (2003) found the decreasing value of effective stress below the water table that 480 leads to smaller changes in peat volume, which supports our simulation results.

The inclusion of spatial heterogeneity provides crucial feedback on peat thickness and carbon stock, as shown by the comparison between MPeat2D and MPeat (Figs. 9 and 10). MPeat2D simulates spatial variations in the peat physical properties and produces a non-uniform hydraulic gradient, including a lower hydraulic conductivity at the margin and nearly flat topography at the centre, which supports the water accumulation. In contrast, as a 1D model, MPeat assumes constant peat 485 physical properties in the lateral direction and a uniform hydraulic gradient, resulting in the omission of peatland processes that affect the water balance. Consequently, MPeat2D simulates a shallower water table position than MPeat, leading to the shorter residence time of organic matter in the unsaturated zone and providing positive feedback on the peat and carbon accumulation (Evans et al., 2021; Huang et al., 2021; Ma et al., 2022). The differences between MPeat2D and MPeat are more

pronounced under a non-constant climate (Fig. 10), indicating the potential importance of spatial variability to understand the influence of climate change on the peatland carbon balance and resilience.

Our simulation results are in agreement with Lapen et al. (2005), who found that lateral variations of hydraulic conductivity encourage water accumulation and produce a more significant peat thickness. However, Lapen et al. (2005) based their finding on a sensitivity analysis of a steady-state groundwater model, which omits the complex feedback from the peatland. Conversely, MPeat2D provides a comprehensive approach incorporating mechanical, ecological, and hydrological feedback to highlight the influence of spatial variations of physical properties and water table position on the peat and carbon accumulation during development process.

5.1 Comparison with field measurements

We compare the simulation results from MPeat2D under a non-constant climate that provides a more realistic condition with field observation for peat physical properties, peat thickness, and carbon accumulation. The comparison of peat physical properties is conducted in the horizontal and vertical directions due to the spatial heterogeneity of bulk density, active porosity, and hydraulic conductivity. In the horizontal direction, we use the data from Lewis et al. (2012), who measured the lateral variabilities of hydraulic conductivity and bulk density at a depth of 30 to 40 cm from a blanket peatland in Ireland as a comparison. Although the peatland type from Lewis et al. (2012) is different from our simulations, the main reason for the comparison is to demonstrate the ability of the model to produce reasonable outputs of the spatial variability on peat physical properties. Lewis et al. (2012) found that the average values of hydraulic conductivity at the margin and the centre are around 10^{-6} and 10^{-4} m s^{-1} , respectively. After 5000 years of development, MPeat2D produces hydraulic conductivity (Fig. 7f) with a similar value at the margin (6.4×10^{-6} m s^{-1}) but higher at the centre (1.3×10^{-3} m s^{-1}) compared to the Lewis et al. (2012) observations. Moreover, the bulk density values obtained from Lewis et al. (2012) are around 55 and 110 kg m^{-3} , while our simulation (Fig. 7b) provides values of about 59 and 101 kg m^{-3} at the centre and margin, respectively. In the vertical direction, the changes in bulk density simulated from MPeat2D (Fig. 7b) are in the range of 50 – 104 kg m^{-3} , consistent with the reported measurements of bulk density about 30 – 120 kg m^{-3} (Lunt et al., 2019; Loisel et al., 2014; Clymo, 1984). Furthermore, the simulation results of active porosity (Fig. 7d) and hydraulic conductivity (Fig. 7f) fluctuate between 0.47 – 0.8 and 3.7×10^{-6} – 1×10^{-2} m s^{-1} , which are in accord with the field observations of active porosity and hydraulic conductivity about 0.1 – 0.8 and 7×10^{-9} – 1.6×10^{-2} m s^{-1} , respectively (Quinton et al., 2008; Quinton et al., 2000; Clymo, 2004; Fraser et al., 2001; Hoag and Price, 1997; Hoag and Price, 1995). Therefore, in general, MPeat2D can model the spatial variability of peat physical properties in the horizontal and vertical directions with reasonable outputs. The discrepancies between simulation results and the field measurement mainly related to the site specific characteristics, including peat stiffness, PFT composition, and substrate topography that result in the variations of compaction effect (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022; Whittington et al., 2007; Malmer et al., 1994).

520 The peat thickness and carbon accumulation rate from MPeat2D, which are calculated based on the total amount of peat and carbon with the total simulation time, appear to be realistic. MPeat2D produces an average growth rate of about 0.52 mm yr^{-1} , which leads to a height of 2.61 m after 5000 years (Fig. 10b). Charman (2002) found that the average growth rates of the blanket and raised peatland are about 0.65 and 1 mm yr^{-1} , respectively, based on the relation between peatland carbon accumulation and age-depth curves. Aaby and Tauber (1975) analysed the correlation between the rate of peat accumulation and the degree of humification that produced the growth rate of raised peatland in the range of $0.16 - 0.80 \text{ mm yr}^{-1}$ with an average value of 0.44 mm yr^{-1} . Aaby and Tauber (1975) suggested that the relationship between the degree of humification and the growth rate is affected significantly by mechanical compaction. A more decomposed peat experiences a higher compaction effect due to the reduction in Young's modulus and strength (Mahdiyasa et al., 2023; Mahdiyasa et al., 2022), which results in a lower peat thickness. Furthermore, the average value of the net rate of carbon accumulation obtained from MPeat2D is about $0.0183 \text{ kg C m}^{-2} \text{ yr}^{-1}$ (Fig. 10c), which is in agreement with the reported measurements of northern peatlands during the Holocene with an average value around $0.0186 \text{ kg C m}^{-2} \text{ yr}^{-1}$ (Yu et al., 2010; Yu et al., 2009).

