Stochastic properties of coastal flooding events – Part 2: Probabilistic analysis

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Abstract. Low-intensity but high-frequency coastal flooding, also known as nuisance flooding, can negatively affect low-lying coastal communities with potentially large socioeconomic effects. This flooding also can greatly affect post-storm coastal dune recovery and reduce the long-term resilience of the back-barrier ecosystem. Recent analytical work has hypothesized that these frequent flooding events are uncorrelated in time and can be modeled as a marked Poisson process with exponentially distributed sizes, a result with important implications for the prediction of coastal flooding. Here we test this proposition using high-temporal-resolution field measurements of an eroding beach on Partially driven by wave runup, this type of flooding is difficult to predict due to the complexity of the processes involved. Here, we present the results of a probabilistic analysis of flooding events measured on an eroded beach at the Texas coast. A high-resolution time series of the flooded area was obtained from pictures using Convolutional Neural Network (CNN)-based semantic segmentation methods. As described in the first part of this contribution, After defining the flooding events using a peak-over-threshold method, we found that the size of the flooding events indeed followed their size follows an exponential distribution as hypothesized. Furthermore, the consecutive flooding events were uncorrelated with one another at daily time scales but correlated at hourly time scales. Finally, as expected from tidal and day-night cycles. Our measurements confirm the broader findings of a recent multi-site investigation of the probabilistic structure of high-water events that used a semi-empirical formulation for wave runup. Indeed, we found a relatively good statistical agreement between our CNN-based empirical flooding data and run-up predictions. Our results formalize the first predictions using total-water-level estimations. As a consequence, our work supports the validity of a relatively simple probabilistic model of coastal flooding events high-frequency coastal flooding driven by wave runup which runup that can be used in coastal risk management and landscape evolution models.

1 Introduction

Most coastal...

Coastal flooding is induced by a short-term rise in water levels caused by a mix of stochastic and deterministic events such as storm surges, wave runup, tides or river discharge due to heavy precipitation (Muis et al., 2016; Ward et al., 2018; Bevacqua et al., 2019). In addition to the extreme coastal extreme hurricane-driven flooding events with return periods in the order of ten
or more years, the importance of low-intensity high-frequency coastal flooding events, with return periods in the order of months, has recently became clear (Sweet et al., 2014; Moftakhari et al., 2017, 2018a). When accumulated over time, the social cost of nuisance flooding can outweigh the costs from the large-scale flooding (Kriebel and Dean, 1993; Moftakhari et al., 2018b) (Moftakhari et al., 2017, 2018a). This low-intensity flooding also control the formation and post-storm recovery of coastal dunes, which are essential for the stability of barrier islands (Durán Vincent et al., 2021).

High-frequency and low-intensity coastal flooding is mostly driven by extreme values of wave runup superimposed to the tidal signal (Serafin and Ruggiero, 2014; Serafin et al., 2017) overtopping a characteristic beach elevation, or any other feature close to the shoreline. As the characteristic elevation of natural beaches typically adjusts to the average wave runup during high tide, they are only flooded during extreme events (Rinaldo et al., 2021). Therefore it is very difficult to predict in detail and has to be described statistically. This probabilistic description can ideally lead to the estimation of both, the overtopping frequency \( \lambda(Z) \) (or return period \( T = \lambda^{-1} \)) of a given elevation \( Z \); and the average size \( S(Z) \) of events overtopping \( Z \). This information can then be used to assess the vulnerability of coastal features and coastal infrastructure and plan accordingly.

Recently, Rinaldo et al. (2021) studied investigated the stochastic properties of high-water events (HWEs), which are associated with coastal flooding on a beach, defined as cluster, on several locations along the US and the world. These events were defined as clusters of consecutive days when total water levels exceeded a given threshold. In this work, the total water level was calculated by adding the still water level from a buoy dataset to the wave runup estimated from an empirical formula based on data, containing tides and surges, to predicted wave runup data. Wave runup was estimated empirically as function of the deep-water significant wave height and wavelength (Stockdon et al., 2006, 2014). They found that the high-water events overtopping the beach (i.e., flooding events HWEs overtopping a characteristic beach elevation (and thus leading to coastal flooding) were uncorrelated and occurred randomly in time; by definition, these events can and can thus be modeled as a Poisson process. They also found that the size and intensity of an event, defined by their size, defined as the maximum total water level during the event, does not vary with increasing elevation relative to the beach elevation, follows an exponential distribution. These findings can be summarized in an equation for the overtopping frequency of a threshold elevation \( Z \): \( \lambda(Z) = \lambda_0 \exp[-(Z - Z_c)/\bar{S}] \), where \( \lambda_0 = 18 \text{yr}^{-1} \), \( \bar{S} \) is the site-dependent average size of HWEs (\( \bar{S} \approx 0.3 \text{m} \)) and \( Z_c \) is a reference elevation that depends on the tidal amplitude and average wave runup and can be interpreted as a characteristic beach elevation (Rinaldo et al., 2021).

