The manuscript uses an optimization method to establish a retrieval technique of the drop size distribution (DSD) based on the assumption of a simple 3-parameter gamma functional form and leveraging on the observations of 2 ground-based S- and C-band polarimetric weather radars. The paper is interesting as it finds application for a rather peculiar observational setup, however, I found some issues in the study that I would like to present to the authors to contribute to their work.

Response: We'd like to thank the reviewer's effort to help us further improve this work. We will address the reviewer's comments in the following section and modify the manuscript according to the reviewer's suggestions.

1) A great emphasis is given to the fact that the proposed method does not require assumptions on the relation between DSD parameters. I would like to point out that this is not very convincing because:

a - assuming a gamma distribution for the DSD is already an assumption by itself. Why not a 4-parameter gamma, a log-normal distribution or perhaps a normalized gamma?

Response: Thank you for your insightful observations regarding the assumptions underlying our method. Upon reflection, the current language may overstate the flexibility of our approach. While our method avoids imposing a μ-Λ relationship, we recognize that choosing any specific distribution model, including the gamma distribution, inherently involves assumptions. We appreciate your pointing this out and will carefully revise the manuscript to eliminate any misleading implications about the absence of assumptions.

b - the parameters of an unnormalized gamma distribution are indeed mathematically co-dependent. As an example, one can see just the measuring units of N0 (which, by the way, have not been written in line 32), those should be 1/mm**(mu). Just by noticing that the measuring units of N0 depend on mu, one can realize that the parameters cannot be independent. Some of the drawbacks of using such a size distribution are discussed Testud 2001 and Illingworth 2002 among others.

Response: Regarding the units for N_0, we realize this omission and will make the necessary corrections to include them in the manuscript.

As to your question about our choice of the gamma distribution: our decision to utilize the three-parameter gamma distribution was driven by a balance between complexity and interpretability. We found it offers a higher degree of complexity that we can feasibly manage, while also providing intuitively interpretable features. This choice was made after considering alternative distributions and aligns with our goal of maintaining a manageable level of model complexity.

We totally agree with the reviewer that there are some drawbacks of using the gamma DSD. We will present these limitations in the revised manuscript and refer to the past works such as Testud 2001 and Illingworth 2002.

2) The rationale for the selection of the used radar parameters is not clear. The term multifrequency radar is usually related to the leveraging of either differential scattering or differential absorption properties of the hydrometeors (see also the cited literature in the introduction), however here reflectivity at S-band is used (because it is considered unaffected by attenuation) and phase shift at S- and C-band. I am not sure if a signal difference in Kdp is to be expected at the S- and C- band at all apart from the expected 1/wavelength scaling for Rayleigh scatterers. Some points:

a - Due to the 1/wavelength scaling C-band Kdp is more sensitive than S-band, but at the same time, it does not contain additional information. This means that the retrieval of 3 parameters DSDs would be again ill-posed. One might also test dropping the least sensitive Kdp information (S-band) and see what happens to the results

Response: Thank you for your insightful questions regarding our selection of radar parameters. We acknowledge that our explanation in the manuscript may not have been sufficiently clear and will strive to clarify these points in the revision. We'd like to address this comment from following 2 aspects:

1.) We did not intend to imply that S-band returns are entirely unattenuated; rather, our point was that they are less affected by attenuation compared to C-band returns.

We will correct any misleading language in the manuscript to reflect this more accurately.

2.) We realized that that ideal dual-frequency technology should utilize the frequencies from different scattering region, such as the Ku-band and Ka-band radars used in the GPM. These two co-located S-band and C-band dual-polarization radars provide us unique data to investigate their usage in hydrometeorology study. Within different purposes, one goal is to understand whether S-band and C-band dual-polarization variables such as Z_{DR} and K_{DP} can reveal DSD information. We started this research from simulation, and we found that the differences in K_{DP} between these bands are still significant. In the simulation, we first calculated the forward/backward scattering amplitude using T-matrix method, and then calculated their reflectivity (Z), attenuation (A), and specific differential phase (K_{DP}) using the gamma DSD assumption with $N_0 = 8000$. The simulated results as are demonstrated in Figure 1 of this reply. In this simulation, the value of μ changes from 0 to 9.5, and three values of Λ, 2, 4, and 6 are selected. From the plots we can find that both attenuation A and specific differential phase K_{DP} show significant difference from S-band and C-band radar especially for small Λ or large μ.