5.2 Comparison with the other two-dimensional peatland development model

We emphasise the critical function of mechanical-ecohydrological feedback to simulate peatland development in 2D by comparing MPeat2D with the other ecohydrological model DigiBog (Baird et al., 2012; Morris et al., 2012). Using the same assumption of the flat and impermeable substrate with constant climate, both models produce dome shapes of the peatland over 5000 years (Fig. 11a). However, the inclusion of mechanical-ecohydrological feedback on MPeat2D provides a plausible profile of bulk density (Fig. 5b) and active porosity (Fig. 5d) that are assumed to be a constant by DigiBog. Consequently, in the early stage of development, the value of bulk density from MPeat2D is lower than DigiBog, producing a more rapid increase in peat thickness and a faster appearance of the unsaturated zone (Fig. 11b). As peatland grows, the changes in peat physical properties and the discrepancy in the hydraulic gradient obtained from MPeat2D lead to the spatial variation of water table depth, which is in line with the field observations (Reeve et al., 2006; Lewis et al., 2012; Kværner and Snilsberg, 2011; Regan et al., 2019). Furthermore, the water table depth decreases as the peatland develops in the MPeat2D, resulting in wetter conditions of the peatland due to a more significant effect of compaction that supports the water accumulation. Contrastingly, DigiBog produces a relatively uniform hydraulic gradient, leading to the constant water table depth between the centre and the margin during the simulation period. This condition limits the capabilities of DigiBog to analyse the lateral variation in peat production, decomposition, and PFT proportion because these peatland characteristics depend on the water table depth (Clymo, 1984; Belyea and Clymo, 2001; Moore et al., 2002; Kokkonen et al., 2019; Laine et al., 2021).

The inclusion of mechanical-ecohydrological feedback also produces a more plausible shape of the peatland in 2D. Although MPeat2D suffers from the appearance of a cliff at the margin as DigiBog (Fig. 11a), the influence of mechanical compaction on the MPeat2D results in a lower cliff height than DigiBog. The peat cliff at the margin does not appear in the natural condition except due to extraction or erosion (Tuukkanen et al., 2017; Tarvainen et al., 2022). Furthermore, the continuum

concept (Irgens, 2008; Jog, 2015) employed by MPeat2D produces continuous deformation of peat pore space, resulting in the smoother profile of peatland shape, especially near the margin, than DigiBog that uses linked vertical column (Fig. 11a). The comparison of the predicted peatland shape from MPeat2D with the Groundwater Mound Hypothesis (GMH) (Ingram, 1982), indicates that MPeat2D produces a lower hydraulic gradient at the margin compared to the hemi-ellipse shape from GMH. The hemi-ellipse shape is obtained by assuming constant hydraulic conductivity throughout the peatland area, which is not true because the field observation from Baird et al. (2008) and Lewis et al. (2012) showed that hydraulic conductivity change in the vertical and horizontal directions. Armstrong (1995) modified GMH by proposing non-uniform hydraulic conductivity that exponentially decreases with depth, resulting in a lower hydraulic gradient at the margin and a more significant thickness at the centre, which agrees with MPeat2D.

5.3 Limitations and future development

MPeat2D assumes a uniform distribution of peat throughout the fixed horizontal domain in the initial stages of development, which results in the omission of the lateral expansion process as the peatland grows. The lateral expansion is crucial to model the paludification that influences peatland behaviour because the transition from forest to peatland involves changes in vegetation, nutrient availability, and peat physical properties (Charman, 2002; Anderson et al., 2003; Rydin and Jeglum, 2006). Peatland lateral expansion requires an evolving domain that results in moving boundary problems (Tezduyar, 2001; Gawlik and Lew, 2015). Simplifying assumptions may be necessary to involve the moving boundary conditions into MPeat2D, including providing the rate of lateral expansion that determines the boundary motion and the changing domain. Moreover, to improve the numerical stability of the model, a smaller grid size might be required, particularly around the boundaries, due to significant differences in the internal stresses.

The assumption of a flat substrate employed by MPeat2D could be improved by introducing a more general landscape condition consisting of upland, sloping area, and lowland based on the theoretical landscape model proposed by Winter (2001). The landscape variations, together with the feedback from mechanical, ecological, and hydrological processes, affect the stresses on the peat body that control the occurrence of failure conditions on the peatland. The peatland failure involving mass movement (Dykes and Selkirk-Bell, 2010; Dykes, 2022; Dykes, 2008) influences the estimation of carbon accumulation on the peatland because it might result in the formation of water channels that facilitate the drainage and oxidation processes (Warburton et al., 2003; Evans and Warburton, 2007). Potentially, this phenomenon could determine the maximum limit to peatland carbon accumulation in a landscape (Large et al., 2021).

The entrapped gas bubbles that are neglected by the current version of MPeat2D might have a significant influence on the peatland mechanics and behaviour. The presence of gas bubbles influences hydraulic conductivity (Baird and Waldron, 2003; Beckwith and Baird, 2001; Reynolds et al., 1992) and pore pressure (Kellner et al., 2004), which results in variations of effective stress. Consequently, the mechanical deformation of peat pore space, including the shrinking or swelling, is also affected by the presence of gas bubbles. The simulation from Reeve et al. (2013) suggested that a higher gas content results in

a more significant peatland surface deformation. We could expand the poroelasticity formulation in the MPeat2D to accommodate the gas bubbles by introducing another fluid below the water table, for example, a water and gas mixture (Kurzeja and Steeb, 2022). This modification requires generalisation in Biot's theory of consolidation to model multiphase fluid saturation.

590 The current version of MPeat2D is focused on modelling raised ombrotrophic peatland, which grows in temperate climates. However, it should be possible to develop MPeat2D to model the other peatland types, for example, the tropical peatland. Modifications of some processes are required before applying MPeat2D to analyse the peatland in tropical areas, including the variation in the rate of peat production, peat physical properties characteristics, and loading behaviour. MPeat2D uses the empirical relationship between peat production and water table depth, which is formulated based on the data from Ellergower Moss, Scotland (Belyea and Clymo, 2001). The rate of peat production in the tropical peatland should be different from the northern temperate peatland due to the variations in the vegetation composition. The hydraulic conductivity of tropical peatlands is relatively high compared to the northern temperate peatland (Baird et al., 2017), which might lead to different hydrological processes. Moreover, the loading from trees and the influence of roots for maintaining mechanical stability are significant processes in tropical peatlands, which requires an additional formulation in the MPeat2D. For example, the weight of trees with a root system could be modelled through the data of above-ground and below-ground biomass. In this case, the loading is applied not only at the top surface but also at the specific depth of the peatland, depending on the root characteristics. Finally, the development of MPeat2D into a three-dimensional (3D) model provides a more comprehensive analysis of the peatland carbon accumulation process and phenomena that require explicit spatial interactions in 3D. The comparison between MPeat2D and MPeat indicates the crucial function of adding a second dimension to estimate peatland carbon accumulation. MPeat2D produces greater cumulative carbon, particularly under non-constant climate, due to the lateral variability of water table depth and peat physical properties incorporated by the 2D model. Based on this preliminary result, it might be possible that a 3D model of peatland development might result in a more significant carbon accumulation than the 1D or 2D models because of the more complex feedback mechanisms involved by a higher dimensional model. The 3D model is also required to understand the patterning phenomena on the peatland surface, which is highly directional and affected by spatial characteristics. The analysis of surface patterning is typically developed based on ecohydrological feedback, which encompasses the interactions between water table position, vegetation communities, nutrient availability, and peat hydraulic properties (Eppinga et al., 2009; Eppinga et al., 2008; Morris et al., 2013; Béguin et al., 2019). However, as a porous medium with relatively low shear and tensile strength (Long, 2005; Boylan et al., 2008; O'Kelly, 2017; Dykes, 2008), mechanical instability also determines the process of surface patterning on the peatland. The simulation from Briggs et al. (2007) indicates that the peatland surface might experience wrinkles due to the changing pattern between tensile and compressive stresses. Moreover, the peatland surface patterns could appear as a consequence of the tensile or compressive failure condition (Dykes, 2008), which dominantly occurs under a low slope angle (Dykes and Selkirk-Bell, 2010). Therefore, a fully coupled mechanical, ecological, and hydrological model of peatland growth in three dimensions might be suitable for analysing the appearance and impact of surface patterning on the peatland water flow and carbon balance.