However, in spite of the generality and simplicity of Rinaldo et al. (2021)’s results, they were based on empirically estimated wave runup data, and therefore it is not clear how they compare to direct measurements of coastal flooding. Given the empirical nature of the wave runup model underpinning the data, which does not take into account the detailed onshore wave propagation nor the varying nearshore profile, the stochastic properties outlined by Rinaldo et al. (2021) were only as good as the quality of the wave runup predictions.

In the primary goal of the present study, we aim to test the predictions made by Rinaldo et al. (2021) by applying the same is to describe the probabilistic structure of flooding events measured at a recently eroded site in northern Texas. Flooding events were defined applying the peak-over-threshold technique to a high-resolution time series of camera images captured at a beach site on the Texas coast. The image analysis was done water area fraction, obtained from coastal images.
using Convolution Neural Network (CNN)-based image segmentation as explained in a companion contribution to this journal volume (Kang et al., in review). In the present study, we A central outcome of our research is the validation of the results of Rinaldo et al. (2021). As shown in Fig. 1, although our study complements the spatial and temporal range investigated by Rinaldo et al. (2021), it is limited to a single site and roughly half-year data. However, we can use our results to establish the validity of Rinaldo et al. (2021)’s more general predictions.

In what follows we introduce and correct the time series of water data; define flooding events; perform the statistical analysis of both the size and inter-arrival of flooding events, and finally compare it to the results of Rinaldo et al. (2021). We finalize with a presentation of the probabilistic model for low-intensity and high-frequency coastal flooding events summarizing both our results and Rinaldo et al. (2021)’s.

![Figure 1](image.png)

**Figure 1.** Comparison between this study and Rinaldo et al. (2021).

2 Defining and measuring flooding events

2.1 Field data

As explained in a companion contribution to this journal volume (Kang et al., in review), we installed three solar-powered stationary GoPro cameras, each with a different field of view, on a beach near Cedar Lakes, Texas to monitor the recovery after Hurricane Harvey in 2017. This site is completely eroded the coastal dunes and the back-beach region (see Figure 2). This site was subject to frequent wave runup events due to its low-lying, yet complex, bathymetric-topographic profile. Each camera captured pictures every 5 minutes during a 6 A.M–6 P.M. observation period and turned off automatically during the night. From November 2017 to May 2018, we captured more than 51,000 images.
2.2 Time series of water area fraction

In Kang et al. (in review), we applied CNN-based image segmentation to identify water pixels with an accuracy of more than 90%. Here we used the CNN to generate a time series of the number of water pixels from 24,793 consecutive non-overlapping daylight pictures, while filling the non-observation periods with zeros. For convenience, the number of water pixels was normalized by the total number of pixels in an image to obtain a water area fraction \( A(t) \) (Fig. 3).

Since our observation period was about six months, we ignored the effect of seasonality and only corrected the images to account for three camera rotation events. These events were relatively mild in the angle of deflection, but were due to relatively strong winds. We also corrected for minor camera rotations due to strong winds and a change in position following one camera replacement on March 12th. These four (Fig. 4A). These changes of the camera field-of-view led to different base levels of water area fraction during non-flooding conditions (Fig. 4A). We identified this base level as the most probable value of the water area between camera rotations (or replacements) and thus estimated it from the mode of the water area distribution during that time period (Fig. 5). We then subtracted the base level (horizontal lines in Fig. 4) from \( A(t) \) to obtain the excess water area fraction \( A(t) \) (Fig. 4B). We used \( A(t) \) to denote the time series of this excess. For simplicity, in what follows we refer to \( A(t) \) as simply the water area fraction—or just water area.

In order to study the stochastic properties of flooding events at different timescales, we found the defined a new time series of water area \( A|_{\tau}(t) \) at the timescale \( \tau \), by taking the maximum of \( A(t) \) over a time window \( \tau \). For example, \( A|_{1h} \) corresponds to an hourly time series \( A|_{1h} \) and \( A|_{24h} \) to a daily time series, etc. Note that by definition, \( A|_{5min} \) was equivalent to \( A(t) \) as pictures were taken every 5 minutes.
2.3 Definition of flooding events

We defined a flooding event as the set of consecutive values of the water area fraction $A_{\tau}(t)$ that exceeded the 2% threshold (Fig. 6). This threshold was about twice the standard deviation of the water area distribution (Fig. 5) and allowed a clear separation between typical fluctuations in water area and the extreme events that characterized flooding conditions—values that characterize flooding conditions (Fig. 5B) and can be associated to a characteristic beach elevation above the shoreline. From the definition, flooding events depended on the time window $\tau$, as the consecutive threshold crossings at the sub-hour timescale merged at the daily timescale (it is enough for the water area to be above 2% for a few minutes to count as a threshold-crossing at any larger timescale [see Fig. 6]).

