

Re: Drop Size Distribution Retrieval Using Dual-Polarization Radar Observations at C-Band and S-Band

Dear Editorial Team of AMT,

We continue to be grateful for the support of the editorial team and the efforts of the reviewers who have greatly helped improve our work.

Please find our enclosed point-to-point response. We have made substantial revisions to our manuscript to address each comment received.

Thank you,

Dan Durbin  
Yadong Wang  
Pao-Liang Chang

<b>Reviewer:</b> 1	<b>Comment Number:</b> R1
<b>Comment:</b> The authors carefully answered reviewer's questions and revised the manuscript according to the reviewer's comments and suggestions. I suggest they should take a minor revision. Change the term "phase" in their interactive discussion response and revised manuscript to either "specific differential phase" for KDP or "differential phase" for PhiDP.	
<b>Reply:</b> We have updated any generic "phase" references with either "differential phase" or "specific differential phase" as appropriate.	
<b>Manuscript Update:</b> Revision: Line 395 Figure 4  Redline Version: Line 451 Figure 4	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-1
<b>Comment:</b> Section 2.2: Radar calibration: Add one or two sentences related to the calibration status of the radars (reflectivity) as Z is an input to the PSO algorithm.	
<b>Reply:</b> We have added the following sentences with respect to calibration of RCWF (S-band reflectivity source):  "RCWF is an S-band dual-polarimetric radar that is part of Taiwan's operational Multi-Radar-Multi-Sensor QPE system (Chang et al, 2021). To achieve a QPE accuracy within 10 percent, the reflectivity bias must be kept within 1 dBZ. This is accomplished by regularly calibrating RCWF's reflectivity using a self-consistency algorithm (Le Loh et al., 2022). In contrast, RCMD is a C-band radar used experimentally and for research purposes rather than operationally."	
<b>Manuscript Update:</b> Revision: Lines 79-83  Redline Version: Lines 81-85	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-2
<b>Comment:</b> Section 2.2: Lines 160-165: Are these conditions in range and Z values part of the methodology or is it meant only for validation purposes? Please clarify this in the manuscript and re-arrange accordingly. If these values are part of the methodology (in an operational context), then how should we proceed to have a complete “map” of retrieve DSD? Perhaps this needs to be discussed in the conclusions.	
<b>Reply:</b> These conditions are primarily used for validation purposes to ensure we had "good" data to operate on. However, the range limitation would be critical in minimizing vertical separation between the observation volume and ground position as well as minimizing the effects of accumulated error.  We have updated this paragraph accordingly: “This set of criteria is designed to strike a balance between creating sufficient deviation between C and S-band differential phases for an accurate DSD retrieval, while also preventing excessive error accumulation as well as minimizing the vertical separation between the radar observation volume and ground location. Additionally, a reflectivity threshold of 25 dBZ is imposed to ensure that there is enough observable precipitation in the terminal gate where the disdrometer is located. These criteria are primarily chosen to increase the quality of the data for development and validation, however similar standards would need imposed if the algorithm were to be operationally applied to address the error accumulation and elevation differences.”	
<b>Manuscript Update:</b> Revision: Lines 176-181  Redline Version: Lines 180-185	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-3
<b>Comment:</b> Section 2.2: Lines 168-172: Remove it because all these were mentioned before in section 2.2.1	
<b>Reply:</b> We have removed the duplicated content.	
<b>Manuscript Update:</b> Revision: N/A Redline Version: Lines 186-193	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-4
<b>Comment:</b> Section 2.2: Lines 174-175: Clarify that Z is not yet corrected from attenuation.	
<b>Reply:</b> We have updated the line accordingly:  “Figure 4 shows the pre-processed (dashed) and post-processed Z (solid), without any attenuation correction, and $\phi_{DP}$ fields along the yellow arrow of Figure 3, contrasting the difference between the post-processed differential phase profiles (solid) with the profiles prior to unfolding and smoothing (dashed).”	
<b>Manuscript Update:</b> Revision: Lines 142-146  Redline Version: Lines 145-149	