620 **6 Conclusion**

MPeat2D is a two-dimensional peatland growth model that incorporates mechanical-ecohydrological feedback and the influence of spatial variability on peatland behaviour. This model is developed based on the poroelasticity and continuum concept, resulting in the plausible outputs of peat physical properties, water table position, and vegetation composition. MPeat2D produces a higher bulk density and lower active porosity and hydraulic conductivity at the margin compared to the centre due to the different effects of compaction, which are in accord with field observations. Furthermore, lateral variability of water table depth because of the changes in the hydraulic gradient leads to different vegetation compositions between the margin and the centre. The comparison between MPeat2D and the other peatland growth models, MPeat and DigiBog, indicates the critical function of mechanical, ecological, and hydrological processes together with the feedback from spatial heterogeneity on the peatland shape, carbon stock and resilience.

630 **Code availability**

The codes that support the findings of this study are openly available in zenodo at <https://doi.org/10.5281/zenodo.10050891> (Mahdiyasa, 2023)

Author contribution

AWM and DJL conceptualised and designed the research. AWM developed the model, performed the simulations, and prepared the initial manuscript. DJL, MI, and BPM provided advice on the model interpretation and contributed to the editing and reviewing of the manuscript. BPM provided financial support and computing resources for the simulations.

Competing interests

The authors declare that they have no conflict of interest.

Acknowledgements

640 This work was funded by LPDP RISPRO-UKICIS (United Kingdom-Indonesia Consortium for Interdisciplinary Sciences) green economy with grant number 4345/E4/AL.04 and ITB Research Programme 2024 “Riset Peningkatan Kapasitas Dosen Muda ITB”.

645

References

- Aaby, B. and Tauber, H.: Rates of peat formation in relation to degree of humification and local environment, as shown by studies of a raised bog in Denmark, *Boreas*, 4, 1-17, <https://doi.org/10.1111/j.1502-3885.1975.tb00675.x>, 1975.
- 650 Abousleiman, Y., Cheng, A. H.-D., Cui, L., Detournay, E., and Roegiers, J.-C.: Mandel's problem revisited, *Géotechnique*, 46, 187-195, <https://doi.org/10.1680/geot.1996.46.2.187>, 1996.
- Anderson, R. L., Foster, D. R., and Motzkin, G.: Integrating lateral expansion into models of peatland development in temperate New England, *Journal of Ecology*, 91, 68-76, <https://doi.org/10.1046/j.1365-2745.2003.00740.x>, 2003.
- Armstrong, A. C.: Hydrological model of peat-mound form with vertically varying hydraulic conductivity, *Earth Surface Processes and Landforms*, 20, 473-477, <https://doi.org/10.1002/esp.3290200508>, 1995.
- 655 Baird, A. J. and Waldron, S.: Shallow horizontal groundwater flow in peatlands is reduced by bacteriogenic gas production, *Geophysical Research Letters*, 30, <https://doi.org/10.1029/2003GL018233>, 2003.
- Baird, A. J., Eades, P. A., and Surridge, B. W. J.: The hydraulic structure of a raised bog and its implications for ecohydrological modelling of bog development, *Ecohydrology*, 1, 289-298, <https://doi.org/10.1002/eco.33>, 2008.
- Baird, A. J., Morris, P. J., and Belyea, L. R.: The DigiBog peatland development model 1: rationale, conceptual model, and hydrological basis, *Ecohydrology*, 5, 242-255, <https://doi.org/10.1002/eco.230>, 2012.
- 660 Baird, A. J., Milner, A. M., Blundell, A., Swindles, G. T., and Morris, P. J.: Microform-scale variations in peatland permeability and their ecohydrological implications, *Journal of Ecology*, 104, 531-544, <https://doi.org/10.1111/1365-2745.12530>, 2016.
- Baird, A. J., Low, R., Young, D., Swindles, G. T., Lopez, O. R., and Page, S.: High permeability explains the vulnerability of the carbon store in drained tropical peatlands, *Geophysical Research Letters*, 44, 1333-1339, <https://doi.org/10.1002/2016GL072245>, 2017.
- 665 Ballard, C. E., McIntyre, N., Wheater, H. S., Holden, J., and Wallage, Z. E.: Hydrological modelling of drained blanket peatland, *Journal of Hydrology*, 407, 81-93, <https://doi.org/10.1016/j.jhydrol.2011.07.005>, 2011.
- Bartlett, M. S. and Porporato, A.: A Class of Exact Solutions of the Boussinesq Equation for Horizontal and Sloping Aquifers, *Water Resources Research*, 54, 767-778, <https://doi.org/10.1002/2017WR022056>, 2018.
- 670 Beckwith, C. W. and Baird, A. J.: Effect of biogenic gas bubbles on water flow through poorly decomposed blanket peat, *Water Resources Research*, 37, 551-558, <https://doi.org/10.1029/2000WR900303>, 2001.
- Béguin, C., Brunetti, M., and Kasparian, J.: Quantitative analysis of self-organized patterns in ombrotrophic peatlands, *Scientific Reports*, 9, 1499, <https://doi.org/10.1038/s41598-018-37736-8>, 2019.
- Belyea, L. R. and Clymo, R. S.: Feedback control of the rate of peat formation, *Proceedings of the Royal Society of London. Series B: Biological Sciences*, 268, 1315-1321, <https://doi.org/10.1098/rspb.2001.1665>, 2001.
- 675 Biot, M. A.: General theory of three-dimensional consolidation, *Journal of Applied Physics*, 12, 155-164, <https://doi.org/10.1063/1.1712886>, 1941.
- Borren, W. and Bleuten, W.: Simulating Holocene carbon accumulation in a western Siberian watershed mire using a three-dimensional dynamic modeling approach, *Water Resources Research*, 42, <https://doi.org/10.1029/2006WR004885>, 2006.
- 680 Bourgault, M.-A., Larocque, M., and Garneau, M.: Quantification of peatland water storage capacity using the water table fluctuation method, *Hydrological Processes*, 31, 1184-1195, <https://doi.org/10.1002/hyp.11116>, 2017.
- Boussinesq, J.: Théorie de l'intumescence liquide, appelée onde solitaire ou de translation, se propageant dans un canal rectangulaire, *Comptes Rendus de l'Académie des Sciences.*, 72, 1871.
- 685 Boylan, N., Jennings, P., and Long, M.: Peat slope failure in Ireland, *Quarterly Journal of Engineering Geology and Hydrogeology*, 41, 93-108, <https://doi.org/10.1144/1470-9236/06-028>, 2008.
- Briggs, J., Large, D. J., Snape, C., Drage, T., Whittles, D., Cooper, M., Macquaker, J. H. S., and Spiro, B. F.: Influence of climate and hydrology on carbon in an early Miocene peatland, *Earth and Planetary Science Letters*, 253, 445-454, <https://doi.org/10.1016/j.epsl.2006.11.010>, 2007.
- Charman, D.: *Peatlands and Environmental Change*, John Wiley and Sons Ltd., Chichester, 301 pp.2002.
- 690 Cheng, A. H. D.: A linear constitutive model for unsaturated poroelasticity by micromechanical analysis, *International Journal for Numerical and Analytical Methods in Geomechanics*, 44, 455-483, <https://doi.org/10.1002/nag.3033>, 2020.
- Cheng, A. H. D. and Detournay, E.: A direct boundary element method for plane strain poroelasticity, *International Journal for Numerical and Analytical Methods in Geomechanics*, 12, 551-572, <https://doi.org/10.1002/nag.1610120508>, 1988.