Following Rinaldo et al. (2021), we characterized a flooding event $i$ (for a given $\tau$) by its starting time $t_i$, i.e. the time water area increased above 2%-- its duration $d_i$ and its size $S_i$, defined as the maximum water area relative to the 2% threshold during the duration of the event (Fig. 6). Furthermore, we defined the inter-arrival time $\Delta t_i$ as the time between consecutive
flooding events $t_{i+1} - t_i$. Below, we analyze the probability distribution function of the duration $d$, size $S$ and inter-arrival time $\Delta t$ of flooding events at different timescales $\tau$.

3 Statistical analysis of measured flooding events

3.1 Duration of flooding events

At the lowest timescale (and higher time resolution, $\tau = 5$ min), the probability density function $f(d)$ of the duration $d$ of flooding events lasting up to 2 hours seems followed can be approximated by a power-law distribution (Fig. 7),

$$f(d) = \frac{\beta}{d_{\text{min}}} \left( \frac{d_{\text{min}}}{d} \right)^{1+\beta}$$

with minimum duration $d_{\text{min}} = 3$ minutes and $\beta = 0.7$ and lower limit $d_{\text{min}} = 3$ min. The fact that this lower limit is below the 5-minutes temporal resolution of our data suggests that we are missing many relatively short flooding events. Interestingly, from Fig. 8, short flooding events are not necessarily of small size.
Above 2 hours, the event duration data drastically deviated from the power-law distribution with no event lasting more than 3 hours, which suggested a physical upper limit for sustained flooding conditions in the absence of a large storm, such as a hurricane.

In our nearly six-month-long measurement period (Fig. 7). Furthermore, the size and duration of flooding events was poorly correlated (Fig. 8), as events where water covered around 10% of the images’ pixels (above normal coverage the 2% threshold), i.e. \( S > 10\% \), can lasted last anything from 10 minutes to 2 hours. However, there was a lower temporal bound seems to be a lower limit for the size of the events events lasting more than 10 minutes (Fig. 8).

### 3.2 Distribution of flooding size

The distribution of the size \( S \) of flooding events, obtained from the time series of water area at the lowest timescale (\( \tau = 5 \) min), was obtained from \( A_{5\text{min}}(t) \), is well approximated by the an exponential distribution \( e^{-S/\bar{S}}/\bar{S} \) with average flooding size \( \bar{S} = 3.36\% \) (Fig. 9a), with average flooding size \( \bar{S} = 3.36\% \). The flooding size distribution remained A). As shown in Fig. 9B, the flooding size distribution remains exponential for timescales \( \tau \) up to the maximum value investigated (4.5 days 36 hours), with p-values higher than rejection range for both Lilliefors test (6.28% to 45.4%) and the rejection threshold for both
Figure 6. Definition of the flooding events, their size ($S$) and inter arrivals ($\Delta t$) from the excess water area fraction $A(t)$. Examples shown for the original timescale $\tau = 5\text{min}$ (A), and for a daily timescale ($\tau = 1\text{day} = 24\text{hours}$) (B). Note all the events shown in (A) were clustered in a single event in (B) (shaded region).

The Lilliefors and the Cramér-von Mises test (34.1% to 79.4%), commonly used test for the tests of exponential fit (Lilliefors, 1969; Cramér, 1928).

The exponential distribution of flooding size $S$ agreed with findings by Rinaldo et al. (2021) for the size of beach overtopping events (referred to as high-water events) obtained from the daily time series of total water levels. This agreement suggested that the distribution type was robust with respect to potential variations of the local beach slope during the measurement period and alongshore flooding variations at the spatial scale defined by the camera field-of-view. The average flooding size $\bar{S}$ increases with the timescale $\tau$ but seems to saturate to $\sim 8\%$ at daily or larger timescales (Fig. 9C).

3.3 Distribution of inter-arrivals

The distribution of inter-arrivals $\Delta t$ strongly depended on the timescale $\tau$ and seemed to have converged towards an exponential distribution for timescales above $\sim 10$ hours (Fig. 10A and B). This was evidenced by the sharp increase in the p-values of both the Lilliefors and the Cramér-von Mises tests from around 10% to about 60% for timescales between 10 and 12 hours (Fig. 10B). P-values remained above 30% for larger timescales.

In these statistical tests, the time at which the time-window analysis started was corrected for the time at which the time-window analysis starts changed to avoid biases. For a given timescale $\tau$, we calculated the goodness of the exponential fit $n$ times, where $n = \tau / 5\text{min}$ is the number of every possible initial time possible initial times at which the time-window
**Figure 7.** Complementary cumulative distribution function $1 - F(d)$ ($F$ is the cumulative distribution function) of the duration $d$ of flooding events. The line shows a power-law fit $(d_{\text{min}}/d)^\beta$, with minimum duration $d_{\text{min}} = 3$ min and exponent $\beta = 0.7$ and a lower limit $d_{\text{min}} = 3$ min. The time resolution of the data sets a lower cutoff at $d = 5$ min.