**Reviewer:**  
2

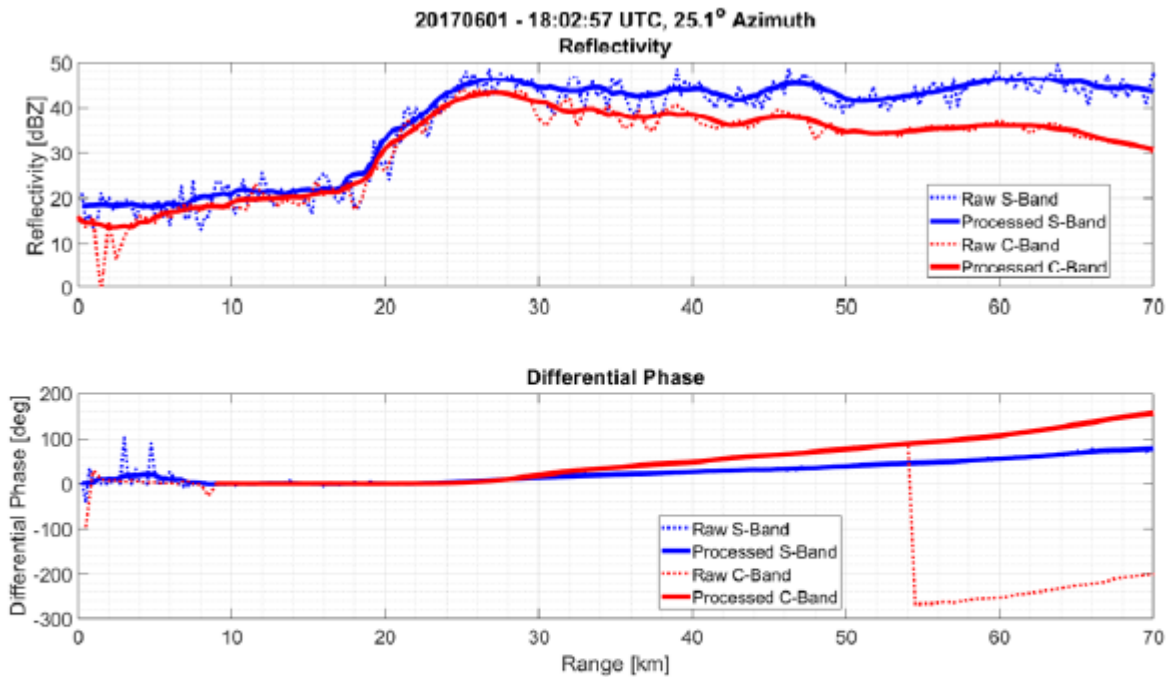
**Comment Number:**  
R2-5

**Comment:**

Section 2.2: Figure 4: Add the raw phidp to compare with the processed phip (FIR filtering). For easy comparison, the initial phase offset can be subtracted.

**Reply:**

We have updated the figure to add the raw phidp. We have also added the raw reflectivity for completeness.



**Figure 4.** The data along the radial indicated in Figure 3 before and after processing. Raw radar data (dashed lines) are contrasted with post-processed data (solid lines). Processed C-band reflectivity is not used as a retrieval input but is useful for validation purposes as discussed in Section 3.1

**Manuscript Update:**

Revision:  
Figure 4

Redline Version:  
Figure 4

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-6
<b>Comment:</b> Section 2.2: Lines 181-183: Clarify that Z (S-band) also suffers from attenuation. The way it read now, tell the reader that Z (C-band) is the only one sensitive to attenuation.	
<b>Reply:</b> We have stated more clearly that S-band also suffers from attenuation:  “Although S-band reflectivity does experience atmospheric attenuation, $Z^C$ is much more vulnerable to this effect and is therefore not used as an input to the algorithm.”	
<b>Manuscript Update:</b> Revision: Line 187  Redline Version: Line 200	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-7
<b>Comment:</b> Line 230, Eq(7): Clarify if the Cost is estimated gate by gate or per radial, i.e., are the variables Z and phidp single values or vectors?	
<b>Reply:</b> We have added the following note to state that the retrieval is for each gate, implying the values are singular rather than vectored:  “For a gate’s retrieval, the cost of every particle is calculated, and the iteration’s current best solution as well as the global best solution of all iterations are recorded.”	
<b>Manuscript Update:</b> Revision: Line 237  Redline Version: Line 251	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-8
<b>Comment:</b> Section 2.2: Eq(9): “four hundred iterations for each retrieval” . This may lead to a large computational time as this retrieval is gate by gate. However, I have not read anything related to this. If this is a burden, then please add a comment about it.	
<b>Reply:</b> For this prototyping phase, minimizing run times was not a priority and the dataset could typically be processed overnight on a desktop computer. Any embedded solution should take advantage of speed improvements gained from parameter tuning or even substituting a more efficient technique in place of PSO.  We have added the following: “‘It should be noted that the setting of four hundred iterations is intended solely for prototype algorithm development, and computational efficiency has not yet been addressed. A simple iteration control algorithm could be implemented to terminate the computation once the root mean square error reaches the predefined threshold. Any embedded solution should take advantage of speed improvements gained from parameter tuning or even substituting a more efficient technique in place of PSO.’”	
<b>Manuscript Update:</b> Revision: Lines 245-249  Redline Version: Lines 260-264	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-9
<b>Comment:</b> Section 2.2: Line 243: “starting from the one nearest to the radar”. Is that the case? Because I thought the minimum range is set to 25 km.	
<b>Reply:</b> Thank you for highlighting this point. We acknowledge that the manuscript lacked clarity, which led to the confusion. We have now clarified that the specified criteria pertain to the terminal gate containing the truth data. The retrieval process still begins at the closest gate to accurately account for all attenuation along the beam. The final (destination) gate is at least 25 km from the radar. To address this issue, we have made the following modifications in the manuscript:  “To achieve reasonable results, the following criteria for candidate data are therefore used for the terminal gate of the retrieval which contains the disdrometer: $25 \text{ km} < \text{Range} < 70 \text{ km}$ $Z^S > 25 \text{ dBZ}$ This set of criteria is designed to strike a balance between creating sufficient deviation between C and S-band differential phases for an accurate DSD retrieval, while also preventing excessive error accumulation as well as minimizing the vertical separation between the radar observation volume and ground location. Additionally, a reflectivity threshold of 25 dBZ is imposed to ensure that there is enough observable precipitation in the terminal gate where the disdrometer is located. These criteria are primarily chosen to increase the quality of the data for development and validation, however similar standards would need imposed if the algorithm were to be operationally applied to address the error accumulation and elevation differences.”	
<b>Manuscript Update:</b> Revision: Line 172 Lines 251-256  Redline Version: Lines 176-179 Lines 270-275	