- 695 Clymo, R. S.: The limits to peat bog growth, *Philosophical Transactions of the Royal Society of London. B, Biological Sciences*, 303, 605-654, <https://doi.org/10.1098/rstb.1984.0002>, 1984.
- Clymo, R. S.: Hydraulic conductivity of peat at Ellergower Moss, Scotland, *Hydrological Processes*, 18, 261-274, <https://doi.org/10.1002/hyp.1374>, 2004.
- Clymo, R. S., Turunen, J., and Tolonen, K.: Carbon Accumulation in Peatland, *Oikos*, 81, 368-388, 10.2307/3547057, 1998.
- 700 Cobb, A. R., Hoyt, A. M., Gandois, L., Eri, J., Dommain, R., Abu Salim, K., Kai, F. M., Haji Su'ut, N. S., and Harvey, C. F.: How temporal patterns in rainfall determine the geomorphology and carbon fluxes of tropical peatlands, *Proceedings of the National Academy of Sciences*, 114, E5187-E5196, <https://doi.org/10.1073/pnas.1701090114>, 2017.
- Coussy, O.: *Poromechanics*, John Wiley & Sons Ltd, Chichester, UK2004.
- de Boer, R.: *Theory of Porous Media*, Springer, Berlin, Heidelberg, Germany, <https://doi.org/10.1007/978-3-642-59637-7>, 2000.
- 705 Detournay, E. and Cheng, A. H. D.: Fundamentals of Poroelasticity, in: *Analysis and Design Methods*, edited by: Fairhurst, C., Pergamon, Oxford, 113-171, 10.1016/B978-0-08-040615-2.50011-3, 1993.
- Duijn, C. J. v. and Mikelic, A.: Mathematical Proof of the Mandel--Cryer Effect in Poroelasticity, *Multiscale Modeling & Simulation*, 19, 550-567, <https://doi.org/10.1137/20m1346043>, 2021.
- Dykes, A. P.: Tensile strength of peat: laboratory measurement and role in Irish blanket bog failures, *Landslides*, 5, 417-429, <https://doi.org/10.1007/s10346-008-0136-1>, 2008.
- 710 Dykes, A. P.: Recent peat slide in Co. Antrim extends the known range of weak basal peat across Ireland, *Environmental Geotechnics*, 9, 22-35, <https://doi.org/10.1680/jenge.17.00088>, 2022.
- Dykes, A. P. and Selkirk-Bell, J. M.: Landslides in blanket peat on subantarctic islands: causes, characteristics and global significance, *Geomorphology*, 124, 215-228, <https://doi.org/10.1016/j.geomorph.2010.09.002>, 2010.
- 715 Dykes, A. P. and Warburton, J.: Failure of peat-covered hillslopes at Dooncarton Mountain, Co. Mayo, Ireland: analysis of topographic and geotechnical factors, *CATENA*, 72, 129-145, <https://doi.org/10.1016/j.catena.2007.04.008>, 2008.
- Edelsbrunner, H.: *Geometry and Topology for Mesh Generation*, Cambridge University Press., Cambridge, United Kingdom2001.
- 720 Eppinga, M. B., Rietkerk, M., Wassen, M. J., and De Ruiter, P. C.: Linking Habitat Modification to Catastrophic Shifts and Vegetation Patterns in Bogs, *Plant Ecology*, 200, 53-68, <https://doi.org/10.1007/s11258-007-9309-6>, 2009.
- Eppinga, M. B., Rietkerk, M., Borren, W., Lapshina, E. D., Bleuten, W., and Wassen, M. J.: Regular Surface Patterning of Peatlands: Confronting Theory with Field Data, *Ecosystems*, 11, 520-536, <https://doi.org/10.1007/s10021-008-9138-z>, 2008.
- Evans, C. D., Peacock, M., Baird, A. J., Artz, R. R. E., Burden, A., Callaghan, N., Chapman, P. J., Cooper, H. M., Coyle, M., Craig, E., Cumming, A., Dixon, S., Gauci, V., Grayson, R. P., Helfter, C., Heppell, C. M., Holden, J., Jones, D. L., Kaduk, J.,
- 725 Levy, P., Matthews, R., McNamara, N. P., Misselbrook, T., Oakley, S., Page, S. E., Rayment, M., Ridley, L. M., Stanley, K. M., Williamson, J. L., Worrall, F., and Morrison, R.: Overriding water table control on managed peatland greenhouse gas emissions, *Nature*, 593, 548-552, <https://doi.org/10.1038/s41586-021-03523-1>, 2021.
- Evans, M. and Warburton, J.: *Geomorphology of Upland Peat: Erosion, Form and Landscape Change*, John Wiley & Sons Ltd2007.
- 730 Fraser, C. J. D., Roulet, N. T., and Moore, T. R.: Hydrology and dissolved organic carbon biogeochemistry in an ombrotrophic bog, *Hydrological Processes*, 15, 3151-3166, <https://doi.org/10.1002/hyp.322>, 2001.
- Gawlik, E. S. and Lew, A. J.: Unified Analysis of Finite Element Methods for Problems with Moving Boundaries, *SIAM Journal on Numerical Analysis*, 53, 2822-2846, <https://doi.org/10.1137/140990437>, 2015.
- George, P.-L.: *Mesh Generation – Application to Finite Elements*, Hermes Science Publishing, Paris2000.
- 735 Glaser, P. H., Hansen, B. C. S., Siegel, D. I., Reeve, A. S., and Morin, P. J.: Rates, pathways and drivers for peatland development in the Hudson Bay Lowlands, northern Ontario, Canada, *Journal of Ecology*, 92, 1036-1053, <https://doi.org/10.1111/j.0022-0477.2004.00931.x>, 2004.
- Hardie, S. M. L., Garnett, M. H., Fallick, A. E., Rowland, A. P., Ostle, N. J., and Flowers, T. H.: Abiotic drivers and their interactive effect on the flux and carbon isotope (^{14}C and $\delta^{13}\text{C}$) composition of peat-respired CO_2 , *Soil Biology and*
- 740 *Biochemistry*, 43, 2432-2440, <https://doi.org/10.1016/j.soilbio.2011.08.010>, 2011.
- Hoag, R. S. and Price, J. S.: A field-scale, natural gradient solute transport experiment in peat at a Newfoundland blanket bog, *Journal of Hydrology*, 172, 171-184, [https://doi.org/10.1016/0022-1694\(95\)02696-M](https://doi.org/10.1016/0022-1694(95)02696-M), 1995.