**Figure 8.** Relation between the size $S$ and duration $d$ of flooding events. Note that the event size $S$ is defined as the maximum value of the water area fraction during the event relative to the 2% threshold. The time resolution of the data is 5 minutes.

Of size $\tau$ could start. For example, the statistical tests were conducted only once for $A_{15\text{min}}(t)A_{15\text{min}}(t)$, but 12 times for $A_{1\text{h}}(t)A_{1\text{h}}(t)$ and 288 times for $A_{24\text{h}}(t)A_{24\text{h}}(t)$. 
Figure 9. (A) Complementary cumulative distribution function $1 - F(S/\bar{S})$ of the flooding event size $S$ normalized by the mean average $\bar{S}$ at three different timescales: $\tau = 5$ min, 1 hour and 1-day 24 hours. The exponential distribution $e^{-S/\bar{S}}/\bar{S}$ is shown for reference (dashed line). (B) P-values testing compatibility with the exponential distribution at different timescales ($\tau$) — 0% (100%) indicate perfect incompatibility (compatibility) with 5% as the typical threshold for passing the test. (C) Average size $\bar{S}$ of flooding events (%) for different timescales $\tau$.

Given that the exponential distribution of inter-arrivals implied that the events were random and independent, indicating a Poisson process. We interpreted that the large deviations from the exponential distribution for timescales less than 10 hours as evidence of correlation between consecutive events (Fig. 10B). At larger timescales, consecutive flooding events seemed to have become independent and could be modelled as a Poisson process, in agreement with the findings of Rinaldo et al. (2021) for beach overtopping events. This change in temporal correlation seemed to be related to the timescale of local weather patterns affected by the daily cycle.

3.4 Frequency of flooding events

The frequency $\lambda$ of flooding events is by definition the inverse of the average inter-arrival $\lambda = 1/\Delta t$ or equivalently $\lambda = N/T$, where $N$ is the total number of flooding events obtained from $A_{\tau}(t)$—the condition $A_{\tau}(t) > 2\%$, and $T = 167$ days is the total duration of the time series. As expected, $\lambda$ decreased with the timescale $\tau$ as flooding events are merged, reaching a plateau at the daily scale of 2.5 events per month, from about 6 events per month at the hourly scale (Fig. 10C). The Most of the decrease in $\lambda$ mirrored roughly takes place at the transition from correlated to uncorrelated events taking place for timescales between 6 to 12 hours (Fig. 10B).
Figure 10. (A) Complementary cumulative distribution function $1 - F(\lambda\Delta t)$ of the inter arrivals ($\Delta t$) of flooding events normalized by the flooding frequency $\lambda = 1/\Delta t$, at timescales $\tau =$5 min, 12 hours and 4 day 24 hours. The exponential distribution is shown for reference (dashed line). (B) Mean P-values with confidence bound ($\pm \sigma$) testing compatibility with the exponential distribution at different timescales ($\tau$) — 0% (100%) indicate perfect incompatibility (compatibility) with 5% as the typical threshold for passing the test. Passing the test, i.e. inter-arrivals are exponentially distributed, means the events are independent, whereas failing the test suggests the events are correlated. (C) Frequency $\lambda$ of flooding events at different timescales ($\tau$), including the 95% confidence interval obtained by $\lambda \times [c_{\text{lower}}, c_{\text{upper}}]$, where $c_{\text{lower}} = \chi^2_{0.025,2n}/2n$, $c_{\text{upper}} = \chi^2_{0.975,2n}/2n$.

4 Comparison with run-up model predictions

The stochastic model of flooding events measured over timescales exceeding 10 hours were in agreement with previous findings for high water events (HWEs) that cause overtopping of the beach, as stated in a previous section (Rinaldo et al., 2021) the findings of Rinaldo et al. (2021) for the size and inter-arrival of events overtopping a characteristic beach elevation (referred to as high-water events or HWEs) obtained from the predicted daily time series of total water levels. However, we sought to ask: Can the HWE predictions also match the measured flooding frequency? How do the predictions compare to the measurements beyond these general stochastic properties? In particular, how the predicted frequency of HWEs compares with the flooding frequency measured from the camera observations, and, if so, are HWE predictions correlated with the? Also, are the predicted flooding from HWEs correlated to flooding measurements at the daily level timescale?