<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-10
<b>Comment:</b> Section 2.2: Line 245: “once a representative DSD” Is this the same as the best DSD or still under the ptimization search. Please clarify it in the text.	
<b>Reply:</b> We have removed this ambiguity to clearly state the optimization search has ceased:  “After completing the retrieval for a gate, attenuation is calculated using Equation 5, and the reflectivity for the next farthest gate is adjusted.”	
<b>Manuscript Update:</b> Revision: Line 253 Redline Version: Lines 271	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-11
<b>Comment:</b> Section 2.2: On the same line: “the attenuation is calculated”. Please add something like .. Is calculated based on Equation (5). So far Equation (5) was not indicated yet.	
<b>Reply:</b> We have updated to include reference to Eq 5: “After completing the retrieval for a gate, attenuation is calculated using Equation 5, and the reflectivity for the next farthest gate is adjusted.”	
<b>Manuscript Update:</b> Revision: Line 254 Redline Version: Line 272	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-12
<b>Comment:</b> Section 2.2: Line 254-255: Replace “This process continues” by “This gate by gate process continues” or similar. Replace “at which point the final retrieval is performed” by “because the range criterion < 70 km” or something similar.	
<b>Reply:</b> We have updated similarly to the suggestion: “In this manner, each gate’s input reflectivity accounts for attenuation experienced between the radar and that gate. This process, illustrated in Figure 7, continues at 4 km intervals until reaching the disdrometer location.”	
<b>Manuscript Update:</b> Revision: Lines 255-256  Redline Version: Lines 273-275	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-13
<b>Comment:</b> Section 2.2: A related question: why the algorithm stops at the location of the disdrometer that is located 70 km from the radar site? One would like to continue till the end of the radial. Please clarify somewhere in the text.	
<b>Reply:</b>  Thank you for pointing this out. While the algorithm can indeed extend to the end of each radial, we limited the range to within 70 km in this prototype algorithm for two primary reasons:  1) Since the retrieval result from each gate highly depends on the results from previous gates, errors can accumulate up to the final gate. Limiting the range to 70 km helps to effectively mitigate these accumulated errors. The 70 km range was chosen based on the ranges of available disdrometers used for validation.  2) The height of the radar beam increases monotonically along the radar beam. Consequently, at longer distances, the deviation between radar observations (in the air) and disdrometer observations (on the ground) could be significant due to vertical variations in the DSD. For validation purposes, limiting the radar's final gate within a reasonable range helps to minimize differences between retrieval results and disdrometer observations, which serve as the ground truth.  The manuscript has been updated as outlined in response to comment R2-2.	
<b>Manuscript Update:</b> Revision: Lines 176-181  Redline Version: Lines 180-185	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-14
<b>Comment:</b> Section 2.2: A second related question: What happens to the algorithm when rain starts around 25 km, i.e., when the difference between phidp(C-band) and phidp(S-band) is not sufficient (see line 155). Then DSDs are not retrieved? Clarify this somewhere in the text.	
<b>Reply:</b> Please see reply to R2-13.	
<b>Manuscript Update:</b> Revision: Lines 176-181  Redline Version: Lines 180-185	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-15
<b>Comment:</b> Section 2.2: Lines 267-269: Please clarify what is the relation between 10 days and 167 cases, how a case is defined?	
<b>Reply:</b> We have updated the manuscript to include the definition of a case (one 360 degree scan of each radar) and specified that only 167 of the cases met our strict synchronization and data quality requirements.  “A potential case is defined as one plan position indicator (PPI) scan of each radar. The stringent criteria for time synchronization narrowed the dataset to 167 cases for time synchronization narrowed the dataset to 167 cases that not only met the synchronization requirements of the radar scans but also had the requisite data quality and fell within the disdrometer range requirement.”	
<b>Manuscript Update:</b> Revision: Lines 313-316  Redline Version: Lines 340-343	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-16
<b>Comment:</b> Section 3.1 Line 278: “The majority of the cases” ... write how many, 5 out of 8?	
<b>Reply:</b> We have updated as follows, although it is a matter of opinion since this is still for our subjective comparison:  “Six of these eight cases show a high degree of agreement across the spectrum of drop sizes, with particularly strong correlation observed for the measurements of moderate drop sizes.”	
<b>Manuscript Update:</b> Revision: Line 325  Redline Version: Line 352	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-17
<b>Comment:</b> Section 3.1 Line 281: “Deviations can be found...” That is obvious. What reasons would the authors suggest for such deviations?	
<b>Reply:</b> We have combined this paragraph with the following paragraph which primarily focuses on our rationale for the deviations, namely the relative importance of different drop sizes. This update provides a better flow from the statement to the underlying reasons.  “Deviations can be found in cases such as 20170613 - 08:04:49 and 20180107 - 09:26:32 for smaller drops ( $D < 0.5$ mm). This is a predictable result given the retrieval input parameters are heavily dominated by contributions of larger diameters. Not all disdrometer sizes need to be equally prioritized for accurate fitting. Examination of Equations 3 and 4 reveals that the contribution to radar parameters from each diameter increases with the size of the drop. Furthermore, previous research evaluating the accuracy of disdrometers across various drop sizes indicated that drops of 0.6 mm and larger are the first reliably measured sizes by optical disdrometers (Tokay et al., 2001). This finding supports the notion that inaccuracies in measuring smaller drop sizes do not significantly impact the calculations of reflectivity or DSD-derived metrics such as rain rate and attenuation.”	
<b>Manuscript Update:</b> Revision: Lines 327-334  Redline Version: Lines 354-362	

Reviewer:

2

Comment Number:

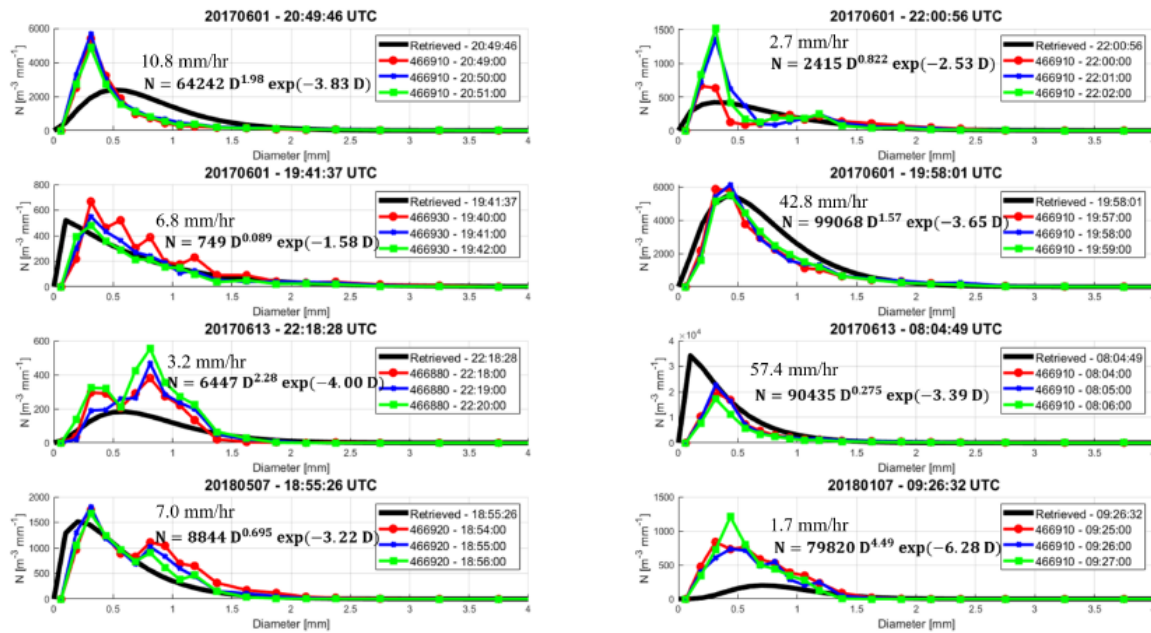
R2-18

**Comment:**

Section 3.1 Figure 8: Add the resulting rainfall rate values from disdrometers (blue line) to each subplot so that we can associate them with a rain intensities.

**Reply:**

We have updated Figure 8 (now Figure 10) to include rainfall rates.



**Figure 10.** The retrieved DSD is shown in black on each plot with its gamma distribution equation. The DSD of the closest disdrometer record is plotted in blue while the previous record and next record time are shown in red and green, respectively. The median rainfall rate of the three disdrometer collections is indicated in mm/hr.

**Manuscript Update:**

Revision:

Figure 10

Redline Version:

Figure 10

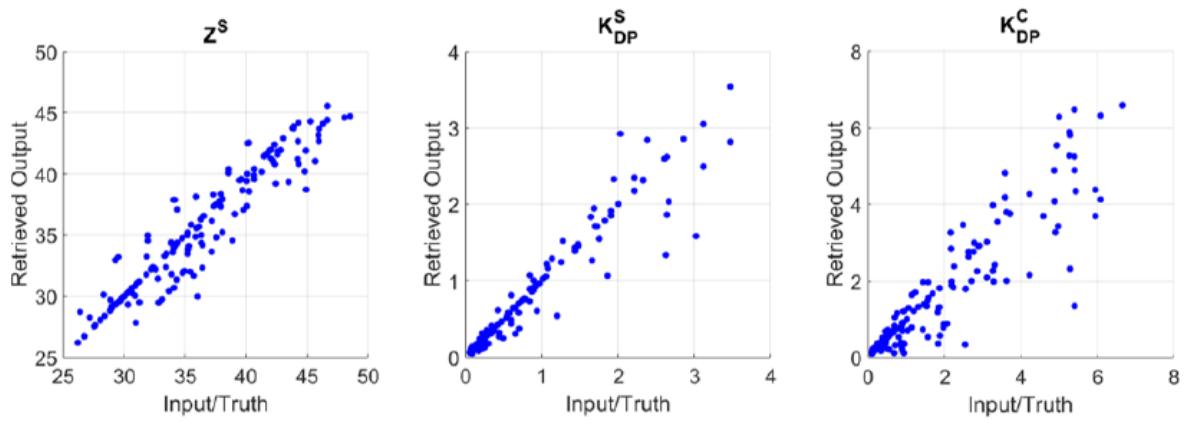
<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-19
<b>Comment:</b> Section 3.1 Lines 290-294 Rewrite to something like this: This simulation involves calculating KDP using equation(4) for varying drop size distributions using a gamma-modeled DSD ...	
<b>Reply:</b> This section has been updated per the reviewer's comments to include which equations/models are being utilized. Additionally, the simulation section has been expanded to include a much more thorough evaluation of the algorithm under simulated ideal conditions.	
<b>Manuscript Update:</b> Revision: Lines 258-301  Redline Version: Lines 281-328	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-20
<b>Comment:</b> Section 3.1 Line 300: Replace "C-band reflectivity excluded" by C-band differential phase excluded.	
<b>Reply:</b> Thank you for catching this error. We have completely removed this example in lieu of a more thorough simulation that addresses one vs. two frequency retrievals.	
<b>Manuscript Update:</b> Revision: Lines 258-301  Redline Version: Lines 281-328	