- Hoag, R. S. and Price, J. S.: The effects of matrix diffusion on solute transport and retardation in undisturbed peat in laboratory columns, *Journal of Contaminant Hydrology*, 28, 193-205, [https://doi.org/10.1016/S0169-7722\(96\)00085-X](https://doi.org/10.1016/S0169-7722(96)00085-X), 1997.
- 745 Hogan, J. M., van der Kamp, G., Barbour, S. L., and Schmidt, R.: Field methods for measuring hydraulic properties of peat deposits, *Hydrological Processes*, 20, 3635-3649, <https://doi.org/10.1002/hyp.6379>, 2006.
- Huang, Y., Ciais, P., Luo, Y., Zhu, D., Wang, Y., Qiu, C., Goll, D. S., Guenet, B., Makowski, D., De Graaf, I., Leifeld, J., Kwon, M. J., Hu, J., and Qu, L.: Tradeoff of CO₂ and CH₄ emissions from global peatlands under water-table drawdown, *Nature Climate Change*, 11, 618-622, <https://doi.org/10.1038/s41558-021-01059-w>, 2021.
- 750 Ingram, H. A. P.: Size and shape in raised mire ecosystems: a geophysical model, *Nature*, 297, 300-303, <https://doi.org/10.1038/297300a0>, 1982.
- Irgens, F.: *Continuum Mechanics*, Springer-Verlag Berlin, Heidelberg, 661 pp., <https://doi.org/10.1007/978-3-540-74298-2>, 2008.
- Jha, B. and Juanes, R.: Coupled multiphase flow and poromechanics: A computational model of pore pressure effects on fault slip and earthquake triggering, *Water Resources Research*, 50, 3776-3808, <https://doi.org/10.1002/2013WR015175>, 2014.
- 755 Jog, C. S.: *Continuum Mechanics: Volume 1: Foundations and Applications of Mechanics 3rd*, Cambridge University Press, Cambridge 2015.
- Kellner, E., Price, J. S., and Waddington, J. M.: Pressure variations in peat as a result of gas bubble dynamics, *Hydrological Processes*, 18, 2599-2605, <https://doi.org/10.1002/hyp.5650>, 2004.
- 760 Kettridge, N., Turetsky, M. R., Sherwood, J. H., Thompson, D. K., Miller, C. A., Benscoter, B. W., Flannigan, M. D., Wotton, B. M., and Waddington, J. M.: Moderate drop in water table increases peatland vulnerability to post-fire regime shift, *Scientific Reports*, 5, 8063, <https://doi.org/10.1038/srep08063>, 2015.
- Kokkonen, N. A. K., Laine, A. M., Laine, J., Vasander, H., Kurki, K., Gong, J., and Tuittila, E.-S.: Responses of peatland vegetation to 15-year water level drawdown as mediated by fertility level, *Journal of Vegetation Science*, 30, 1206-1216, <https://doi.org/10.1111/jvs.12794>, 2019.
- 765 Korhola, A., Alm, J., Tolonen, K., Turunen, J., and Jungner, H.: Three-dimensional reconstruction of carbon accumulation and CH₄ emission during nine millennia in a raised mire, *Journal of Quaternary Science*, 11, [https://doi.org/10.1002/\(SICI\)1099-1417\(199603/04\)11:2<161::AID-JQS248>3.0.CO;2-J](https://doi.org/10.1002/(SICI)1099-1417(199603/04)11:2<161::AID-JQS248>3.0.CO;2-J), 1996.
- Kurzeja, P. and Steeb, H.: Acoustic waves in saturated porous media with gas bubbles, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 380, 20210370, doi:10.1098/rsta.2021.0370, 2022.
- 770 Kværner, J. and Snilsberg, P.: Groundwater hydrology of boreal peatlands above a bedrock tunnel – Drainage impacts and surface water groundwater interactions, *Journal of Hydrology*, 403, 278-291, <https://doi.org/10.1016/j.jhydrol.2011.04.006>, 2011.
- Laine, A. M., Korrensalo, A., Kokkonen, N. A. K., and Tuittila, E.-S.: Impact of long-term water level drawdown on functional plant trait composition of northern peatlands, *Functional Ecology*, 35, 2342-2357, <https://doi.org/10.1111/1365-2435.13883>, 2021.
- 775 Lapen, D. R., Price, J. S., and Gilbert, R.: Modelling two-dimensional steady-state groundwater flow and flow sensitivity to boundary conditions in blanket peat complexes, *Hydrological Processes*, 19, 371-386, <https://doi.org/10.1002/hyp.1507>, 2005.
- Large, D. J., Marshall, C., Jochmann, M., Jensen, M., Spiro, B. F., and Olausson, S.: Time, Hydrologic Landscape, and the Long-Term Storage of Peatland Carbon in Sedimentary Basins, *Journal of Geophysical Research: Earth Surface*, 126, e2020JF005762, <https://doi.org/10.1029/2020JF005762>, 2021.
- 780 Lewis, C., Albertson, J., Xu, X., and Kiely, G.: Spatial variability of hydraulic conductivity and bulk density along a blanket peatland hillslope, *Hydrological Processes*, 26, 1527-1537, <https://doi.org/10.1002/hyp.8252>, 2012.
- Lieth, H.: Modeling the Primary Productivity of the World, in: *Primary Productivity of the Biosphere*, edited by: Lieth, H., and Whittaker, R. H., Springer Berlin Heidelberg, Berlin, Heidelberg, 237-263, 10.1007/978-3-642-80913-2_12, 1975.
- 785 Loisel, J., Yu, Z., Beilman, D. W., Camill, P., Alm, J., Amesbury, M. J., Anderson, D., Andersson, S., Bochicchio, C., Barber, K., Belyea, L. R., Bunbury, J., Chambers, F. M., Charman, D. J., De Vleeschouwer, F., Fialkiewicz-Kozielec, B., Finkelstein, S. A., Gałka, M., Garneau, M., Hammarlund, D., Hinchcliffe, W., Holmquist, J., Hughes, P., Jones, M. C., Klein, E. S., Kokfelt, U., Korhola, A., Kuhry, P., Lamarre, A., Lamentowicz, M., Large, D., Lavoie, M., MacDonald, G., Magnan, G., Mäkilä, M., Mallon, G., Mathijssen, P., Mauquoy, D., McCarroll, J., Moore, T. R., Nichols, J., O'Reilly, B., Oksanen, P., Packalen, M., Peteet, D., Richard, P. J., Robinson, S., Ronkainen, T., Rundgren, M., Sannel, A. B. K., Tarnocai, C., Thom, T., Tuittila, E.-S., Turetsky, M., Välranta, M., van der Linden, M., van Geel, B., van Bellen, S., Vitt, D., Zhao, Y., and Zhou, W.: A database