According to Rinaldo et al. (2021), we obtained HWEs from the daily Following the methodology from Rinaldo et al. (2021) which involved calculating the hourly time series of total water levels defined as the sum of elevation for the same site using a beach slope of 0.02, we generated a new time series of daily total water levels relative to MSL. This required summing
the still water level measured by the tidal gauge, and a semi-empirical estimation of the 2%-exceedance wave run-up (Rinaldo et al., 2021). This semi-empirical estimation relies on off-shore data of values of the significant wave height and peak wave frequency, and the local beach slope (Stockdon et al., 2006, 2014). We used tidal gauge and buoy wave data.

Our data sources included the tidal gauge at Galveston Pier 21 (29.31° N, 94.793W), and wave buoy station 42035 (29.236°N, 94.403°W). Both located in Galveston, Texas, the closest site. Rinaldo et al. (2021) already estimated the hourly time series of total water elevation for this site using a beach slope of they provided hourly measurements of water levels and significant wave heights and peak period. While the water depth of the wave buoy was 15m, we did not consider reverse shoaling to deeper water, as recommended by Stockdon et al. (2006), to maintain the simplicity of our analysis and directly compare to the results of Rinaldo et al. (2021). Since we did not perform measurements of the beach profile at the study site in the observation period, we assumed the beach slope, which is needed to calculate wave runup, was constant and equal to 0.02 (Houwer et al., 2015; Rinaldo et al., 2021).

For consistency, we removed total water levels; ignored total water level values during non-observation hours of flooding monitoring. We then converted the hourly time series of total water level to the daily maximum a time series of daily-maximum total water level \( \eta_d \) by sampling maximum values per each day, by taking the maximum value per day. This removed tidal cycles from the time series. Finally, we defined the HWEs a high-water event (HWE) as the set of consecutive daily total water levels exceeding a given beach elevation threshold, elevation \( Z_c \).

Since flooding events relative to MSL (Rinaldo et al., 2021), Here \( Z_c \) is interpreted as a characteristic beach elevation in which case HWEs represent potential flooding events. Thus, since flooding events and HWEs are equivalent for the purpose of this work, in what follows we will refer to HWEs as ‘predicted flooding events’, in contrast to the ‘measured’ flooding events obtained from our CNN-based analysis of camera observations.

**Frequency of flooding events**

### 4.1 Frequency of predicted vs. measured flooding events

As expected, the frequency of predicted flooding (from HWEs) decreased with beach elevation events decreased with the characteristic beach elevation \( Z_c \), as the number of overtopping events decreased (Fig. 11). The predicted flooding frequency was within the statistical range: 95% confidence interval of the measurements for beach elevations between 0.5mand \( Z_c \) in the range \( Z_c < 0.9m \). This upper limit is consistent with the characteristic beach elevation 0.9m. In particular, for the beach elevation of 0.7m, estimated by Rinaldo et al. (2021) using a digital elevation model (DEM) of the area (Fig. 11). In fact, we expect a lower beach elevation at our site following the large beach erosion after hurricane Harvey in August 2017, in agreement with the trend observed in Fig. 11.

Although we lack measurements of the actual beach elevation profile during our observation period, the value \( Z_c = 0.7m \), at which the predicted frequency matched the measured value of \( \lambda = 2.5 \) month\(^{-1} \) obtained for a daily timescale (\( \tau = 24h \)), as seen in Fig. 10C. This beach elevation was noticeable lower than the characteristic beach elevation 0.9m estimated by...
Figure 11. Frequency—Symbols show the frequency ($\lambda$) of high-water coastal flooding events overtopping the predicted by high-water events above a characteristic beach elevation $Z_c$ obtained from extrapolated water elevation data (Rinaldo et al., 2021), as function of the characteristic beach elevation $Z_c$ (relative to MSL). The measured flooding frequency extracted from the water area daily time series $A|_{1d}(t)$ (mean ± 95% confidence interval) of flooding events at the daily timescale is shown for comparison (solid and dashed black lines). The red solid line is the flooding frequency predicted by Rinaldo et al. (2021) for a characteristic beach elevation around 0.9m (dashed red line) estimated from a digital elevation model (DEM).

Rinaldo et al. (2021) using a digital elevation model of the area (DEM, Fig. 11). However, it was consistent with the large observed beach erosion in our field site after hurricane Harvey in August 2017. Hypothesis that beach erosion can explain potential differences between predicted and measured flooding frequencies. Indeed, the beach scarp visible at the vegetation edge in Fig. 3, panel $A_1$, is about 20cm tall and could help explain the elevation gap.