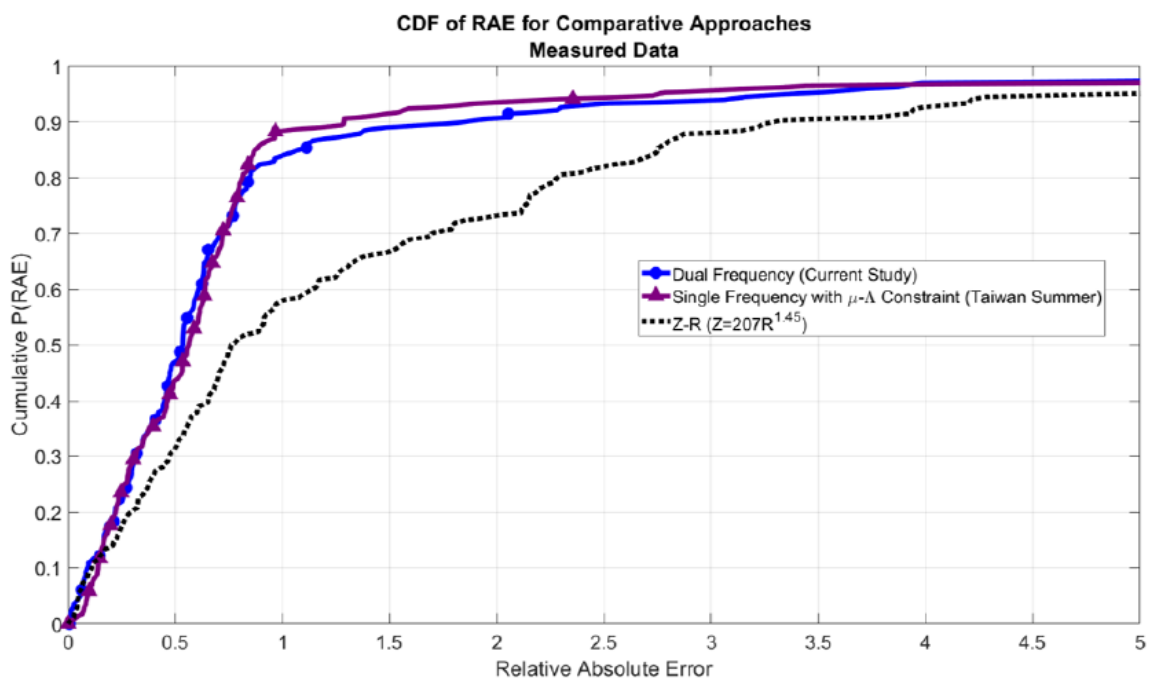


<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-21
<b>Comment:</b> Section 3.1 To strengthen the qualitative assessment. I suggest the following points: <ul style="list-style-type: none"> <li>• As the estimation of accurate phidp from raw phidp is not straightforward, especially at C-band, show some radials of processed phidp (C and S-band) and compare them with optimal DSD-based phidp.</li> <li>• Attenuation correction, Figure 11: Only showing one single radial is not convincing and difficult to suggest conclusions. Similar to my previous point, show some more radials with diverse Z radial profiles (or a PPI).</li> <li>• Lines 305-310: It is not consistent with Line 290 (“key question”). I suggest you either i) make a strong analysis to address this question or ii) Rewrite Line 290 and move Equation (9) and Figures 9 and 10 to Appendix.</li> </ul>	
<b>Reply:</b> We have addressed the reviewer's comments through the following updates: <ol style="list-style-type: none"> <li>1) We updated this figure to include more examples. A PPI plot is not feasible with our processed data, but we have expanded the figure to include 3 examples with the space available.</li> <li>2) We have included the input and retrieved phi_dp profiles for these examples.</li> <li>3) We have provided a stronger analysis on one of the "key questions" (one vs two frequency) through a more thorough simulation.</li> </ol>	
<b>Manuscript Update:</b> <ol style="list-style-type: none"> <li>1) 3 examples  Revision:  Figure 11  Redline Version:  Figure 11</li> <li>2) Inclusion of phidp  Revision:  Figure 11  Redline Version:  Figure 11</li> <li>3) Updated simulation analysis  Revision:  Lines 258-301  Redline Version:  Lines 281-328</li> </ol>	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-22
<b>Comment:</b> Section 3.2 This sections lack of statistical results. Showing a time series plot is not so relevant in this study. I would suggest using scatterplots and show metrics such as RMSE, Bias, correlation, etc. For instance, a scattering plot related to the attenuation correction: Z(retrieved DSD) vs Z(disdrometer). And two more related to KDP and Rainfall rate.	
<b>Reply:</b> We appreciate the reviewer’s suggestions. To address the reviewer’s comment, we have updated the manuscript to include scatter plots of retrieved vs. input values for $Z$ , $K_{DP}^S$ , and $K_{DP}^C$ . We have also included the calculated correlation coefficients in the manuscript text. We recognize that the statistical results of rainfall are normally evaluated either in the accumulated (e.g., hourly based or days based) format (e.g. Wang et al. 2013, Zhang et al. 2016, Zhang et al. 2020), or in a relatively large scale of rain rate (Scarchilli et al. 1993). However, based on our current available data, such statistical analysis is not feasible. Currently, we are collecting more data for such a goal, and will present additional statistical results in future work. The time-series data (relative absolute error) has been reformatted in terms of a cumulative error distribution which is much more interpretable. <p>In the revised manuscript, new figures and discussions are included.</p> <p>Wang, Y. J. Zhang, A. Ryzhkov, L. Tang, 2013: C-band polarimetric radar QPE based on specific differential propagation phase for extreme typhoon rainfall. JTECH. <a href="https://doi.org/10.1175/JTECH-D-12-00083.1">https://doi.org/10.1175/JTECH-D-12-00083.1</a></p> <p>Jian Zhang, et al., 2016: Multi-Radar Multi-Sensor (MRMS) Quantitative Precipitation Estimation: Initial Operating Capabilitie. BAMS</p> <p>Gianfranco Scarchilli, et al. 1993: Rainfall estimation using polarimetric techniques at C-band frequencies. JAMC.<a href="https://doi.org/10.1175/1520-0450(1990)029&lt;0561:MRTSUC&gt;2.0.CO;2">https://doi.org/10.1175/1520-0450(1990)029&lt;0561:MRTSUC&gt;2.0.CO;2</a></p> <p>Jian Zhang, et al. 2020: A dual-polarization radar synthetic QPE for operations. JHM.<a href="https://doi.org/10.1175/JHM-D-19-0194.1">https://doi.org/10.1175/JHM-D-19-0194.1</a></p>	



**Figure 12.** The retrieved calculated value of each parameter (y-axis) is shown relative to the algorithm’s input (x-axis) based on the radar measurements at the final gate (disdrometer location).