In this work, we also studied the necessity of including C-band variable. We found that the retrieval results become significantly worse if only S-band variables are used.

Based on the above study through simulation and real cases, we believe C-band K_{DP} can provide extra information related to DSD.

Figure 1

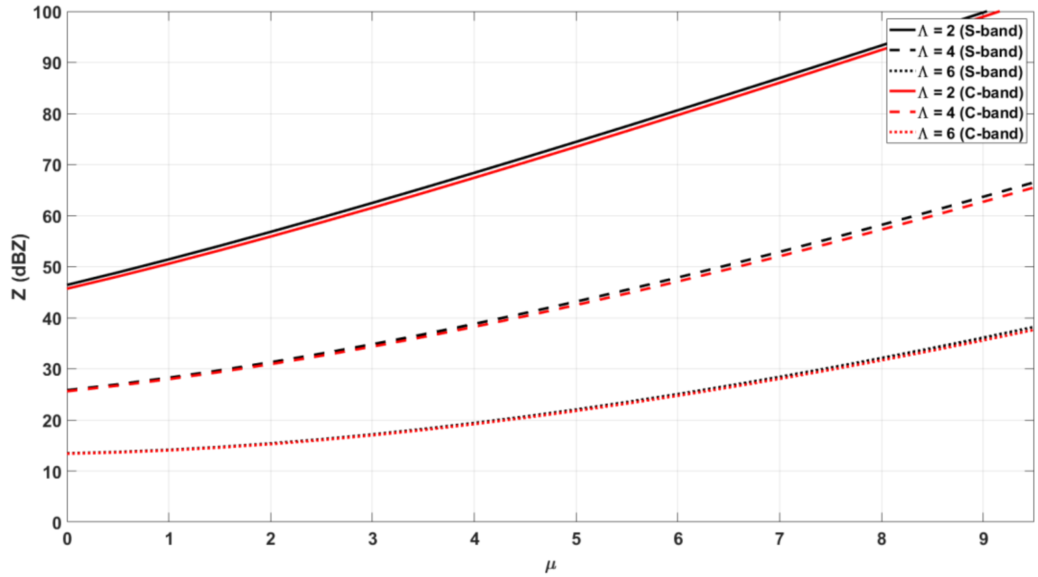

Figure 1 –Simulated Reflectivity For Various Parameters – The S and C-band returns are too similar to provide independent information for retrieval purposes.

Figure 2

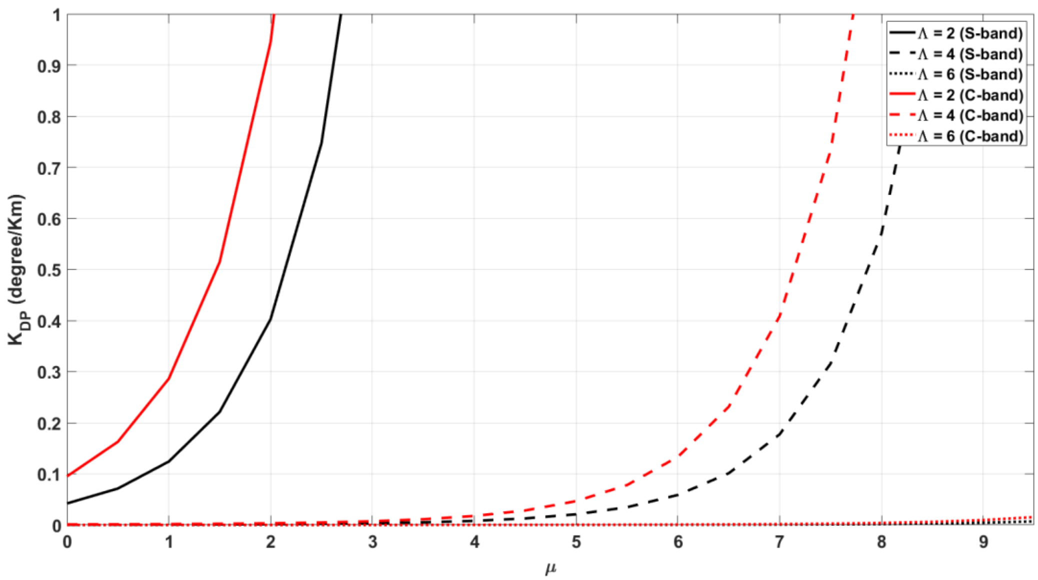

Figure 2 –KDP Variation For Various Parameters – The S and C-band returns vary in KDP values which is a key premise of the algorithm. KDP values separate at the two frequencies at low and moderate values of Λ. The high values of Λ do not provide useful separation as can be imagined by such a quickly collapsing distribution.