- and synthesis of northern peatland soil properties and Holocene carbon and nitrogen accumulation, *The Holocene*, 24, 1028-1042, <https://doi.org/10.1177/0959683614538073>, 2014.
- 795 Long, M.: Review of peat strength, peat characterisation and constitutive modelling of peat with reference to landslides, *Studia Geotechnica et Mechanica*, 27, 67-90, 2005.
- Lunt, P. H., Fyfe, R. M., and Tappin, A. D.: Role of recent climate change on carbon sequestration in peatland systems, *Science of The Total Environment*, 667, 348-358, <https://doi.org/10.1016/j.scitotenv.2019.02.239>, 2019.
- 800 Ma, L., Zhu, G., Chen, B., Zhang, K., Niu, S., Wang, J., Ciais, P., and Zuo, H.: A globally robust relationship between water table decline, subsidence rate, and carbon release from peatlands, *Communications Earth & Environment*, 3, 254, <https://doi.org/10.1038/s43247-022-00590-8>, 2022.
- Mahdiyasa, A. W.: MPeat2D (1.0), Zenodo [code], <https://doi.org/10.5281/zenodo.10050891>, 2023.
- Mahdiyasa, A. W., Large, D. J., Muljadi, B. P., and Icardi, M.: Modelling the influence of mechanical-ecohydrological feedback on the nonlinear dynamics of peatlands, *Ecological Modelling*, 478, 110299, <https://doi.org/10.1016/j.ecolmodel.2023.110299>, 2023.
- 805 Mahdiyasa, A. W., Large, D. J., Muljadi, B. P., Icardi, M., and Triantafyllou, S.: MPeat—A fully coupled mechanical-ecohydrological model of peatland development, *Ecohydrology*, 15, e2361, <https://doi.org/10.1002/eco.2361>, 2022.
- Malhotra, A., Roulet, N. T., Wilson, P., Giroux-Bougard, X., and Harris, L. I.: Ecohydrological feedbacks in peatlands: an empirical test of the relationship among vegetation, microtopography and water table, *Ecohydrology*, 9, 1346-1357, <https://doi.org/10.1002/eco.1731>, 2016.
- 810 Malmer, N., Svensson, B. M., and Wallén, B.: Interactions between Sphagnum mosses and field layer vascular plants in the development of peat-forming systems, *Folia Geobotanica & Phytotaxonomica*, 29, 483-496, <https://doi.org/10.1007/BF02883146>, 1994.
- Mandel, J.: Consolidation Des Sols (Étude Mathématique), *Géotechnique*, 3, 287-299, <https://doi.org/10.1680/geot.1953.3.7.287>, 1953.
- 815 McNeil, P. and Waddington, J. M.: Moisture controls on Sphagnum growth and CO₂ exchange on a cutover bog, *Journal of Applied Ecology*, 40, 354-367, <https://doi.org/10.1046/j.1365-2664.2003.00790.x>, 2003.
- Mesri, G. and Ajlouni, M.: Engineering properties of fibrous peats, *Journal of Geotechnical and Geoenvironmental Engineering*, 133, 850-866, [https://doi.org/10.1061/\(ASCE\)1090-0241\(2007\)133:7\(850\)](https://doi.org/10.1061/(ASCE)1090-0241(2007)133:7(850)), 2007.
- 820 Moore, P. A., Lukenbach, M. C., Thompson, D. K., Kettridge, N., Granath, G., and Waddington, J. M.: Assessing the peatland hummock-hollow classification framework using high-resolution elevation models: implications for appropriate complexity ecosystem modeling, *Biogeosciences*, 16, 3491-3506, <https://doi.org/10.5194/bg-16-3491-2019>, 2019.
- Moore, T. R., Bubier, J. L., Frolking, S. E., Lafleur, P. M., and Roulet, N. T.: Plant biomass and production and CO₂ exchange in an ombrotrophic bog, *Journal of Ecology*, 90, 25-36, <https://doi.org/10.1046/j.0022-0477.2001.00633.x>, 2002.
- 825 Moore, T. R., Trofymow, J. A., Siltanen, M., Prescott, C., and Group, C. W.: Patterns of decomposition and carbon, nitrogen, and phosphorus dynamics of litter in upland forest and peatland sites in central Canada, *Canadian Journal of Forest Research*, 35, 133-142, <https://doi.org/10.1139/x04-149>, 2005.
- Morris, P. J., Baird, A. J., and Belyea, L. R.: The DigiBog peatland development model 2: ecohydrological simulations in 2D, *Ecohydrology*, 5, 256-268, <https://doi.org/10.1002/eco.229>, 2012.
- 830 Morris, P. J., Baird, A. J., and Belyea, L. R.: The role of hydrological transience in peatland pattern formation, *Earth Surf. Dynam.*, 1, 29-43, <https://doi.org/10.5194/esurf-1-29-2013>, 2013.
- Morris, P. J., Belyea, L. R., and Baird, A. J.: Ecohydrological feedbacks in peatland development: a theoretical modelling study, *Journal of Ecology*, 99, 1190-1201, <https://doi.org/10.1111/j.1365-2745.2011.01842.x>, 2011.
- 835 Morris, P. J., Baird, A. J., Young, D. M., and Swindles, G. T.: Untangling climate signals from autogenic changes in long-term peatland development, *Geophysical Research Letters*, 42, 10,788-710,797, <https://doi.org/10.1002/2015GL066824>, 2015.
- Morris, P. J., Davies, M. L., Baird, A. J., Balliston, N., Bourgault, M.-A., Clymo, R. S., Fewster, R. E., Furukawa, A. K., Holden, J., Kessel, E., Ketcheson, S. J., Kløve, B., Larocque, M., Marttila, H., Menberu, M. W., Moore, P. A., Price, J. S., Ronkanen, A.-K., Rosa, E., Strack, M., Surridge, B. W. J., Waddington, J. M., Whittington, P., and Wilkinson, S. L.: Saturated Hydraulic Conductivity in Northern Peats Inferred From Other Measurements, *Water Resources Research*, 58, e2022WR033181, <https://doi.org/10.1029/2022WR033181>, 2022.
- 840