**Figure 13.** The performance of the dual frequency approach (blue circles) shows roughly equivalent performance to the single frequency approach employing a relevant  $\mu - \Lambda$  constraint (purple triangles). Both methods utilizing DSD information outperform the region-specific Z-R derived rainrates (black dash).

**Manuscript Update:**

Scatter Plots  
Revision:

Figure 12

Redline Version:

Figure 12

Cumulative Error Distribution

Revision:

Figure 13

Redline Version:

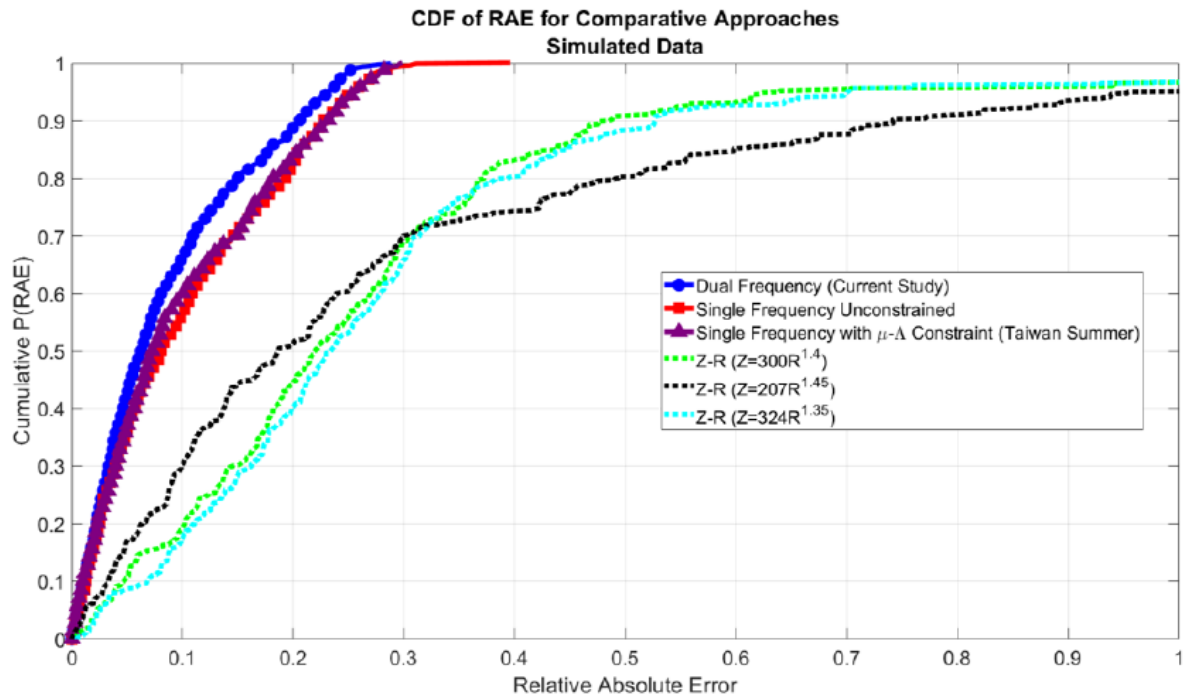
Figure 13

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-23
<b>Comment:</b> Section 3.2 Lines 345-347: “Deviations from this correlation can mislead the algorithm” These lines are somehow “in the air”. I suggest the authors to add some related references for the estimation of KDP that use self-consistency. For example: <ul style="list-style-type: none"> <li>• Park, S., V. N. Bringi, V. Chandrasekar, M. Maki, and K. Iwanami, 2005: Correction of Radar Reflectivity and Differential Reflectivity for Rain Attenuation at X Band. Part I: Theoretical and Empirical Basis. J. Atmos. Oceanic Technol., 22, 1621–1632, <a href="https://doi.org/10.1175/JTECH1803.1">https://doi.org/10.1175/JTECH1803.1</a>.</li> <li>• Giangrande, S. E., McGraw, R., and Lei, L.: An application of linear programming to polarimetric radar differential phase processing, J. Atmos. Ocean. Tech., 30, 1716–1729, 2013.</li> <li>• Gorgucci, E., Scarchilli, G., and Chandrasekar, V.: Specific Differential Phase Estimation in the Presence of Nonuniform Rainfall Medium along the Path, J. Atmos. Ocean. Tech., 16, 1690–1697, 1999.</li> <li>• Reinoso-Rondinel, R., Unal, C., and Russchenberg, H.: Adaptive and high-resolution estimation of specific differential phase for polarimetric X-band weather radars, J. Atmos. Ocean. Tech., 35, 555–573, 2018.</li> </ul>	
<b>Reply:</b> We have updated the manuscript to include the need for a self-consistency criteria and the suggested references have been added into the revised manuscript.  “Therefore, incorporating self-consistency relations between the input variables stands out as a promising direction for enhancing algorithmic accuracy in future research endeavors. (Park et al., 2005; Giangrande et al., 2013; Gorgucci et al., 1999; Reinoso-Rondinel et al., 2018)”	
<b>Manuscript Update:</b> Revision: Lines 376-381  Redline Version: Lines 435-436	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-24
<b>Comment:</b> Section 4 Discuss more the generalization of the proposed algorithm to retrieve DSD in terms of dual frequency systems and the range constraint ( $> 25$ , $< 75$ km).	
<b>Reply:</b> Please see reply to RC2-2.	
<b>Manuscript Update:</b> N/A	