Figure 3

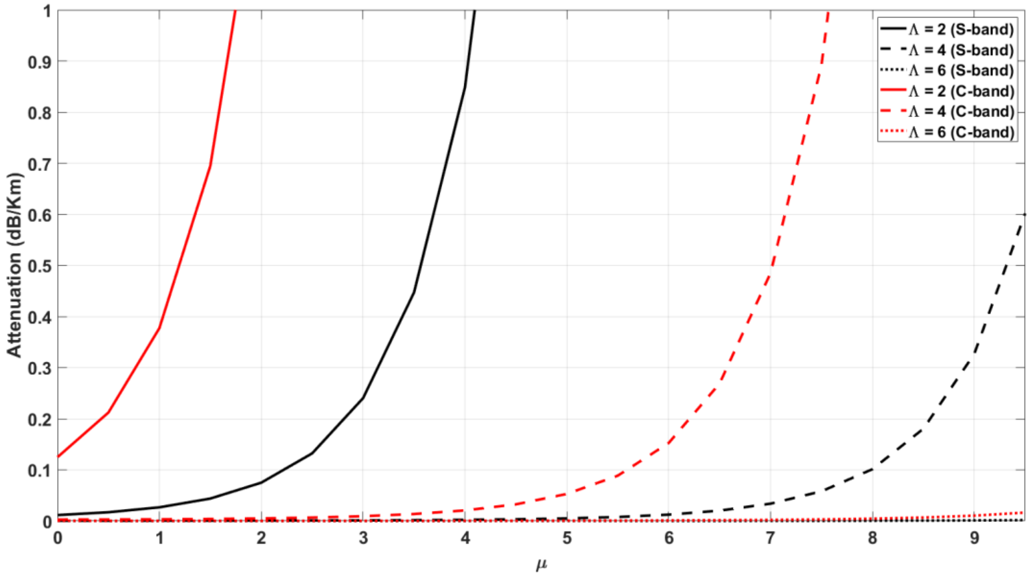

Figure 3 – Attenuation Variation For Various Parameters – As expected, the attenuation is much greater at C-band relative to S-band. Large separation in attenuation values are present for identical gamma parameters.

In the revised manuscript, we will add simulation results with discussion. We will also provide more discussion related to the impact of radar frequencies in DSD retrieval.

b - it is not clear to me why Kdp at two frequencies is used and not some other polarimetric/multifrequency quantity. ZDR is a straightforward example, LDR if available. If one assumes S-band reflectivity to be unattenuated (which is also a core assumption in this study) then one can estimate differential attenuation at the S- and C-band which would also be a nice proxy for the total liquid water content. I would have expected a better explanation of why certain radar variables have been used and not others. Perhaps, one might have conducted a theoretical sensitivity study with T-matrix to identify the best choice of observations to include in the retrieval technique given a climatology of observed DSDs... just some suggestions.

Response: This is a very good suggestion. We realized that K_{DP} may not be the optimal variables in the DSD retrieval. However, two reasons limit our selection for other variables. First, LDR is not available for both radars used in this work. Second, through our simulation, we knew that Z_{DR} show more obvious features under different frequencies. However, the S-band radar (RCWF) is operationally calibrated, ensuring reliability in its measurements. In contrast, the C-band radar (RCMD) is used primarily for research, and we had concerns about the accuracy of its values especially the calibration bias. We believe that more uncertainties will be brought into the retrieval results with a questionable variable. That is the reason that we did not use Z_{DR} in this work. For any future work, we will closely work with the radar engineers to evaluate any Z_{DR} calibration issues, and hopefully could include such a variable as it appears to be very promising.

In the revised manuscript, we will provide more discussions related to the polarimetric radar variable selection.