4.2 Synchronicity of measured and predicted flooding events

We compared to the measurements at the daily level by defining a corrected and normalized rescaled daily time series of the measured flood area-flooded area fraction ($R_m$) relative to the 2% threshold and the predicted water elevation above the beach ($R_p$) above a characteristic beach elevation $Z_c$ as follows:

$$R_m(t) = \frac{A|_{1d}(t) - 2\%}{S_m} \Theta(A|_{1d}(t) - 2\%) \max\left(\frac{A|_{24h}(t) - 2\%}{S_m}, 0\right)$$

$$R_p(t) = \frac{\eta_d(t) - Z_c}{S_p} \Theta(\eta_d(t) - Z_c) \max\left(\frac{\eta_d(t) - Z_c}{S_p}, 0\right)$$

(2)
Figure 12. Daily time series of the rescaled measured flooded area ($R_m$) and the rescaled predicted water elevation ($R_p$) above a beach elevation $Z_c = 0.7\text{m}$, from 23 Nov 2017 to 31 Mar 2018 (see definition in the main text). At the selected elevation ($Z_c = 0.7\text{m}$) the predicted flooding frequency from water elevation data equals the measured one from flooded area (see Fig. 11) however the duration of the predicted flooding events is much longer.

In both cases, $A_t$ is the area under the curve of $\frac{\partial\eta}{\partial t}$ for the measurements. $A_t^{\text{expected}}$ was the area under the curve of $\frac{\partial\eta}{\partial t}$ for the expected scenario. The difference between the two areas is the area for which the difference between $\eta$ and the expected $\eta$ is greater than $\tau$. The mean area under the curve of $\frac{\partial\eta}{\partial t}$ was the mean size of the measured flooding events for $\tau = 24\text{h}$ was $S_m = 7.5\%$ (see Fig. 9C).

For the predictions, $\eta_d$ was the estimated daily maximum of the total water level at the shoreline. $Z_c$ was the characteristic beach elevation and $S_p = 0.3\text{m}$ was the mean size of the predicted HWEs (Rinaldo et al., 2021). In both cases, $\Theta(x)$ is the step function (1 for $x > 0$ and 0 otherwise) the function $\max\{R_m, R_p\}$ ensures $R_m$ and $R_p$ are positive. Due to lack of data, we could only generate predictions for the first 130 days of our total 170 day measurement period (Fig. 12).

In spite of the numerous uncertainties in the estimation of the actual total water level from off-shore wave data and its application to estimate local run-up and beach overtopping, both time series, the time series given by Eqs. 2 were remarkably similar for the characteristic beach elevation $Z_c = 0.7\text{m}$ at which the predicted flooding frequency equals the measured one (Fig. 12). Indeed, most measured events were accurately captured by the prediction, including their relative intensity.

We evaluated the performance of the run-up model in predicting the measurements at the daily level using the conditional probabilities $P(m|p)$ and $P(p|m)$. The first one was where $P(m|p)$ is the probability of measuring an event during a day when flooding conditions were predicted ($p$), and the second one was the probability of predicting an event that was measured during a day when flooding was observed/measured. Figure 13 shows the rates of false positives $\frac{1}{1 - P(m|p)} P(p|m) = 1 - P(m|p)$, when flooding was
Figure 13. Performance of the model predictions when compared to measurements for different beach elevations ($Z_c$). False positive and false negative rates are defined in terms of conditional probabilities as the probability of flooding conditions being predicted but not measured, and the probability of flooding conditions being measured but not predicted, respectively (see text for detailed definition).

Predicted but not measured, and false negatives $1 - P(p|m)$ of the prediction $P(\neg p|m) = 1 - P(p|m)$, when flooding was measured but not predicted, for the time series given by Eqs. 2 as function of the characteristic beach elevation $Z_c$.

As it is already apparent in Fig. 12, for $Z_c = 0.7$ m the rate of false negatives was relatively low $\sim 25\%$ whereas the rate of false positives was quite high $\sim 75\%$. Since the predicted and measured frequency of flooding events are equal, the large rate of false positives implies the duration of the predicted flooding events is much longer than the observed ones. As the beach elevation increased, the rate of false negatives drastically increased, which suggests that our somehow supports our indirect estimation of the local beach elevation characteristic beach elevation at our site by comparing the predicted (using a run-up model) and measured (using our camera data) flooding frequencies consistent with the daily time series of total water elevation predicted from the run-up model (Fig. 12) and measured flooding frequencies. However, no similar improvement occurred for the rate of false positives, as the run-up model consistently overpredicted the number of flooding days at all beach elevations.