- Munir, T. M., Perkins, M., Kaing, E., and Strack, M.: Carbon dioxide flux and net primary production of a boreal treed bog: Responses to warming and water-table-lowering simulations of climate change, *Biogeosciences*, 12, 1091-1111, <https://doi.org/10.5194/bg-12-1091-2015>, 2015.
- 845 O'Kelly, B. C.: Measurement, interpretation and recommended use of laboratory strength properties of fibrous peat, *Geotechnical Research*, 4, 136-171, <https://doi.org/10.1680/jgere.17.00006>, 2017.
- Peltoniemi, K., Laiho, R., Juottonen, H., Bodrossy, L., Kell, D. K., Minkkinen, K., Mäkiranta, P., Mehtätalo, L., Penttilä, T., Siljanen, H. M. P., Tuittila, E.-S., Tuomivirta, T., and Fritze, H.: Responses of methanogenic and methanotrophic communities to warming in varying moisture regimes of two boreal fens, *Soil Biology and Biochemistry*, 97, 144-156, <https://doi.org/10.1016/j.soilbio.2016.03.007>, 2016.
- 850 Phillips, P. J. and Wheeler, M. F.: A coupling of mixed and continuous Galerkin finite element methods for poroelasticity I: the continuous in time case, *Computational Geosciences*, 11, 131-144, <https://doi.org/10.1007/s10596-007-9045-y>, 2007.
- Potvin, L. R., Kane, E. S., Chimner, R. A., Kolka, R. K., and Lilleskov, E. A.: Effects of water table position and plant functional group on plant community, aboveground production, and peat properties in a peatland mesocosm experiment (PEATcosm), *Plant and Soil*, 387, 277-294, <https://doi.org/10.1007/s11104-014-2301-8>, 2015.
- 855 Price, J. S.: Role and character of seasonal peat soil deformation on the hydrology of undisturbed and cutover peatlands, *Water Resources Research*, 39, <https://doi.org/10.1029/2002WR001302>, 2003.
- Price, J. S. and Schlotzhauer, S. M.: Importance of shrinkage and compression in determining water storage changes in peat: the case of a mined peatland, *Hydrological Processes*, 13, 2591-2601, [https://doi.org/10.1002/\(SICI\)1099-1085\(199911\)13:16<2591::AID-HYP933>3.0.CO;2-E](https://doi.org/10.1002/(SICI)1099-1085(199911)13:16<2591::AID-HYP933>3.0.CO;2-E), 1999.
- 860 Price, J. S., Cagampan, J., and Kellner, E.: Assessment of peat compressibility: is there an easy way?, *Hydrological Processes*, 19, 3469-3475, <https://doi.org/10.1002/hyp.6068>, 2005.
- Quinton, W. L., Gray, D. M., and Marsh, P.: Subsurface drainage from hummock-covered hillslopes in the Arctic tundra, *Journal of Hydrology*, 237, 113-125, [https://doi.org/10.1016/S0022-1694\(00\)00304-8](https://doi.org/10.1016/S0022-1694(00)00304-8), 2000.
- Quinton, W. L., Hayashi, M., and Carey, S. K.: Peat hydraulic conductivity in cold regions and its relation to pore size and geometry, *Hydrological Processes*, 22, 2829-2837, <https://doi.org/10.1002/hyp.7027>, 2008.
- 865 Reeve, A. S., Glaser, P. H., and Rosenberry, D. O.: Seasonal changes in peatland surface elevation recorded at GPS stations in the Red Lake Peatlands, northern Minnesota, USA, *Journal of Geophysical Research: Biogeosciences*, 118, 1616-1626, <https://doi.org/10.1002/2013JG002404>, 2013.
- Reeve, A. S., Evensen, R., Glaser, P. H., Siegel, D. I., and Rosenberry, D.: Flow path oscillations in transient ground-water simulations of large peatland systems, *Journal of Hydrology*, 316, 313-324, <https://doi.org/10.1016/j.jhydrol.2005.05.005>, 2006.
- Regan, S., Flynn, R., Gill, L., Naughton, O., and Johnston, P.: Impacts of Groundwater Drainage on Peatland Subsidence and Its Ecological Implications on an Atlantic Raised Bog, *Water Resources Research*, 55, 6153-6168, <https://doi.org/10.1029/2019WR024937>, 2019.
- 875 Reynolds, W. D., Brown, D. A., Mathur, S. P., and Overend, R. P.: Effect of in-situ gas accumulation on the hydraulic conductivity of peat, *Soil Science*, 153, 397-408, 1992.
- Rydin, H. and Jeglum, J.: *The Biology of Peatlands*, Oxford University Press, Oxford 2006.
- Shewchuk, J. R.: Delaunay refinement algorithms for triangular mesh generation, *Computational Geometry*, 22, 21-74, [https://doi.org/10.1016/S0925-7721\(01\)00047-5](https://doi.org/10.1016/S0925-7721(01)00047-5), 2002.
- 880 Swinnen, W., Broothaerts, N., and Verstraeten, G.: Modelling long-term blanket peatland development in eastern Scotland, *Biogeosciences*, 16, 3977-3996, <https://doi.org/10.5194/bg-16-3977-2019>, 2019.
- Tarvainen, O., Hökkä, H., Kumpula, J., and Tolvanen, A.: Bringing back reindeer pastures in cutaway peatlands, *Restoration Ecology*, 30, e13661, <https://doi.org/10.1111/rec.13661>, 2022.
- Terzaghi, K.: *Theoretical soil mechanics*, John Wiley & Sons, Inc, New York 1943.
- 885 Tezduyar, T. E.: Finite element methods for flow problems with moving boundaries and interfaces, *Archives of Computational Methods in Engineering*, 8, 83-130, <https://doi.org/10.1007/BF02897870>, 2001.
- Tuukkanen, T., Marttila, H., and Kløve, B.: Predicting organic matter, nitrogen, and phosphorus concentrations in runoff from peat extraction sites using partial least squares regression, *Water Resources Research*, 53, 5860-5876, <https://doi.org/10.1002/2017WR020557>, 2017.