<b>Reviewer:</b> 2	<b>Comment Number:</b> R2-25
<b>Comment:</b> Section 4 Clarify if this algorithm can be applied in real-time operations. If not, then what aspects would need further work?	
<b>Reply:</b>  Yes, this algorithm is developed to applied in real-time operation. We clarify this and point out some proposed improvements for doing such in the conclusion.  “The optimization approach involves various adjustable parameters, including swarm size, number of iterations, and acceleration coefficients. Fine-tuning these parameters could result in faster and more optimal results. Moreover, adapting the algorithm to function as an embedded application for field testing is also a promising area for further development.”	
<b>Manuscript Update:</b> Revision: Lines 402-404  Redline Version: Lines 458-460	

<b>Reviewer:</b> 3	<b>Comment Number:</b> R3-1
<b>Comment:</b> 1. The authors addressed my question #5 in their Response (See their Reply to RC1, Comment Number 6), which suggested that the proposed method be compared with single-wavelength radar observations and a mu-lambda constraint. The Response states that the proposed method and the mu-lambda constraint method produce similar results. Thus, the proposed method is not better than a single frequency method using a mu-lambda constraint. However, this comparison is not included in the manuscript. The manuscript should include the single frequency mu-lambda constraint method as a benchmark to show that the proposed method produces similar results.	
<b>Reply:</b> A comparison with the mu-Lambda constraint is now included with both simulated data and the measured/truthed data. The errors of each are much more clearly demonstrated in the form of a cumulative distribution rather than a time series plot.  Update for simulated data: “Figure 8 illustrates the cumulative distribution of errors for each method. In this plot, the cumulative portion of errors is shown on the y-axis, while the sorted error values of each method are displayed on the x-axis. As an example interpretation of the plot, 90% of current study’s errors (blue) are less than 0.2 in terms of RAE, while 90% of the Tawain Z- R based errors(black dashed) are not contained until an error level of 0.78 RAE. Interpreting the plot from the constant RAE perspective, 65% of the current study’s errors are below 0.1, while 60% of $\mu - \Lambda$ (purple) method’s errors and 30% of the Tawain Z- R (black) method’s errors are below 0.1 RAE. The conclusion of the simulations can be drawn from the plots: the dual frequency approach provides a modest improvement over single frequency retrievals, even with a relevant $\mu - \Lambda$ constraint, and a significant improvement compared to all Z-R based rates. Under these ideal simulated conditions, the median RAE of the dual-frequency approach was 0.0623 while the $\mu - \Lambda$ single-frequency approach was 0.0725. The median RAE of the best performing (at the 50th percentile) Z-R relationship ( $Z = 207R^{1.45}$ ), was 0.1861.”	

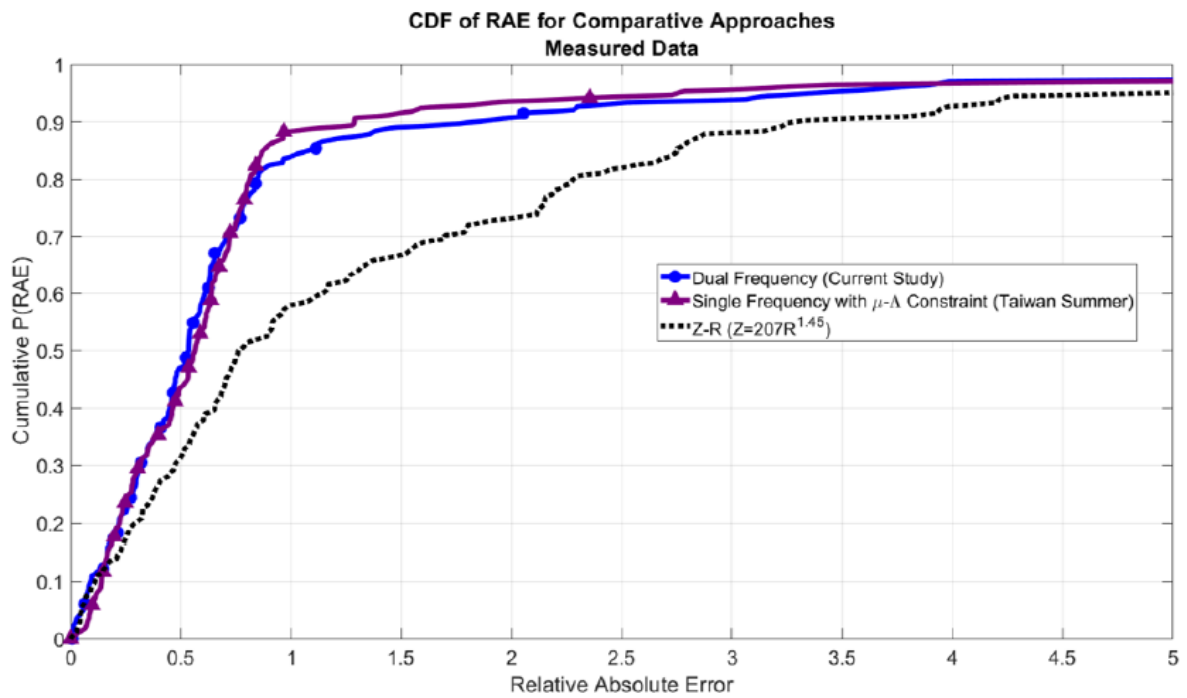


**Figure 8.** The distribution of errors (cumulative) associated with each approach are plotted. The results of this study’s proposed methodology using two frequencies (blue circles) shows modest improvement when compared to single frequency retrievals with (purple triangle) and without (red square) assumed  $\mu - \Lambda$  constraints. A more significant improvement is seen when compared to the various errors associated with Z-R rain rate estimations are indicated with dashed lines.