Given the nature of the run-up predictions using off-shore data (Stockdon et al., 2006, 2014), that in a first approximation neglects the alongshore variability of the bathymetry or the details of wave shoaling (García-Medina et al., 2017; Atkinson et al., 2017) one can argue that the prediction only indicates conditions favorable to flooding events somewhere along the shoreline and not necessarily the actual occurrence of a flooding event at a precise location. This statistical interpretation would be inline with our findings.
Corrected and normalized daily time series of measured flooded area \( (P_m) \) and predicted water elevation above the beach \( (P_r) \) from 23 Nov 2017 to 31 Mar 2018, when both time series are available (see definition in the main text). At the selected beach elevation \( (Z_r = 0.7 \text{m}) \) the predicted flooding frequency from water elevation data equals the measured one from flooded area (see Fig. 11):

5 Towards a probabilistic model of low-intensity and high-frequency flooding events

Performance of the model predictions when compared to the measurements. False positive and false negative rates are defined in terms of conditional probabilities as the probability of flooding predicted but not measured, and the probability of flooding measured but not predicted, respectively (see text for detailed definition). Summarizing our findings, flooding events obtained from the daily time series of water area \( A_{24h}(t) \) were uncorrelated and their size followed an exponential distribution with average \( \bar{S} = 7.5\% \) (Figs. 9 and 10). Therefore, the frequency of a flooding event of at least a size \( S_c \) (in percent of water pixels) is given by:

\[
\lambda(S_c) = \lambda_{2\%} e^{-(S_c-2\%)/\bar{S}}
\]

(3)

where \( \bar{S} = 7.5\% \) is the average size and \( \lambda_{2\%} = (2.5 \pm 1) \text{month}^{-1} \) is the frequency of all measured flooding events at the daily scale. Note that \( \lambda_{2\%} \) depends on the selected 2% threshold for the water area fraction (also appearing in the exponent) separating flooding and non-flooding conditions.

Similarly, as was already mentioned in the introduction, Rinaldo et al. (2021) found the overtopping (i.e., flooding) frequency of an elevation \( Z \) relative to MSL, can be approximated as:

\[
\lambda(Z) = \lambda_b e^{-(Z-Z_r)/\bar{S}}
\]

(4)

where \( \bar{S} \approx 0.3 \text{m} \) is the approximated average size of HWEs (it was found to be mildly site-dependent) and \( \lambda_b = 1.5 \text{month}^{-1} \) is the overtopping frequency at the reference elevation \( Z_r \), relative to MSL. This elevation was found to roughly correspond to the characteristic beach elevation at a given site and depends on the local tidal amplitude \( A_t \) and average predicted wave runup \( \bar{H}_w \), as,

\[
Z_r \approx A_t + 1.6 \bar{H}_w.
\]

(5)

The average wave runup, predicted using the formulation of Stockdon et al. (2006), can in turn be expressed in terms of the deep-water significant wave height \( H_s \) and wavelength \( L_0 \), as,

\[
\bar{H}_w = a(\beta) \sqrt{H_s L_0}
\]

(6)

where the overline means average over the time period analyzed, \( L_0 \) is calculated from the peak wave period \( T_p \) using the deep-water dispersion relation \( L_0 = gT_p^2/(2\pi) \), and the factor \( a(\beta) \) is function of the beach slope \( \beta \) and can be written as,

\[
a(\beta) = 0.033 \left( \sqrt{1 + 1.2\beta/\beta_c} + \beta/\beta_c \right)
\]

(7)
with constant $\beta_c = 0.087$ (Rinaldo et al., 2021). Equations 4–7 provide a relatively simple and widely applicable probabilistic model of low-intensity and high-frequency flooding events.

Although our Eq. 3 only gives an indirect measure of actual flooded area, by using the fraction of water pixels in an image, and is only valid for our field site, it does support the validity of Eq. 4 in describing actual flooding using HWEs.

6 Discussion and Conclusion

We studied the stochastic properties of coastal-flooding events monitored via 5-minutes time-lapse imagery for more than 160 days and processed using CNN-based image segmentation. When analyzing the data at the daily timescale, we find flooding events can be modeled as a Poisson process with exponentially distributed sizes, in agreement with recent findings using a run-up model to predict beach overtopping (Rinaldo et al., 2021). The frequency of flooding events depended strongly on the timescale at which data was analyzed, and decreased from about 6 events per month at the hourly timescale to a plateau of 2.5 events per month at the daily timescale. Furthermore, the correlation between consecutive events also depended on the timescale, at which we defined the flooding events. Following our statistical analysis of event inter-arrivals, flooding events seems to be correlated for timescales smaller than 10 hours, while events are random and independent at larger timescales, thereby following a Poisson process.

The frequency of flooding events depended strongly on the timescale at which data was analyzed, and decreased from about 6 events per month at the hourly timescale to a plateau of 2.5 events per month at the daily timescale. Although this value is larger than the frequency of 1.5 events per month predicted by HWEs (Rinaldo et al., 2021) for natural beaches, the latter value was within the confidence bounds of our data, which were relatively large due to the short-time period analyzed. Nevertheless, a higher measured flooding frequency was expected because of beach erosion induced by hurricane Harvey. This change in temporal correlation for timescales around 10 hours could be related to the tidal period (which is about 12h at this location) and the day-night cycle potentially disrupting any local weather pattern behind the flooding event.