- 890 Waddington, J. M., Kellner, E., Strack, M., and Price, J. S.: Differential peat deformation, compressibility, and water storage between peatland microforms: Implications for ecosystem function and development, *Water Resources Research*, 46, <https://doi.org/10.1029/2009WR008802>, 2010.
Wang, H. F.: *Theory of linear poroelasticity with applications to geomechanics and hydrogeology*, Princeton University Press, Princeton, NJ, USA2000.
- 895 Warburton, J., Higgitt, D., and Mills, A.: Anatomy of a Pennine peat slide, Northern England, *Earth Surface Processes and Landforms*, 28, 457-473, <https://doi.org/10.1002/esp.452>, 2003.
Whittington, P., Strack, M., Haarlem, R. v., Kaufman, S., P. Stoesser, Maltez, J., Price, J. S., and Stone, M.: The influence of peat volume change and vegetation on the hydrology of a kettle-hole wetland in Southern Ontario, Canada, *Mires and Peat*, 2, 2007.
- 900 Whittington, P. N. and Price, J. S.: The effects of water table draw-down (as a surrogate for climate change) on the hydrology of a fen peatland, Canada, *Hydrological Processes*, 20, 3589-3600, <https://doi.org/10.1002/hyp.6376>, 2006.
Winston, R. B.: Models of the geomorphology, hydrology, and development of domed peat bodies, *GSA Bulletin*, 106, 1594-1604, [https://doi.org/10.1130/0016-7606\(1994\)106<1594:Motgha>2.3.Co;2](https://doi.org/10.1130/0016-7606(1994)106<1594:Motgha>2.3.Co;2), 1994.
Winter, T. C.: The concept of hydrologic landscapes, *Journal of the American Water Resources Association*, 37, 335-349, <https://doi.org/10.1111/j.1752-1688.2001.tb00973.x>, 2001.
- 905 Xiang, W. and Freeman, C.: Annual variation of temperature sensitivity of soil organic carbon decomposition in North peatlands: implications for thermal responses of carbon cycling to global warming, *Environmental Geology*, 58, 499-508, [10.1007/s00254-008-1523-6](https://doi.org/10.1007/s00254-008-1523-6), 2009.
Young, D. M., Baird, A. J., Gallego-Sala, A. V., and Loisel, J.: A cautionary tale about using the apparent carbon accumulation rate (aCAR) obtained from peat cores, *Scientific Reports*, 11, 9547, <https://doi.org/10.1038/s41598-021-88766-8>, 2021.
- 910 Young, D. M., Baird, A. J., Morris, P. J., and Holden, J.: Simulating the long-term impacts of drainage and restoration on the ecohydrology of peatlands, *Water Resources Research*, 53, 6510-6522, <https://doi.org/10.1002/2016WR019898>, 2017.
Young, D. M., Baird, A. J., Charman, D. J., Evans, C. D., Gallego-Sala, A. V., Gill, P. J., Hughes, P. D. M., Morris, P. J., and Swindles, G. T.: Misinterpreting carbon accumulation rates in records from near-surface peat, *Scientific Reports*, 9, 17939, <https://doi.org/10.1038/s41598-019-53879-8>, 2019.
- 915 Yu, Z., Beilman, D. W., and Jones, M. C.: Sensitivity of Northern Peatland Carbon Dynamics to Holocene Climate Change, in: *Carbon Cycling in Northern Peatlands*, 55-69, <https://doi.org/10.1029/2008GM000822>, 2009.
Yu, Z., Loisel, J., Brosseau, D. P., Beilman, D. W., and Hunt, S. J.: Global peatland dynamics since the Last Glacial Maximum, *Geophysical Research Letters*, 37, <https://doi.org/10.1029/2010GL043584>, 2010.
- 920 Zhu, J., Wang, Y., Wang, Y., Mao, Z., and Langendoen, E. J.: How does root biodegradation after plant felling change root reinforcement to soil?, *Plant and Soil*, 446, 211-227, <https://doi.org/10.1007/s11104-019-04345-x>, 2020.
Zhu, X., Wang, Y., Zan, J., and Li, C.: Application of fractal theory in generation and refinement of finite element mesh, *Applied Mathematics and Computation*, 175, 1039-1045, <https://doi.org/10.1016/j.amc.2005.08.017>, 2006.