3) The issue of comparing ground measurements with radar volumes aloft is not discussed enough. From what I understood multiple parsivels on the ground are used to compare their simulated radar quantities with radar variables, those parsivels can be up to 70 km away. I did not understand what is the vertical separation between the radar volume and the parsivels. Is this small enough to ensure that the radar and the disdrometers are observing the same DSDs? I assume that different radar elevation angles are used for the comparison with the various disdrometers, is that taken into account in the T-Matrix calculations? if yes, isn't this causing the dataset to be inhomogeneous, what is the effect on the optimization method?

Response: We acknowledge the challenge posed by the vertical separation between radar volumes and ground-based measurements, and this is the challenge for all radar-based DSD retrieval and radar-based QPE algorithms. There is no way to really eliminate the difference from these two observations, rather we can only attempt to mitigate it.

In order to mitigate the retrieval error caused by the DSD vertical variation, we added two constrains in our algorithm. First, our analysis was confined to data from the lowest elevation tilt of 0.5 degrees to minimize vertical separation, which in essence is all we can do. However, to enrich our dataset with more cases that align with our strict time synchronization criteria, we expanded our selection to include data from the next lowest elevation tilt of 1.4 degrees. Given the fact that only the lowest two elevation angles are used, the canting angle effects on the calculated radar variables is very limited. Furthermore, we limited our retrievals to distances under 70 km to avoid complications arising from less favorable geometric conditions at greater ranges. We recognize the importance of transparency in addressing this limitation and will ensure a thorough discussion of this aspect in the revised manuscript, emphasizing our efforts to balance data quality with practical constraints.

We will provide more details related to the measurement differences in the revised manuscript.

3.) What is the effect of the 4km averaging window, isn't this causing the radar quantities to be affected by returns that are up to 4 km away? By judging from figure 4 it seems that the averaging window is not applied as a moving average but rather at discrete points every 4km, does this mean that the bin center of the radar range can be up to 2 km away from the disdrometer position?

Response: The implementation of a 4 km averaging window in our analysis was a deliberate decision. We are aware that this averaging causes radar returns from up to 2 km away to influence the local observation. However, we view this as an acceptable compromise. The nature of both reflectivity and phi_dp is such that they are inherently noisy, necessitating a certain degree of smoothing for meaningful analysis. The choice of a 4 km smoothing length also aligns with our KDP calculation methodology, which utilizes window lengths of 2.25 km and 6.25 km.

To provide further context, and in response to another reviewer's feedback, we have included a figure illustrating the level of noise typically present in the raw reflectivity observations.