Evaluation data:

“Figure 13 features a plot of the cumulative distribution of RAE for all three methods. The retrieval algorithm’s accuracy is depicted by the blue-circles line, while the  $\mu - \Lambda$  constrained retrieval is shown in purple, and the Z-R relationship benchmark is indicated with the dashed black line. This visualization highlights that the proposed method of estimating rainfall rates using retrieved DSD parameters significantly enhances accuracy over a region-specific Z-R relationship (Chang et al., 2021). Specifically, the median RAE for the Z-R method is 0.76, while the retrieval results correspond to a median of 0.53, marking a notable improvement of 30.3 % in this study’s context. The dual-frequency approach is comparable to the single-frequency constrained approach and the only claimed relative benefits are not needing to ascertain a regional  $\mu - \Lambda$  relationship or cases in which the precipitation deviates from the assumed relationship.”





**Figure 13.** The performance of the dual frequency approach (blue circles) shows roughly equivalent performance to the single frequency approach employing a relevant  $\mu - \Lambda$  constraint (purple triangles). Both methods utilizing DSD information outperform the region-specific Z-R derived rainrates (black dash).

**Manuscript Update:**

Revision:

Lines 287-296 and 366-373

Redline Version:

Lines 314-323 and 421-429

<b>Reviewer:</b> 3	<b>Comment Number:</b> R3-2
<b>Comment:</b> 2. Comparing the proposed dual wavelength retrieved rain rates to a generic $z = 300R^{1.4}$ power law relationship is not a valid comparison. The comparison should be performed using a regionally tuned Z-R relationship produced from the disdrometer data. The manuscript should compare the rain rates estimated from the proposed retrieval method with rain rates estimated from a disdrometer derived regional Z-R relationship.	
<b>Reply:</b> We have reevaluated the results using a region-specific Z-R relationship. Additionally, a more comprehensive simulation section has incorporated three Z-R relationships for comparison.  “Equation 14 represents perhaps the most common meteorological radar relationship Ulbrich and Lee (1999), Equation 15 is specific to Taiwan as proposed by Chang et al. (Chang et al., 2021), and Equation 16 is derived from fitting calculated Z and R values found in the initial steps of the simulation.” $Z=300R^{1.4}$ (14) $Z=207R^{1.45}$ (15) $Z=324R^{1.35}$ (16)  Equation 15 (Taiwan specific) is used as the Z-R relationship for the measured data evaluation.	
<b>Manuscript Update:</b> Revision: Lines 280-286  Redline Version: Lines 307-313	

**Reviewer:**

3

**Comment Number:**

R3-3

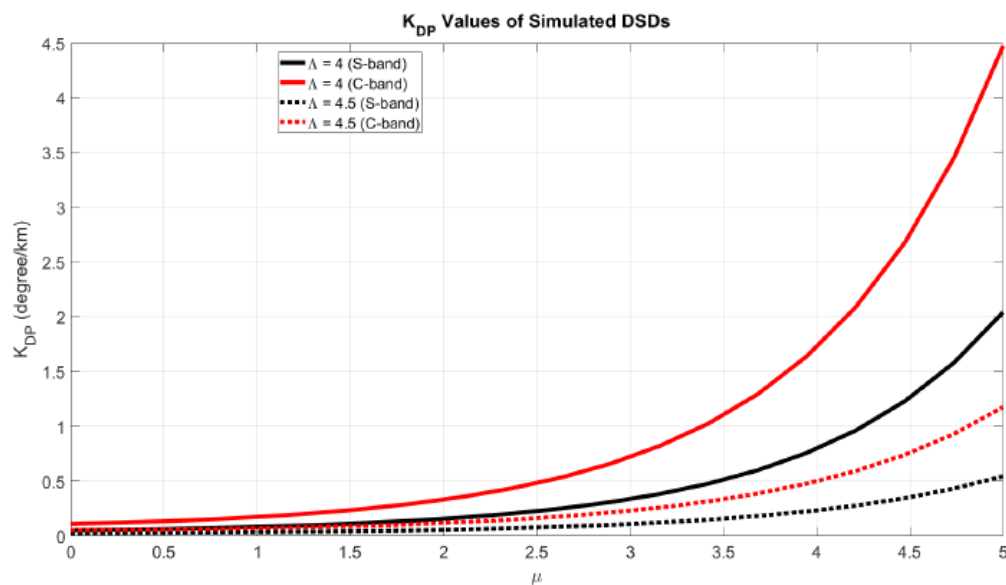
**Comment:**

3. The simulations shown in Figure 9 are misleading because they are incomplete and do not represent the observed rain systems. For example, the simulation values of  $N_0 = 8000$ ,  $\mu = 2$ , and  $\lambda = 2$  will produce reflectivity factors over 55 dBZ (I am using formulas given in Ulbrich 1983 J. Clim. Appl. Meteor.). The manuscript should either remove the simulations (and Figure 9) or add a more thorough simulation analysis that includes rain systems observed by the radars and the possible correlation between DSD parameters.

**Reply:**

We have updated the manuscript to include a dedicated simulation section which involves performing retrievals on simulated data in order to examine the performance in an ideal setting.

We believe the simulation at the parameter level is still valuable and have thus updated it to include values that are more representative of our data set. The reflectivity factor for the DSDs generated by the combined gamma parameters of the plot should not exceed 45 dBZ.



**Figure 9.**  $K_{DP}$  is simulated for DSDs of varying  $\mu$  and  $\Lambda$  values with fixed  $N_0 = 8000$ . A separation between the  $K_{DP}$  values is clear at the two radar bands.

**Manuscript Update:**

Revision:

Lines 257-301 and Figure 9

Redline Version:

Lines 281-328 and Figure 9