We found the size of flooding events was exponentially distributed with average sizes of about 4% of the camera field-of-view when data was analyzed at the hourly timescale to a maximum 8% at the daily or-larger timescale. When estimated at the highest 5-minute resolution, we also found the actual duration of flooding events typically varies between 10 and 100 minutes, and seemed to follow a power-law distribution. However, there was a lack of events longer than 3 hours in our nearly six-month period, during which there was no large storms, seems to suggest a physical upper limit for sustained flooding conditions perhaps related to high tides. However, in this region astronomical tides are relatively small and water levels are mainly affected by waves, which would again point to wave runup driving the observed flooding, as suggested by the high-water event analysis. Furthermore, we found a poor correlation between the size and the duration of flooding events.

Finally, at the daily timescale, we found that flooding events can be modeled as a Poisson process with exponentially distributed sizes, in agreement with recent findings using a run-up model to predict coastal flooding (Rinaldo et al., 2021). The main probabilistic properties of measured and predicted flooding events can thus be described by Eqs. 3 and 4, respectively. One way to understand the similar form of both equations is through the relation between flooded
area and water depth at the shoreline. Assuming the beach slope in our field site is relatively constant then we would expect both to be proportional, in which case the fraction of water pixels would also correlate with water depth at the shoreline. Therefore, our agreement with Rinaldo et al. (2021) suggests that the exponential distribution is robust with respect to potential variations of the local beach slope during the measurement period and alongshore variations of the flooded area at the spatial scale defined by the camera field-of-view.

Going beyond the statistical agreement pointed above, the frequency of 1.5 events per month predicted by HWEs (Rinaldo et al., 2021) for natural beaches, although lower than the 2.5 events per month measured, was within the confidence bounds of our data, which were relatively large due to the short time period analyzed. Nevertheless, a higher measured flooding frequency was expected because of beach erosion induced by hurricane Harvey, which would improve the agreement with the model. When focusing on the daily correlation of predicted and measured flooding, the predictions from the analysis of HWEs (Rinaldo et al., 2021) captured most of the runup model behind the HWEs prediction of Rinaldo et al. (2021) indeed captures the occurrence of local daily flooding, although it noticeably overpredicts them. Furthermore, we found a good agreement between the predicted flooding frequency (from HWEs) and our measurements when taking into account local beach erosion due to hurricane Harvey. The large fraction of false positives in the predicted flooded days (particularly at the end of the measurement period), even after correcting for a different beach elevation, could result from the assumption of a constant beach slope along the whole beach section covered by the camera and for the whole observation period. Since run-up predictions using off-shore data (Stockdon et al., 2006, 2014) are essentially valid for a single transect and thus neglect the alongshore variability of the bathymetry or the details of wave shoaling (García-Medina et al., 2017; Atkinson et al., 2017), it would be difficult to capture the complexity of the site-to-site variability of flooding over a relatively large beach section. On the other hand, it could be that the predicted flooding was taking place somewhere else along the beach and was not captured by our local observations. A final possibility is that our sampling frequency of one picture every 5 minutes is not high enough to capture all possible large runup events (as predicted by the HWEs formulation), in which case the false positive rate could be lower. This is supported by the fact that the distribution function of the duration of flooding events has a lower-limit of 3 minutes.

In summary, regardless of these sources of potential errors, and more in line with the statistical nature of wave runup data and the uncertainty in the calibration of the model parameters in the first place (García-Medina et al., 2017; Atkinson et al., 2017), one can argue that the prediction only indicates conditions favorable to flooding events somewhere along the shoreline and not necessarily the actual occurrence of a flooding event at a precise location. This statistical interpretation would agree with our findings.

In addition to our findings characterizing the probabilistic structure, including frequency, intensity and duration, of coastal flooding at our field site, by validating the predictions of Rinaldo et al. (2021), our work also demonstrates the suitability of HWEs predictions, based on relatively simple run-up models, to estimate the frequency and intensity of beach and dune overtopping. We also provided a detailed characterization of the probabilistic structure of flooding events, including its frequency, intensity and duration, events leading to coastal flooding and dune erosion. Our results thus formalize, i.e. validate and expand, the first probabilistic model of high-frequency low-intensity coastal flooding events driven by wave
run-up. After further estimation (e.g., Eqs. 4–7), After further calibration of the model parameters for different locations, this probabilistic model can be very useful in coastal risk management and landscape evolution models.

Data availability. All data used or generated in this study is available in the Texas Data Repository (TDR), doi.org/10.18738/T8/FLGDS0.

Author contributions. O.D.V. designed the study. O.D.V., R.A.F. and T.H. installed the field equipment and carried out the observations. B.K. performed the time series analysis. B.K and O.D.V. performed the probabilistic analysis. B.K and O.D.V. prepared the manuscript with contributions from all co-authors.

Competing interests. The authors declare that they have no conflict of interest.

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