Figure 4

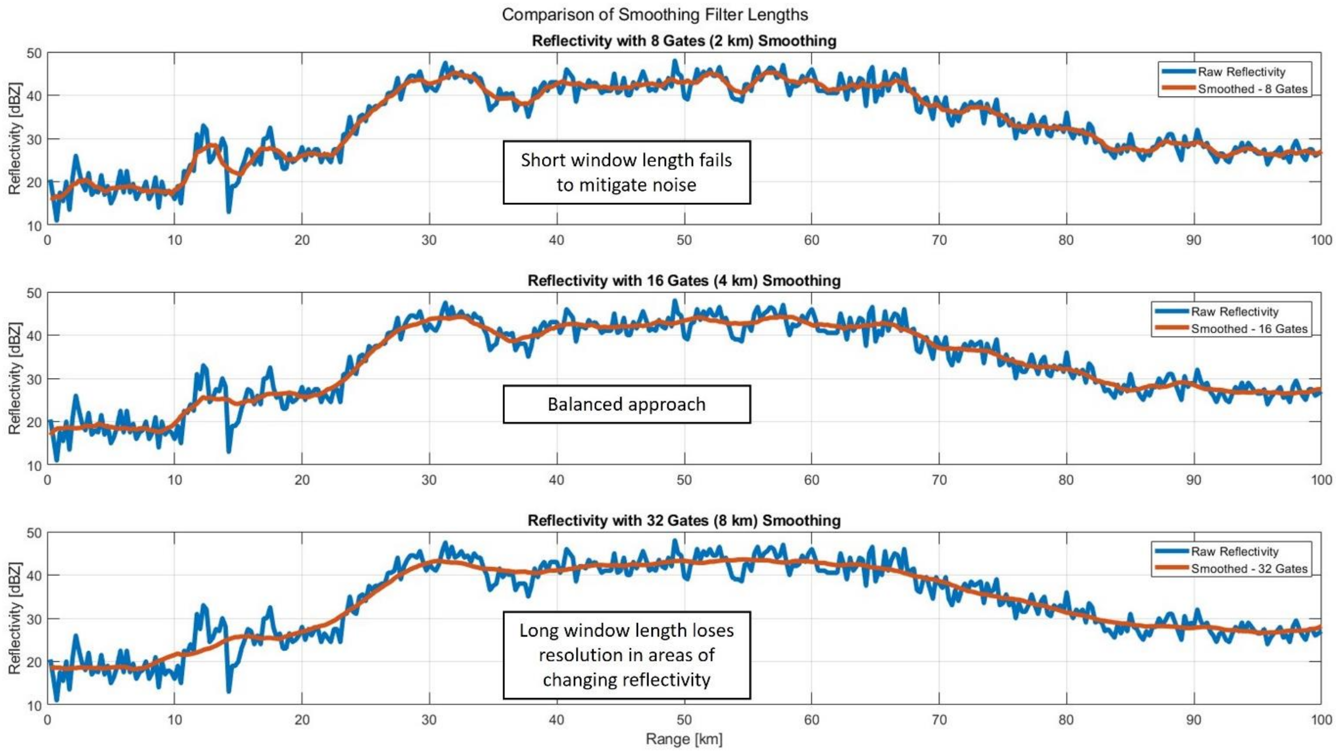

Figure 4 – Parameter Smoothing Example – An approach that balances smoothing and information preservation is clear.

Figure 4 (of the manuscript) may have led the reader to believe filtering was applied only at 4 km points. The filtering was applied as a moving average, however for the along-beam retrieval, discrete points are chosen to operate on. We will add a brief note to the revised manuscript to avoid any confusion.

4) The metric used to test the DSD retrieval is again not clear. It appears that DSDs are compared only qualitatively. If this is the case I strongly recommend plotting DSD in semilog scale, otherwise it would be extremely difficult to evaluate them. In general, it would be nice to first establish what is the goal in terms of retrieval. Alternatively one might report retrieval errors concerning certain moments of the distribution, for example: total number of droplets, total liquid water content, mean size, and distribution width. To do so, one would need a statistically significant sample of retrieved and observed DSDs, I believe that one reviewer already reported on the lack of that.

The main omission and criticism you have outlined is consistent with that of the other reviewers- the lack of a comprehensive performance assessment against an acceptable benchmark. This has prompted us to significantly expand the scope of our study.

To address this, we will update the manuscript to explicitly include following revision:

We quantitatively evaluated the performance of the proposed approach and look forward to including the results in the revised manuscript. In the quantitative evaluation, the rainfall rates were firstly estimated using three different approaches: i.) using the retrieved DSD parameters following equation $R = \frac{\pi}{6}\int_0^{D_{max}} D^3 N(D) v(D) dD$ (Bringi 20002, Zhang 2001, etc); ii) using the S-band radar reflectivity (Z) following the WSR-88D R-Z relationship, $Z = 300\ R^{1.4}$ (Ulbrich and Lee 1999) and iii) using the DSD observed by the Parsivel disdrometer following equation $R = \frac{6\,\pi\times10^4}{\Delta t}\sum_{j=1}^{M}\frac{D_j^3}{S_j^{2DVD}}$ (Raupach and Berne, 2015). The rainfall rates from i and ii were then compared with the iii, which was treated as the ground truth. In the comparison, the relative absolute error (RAE) was calculated as.

$$\epsilon = \frac{|R_d - R|}{R_d}$$

where R_d and R are the rainfall rate from iii and i/ii, respectively.
Total 167 cases were used in the analysis. The criteria of cases selection are:
1.) time difference between S- and C- band scan is within 1 minutes
2.) only the lowest two elevation angle (0.5° and 1.4°) are used.
3.) reflectivity>25
4.) 25<disdrometer range<70 km

The time series plot presented in the following figure illustrates the RAE results for two different approaches. Approach i, our proposed method, is represented by the blue line, while Approach ii, which employs the conventional R(Z) method, is indicated by the red line. The plot demonstrates that estimating rainfall rates using retrieved DSD parameters, as in our proposed approach, yields higher accuracy compared to the traditional Z-R relationship. Specifically, the median RAE for the Z-R approach stands at 0.72, which is notably reduced to 0.53 with our proposed method. This represents a significant improvement of 26.4% as observed in this study.

Figure 5

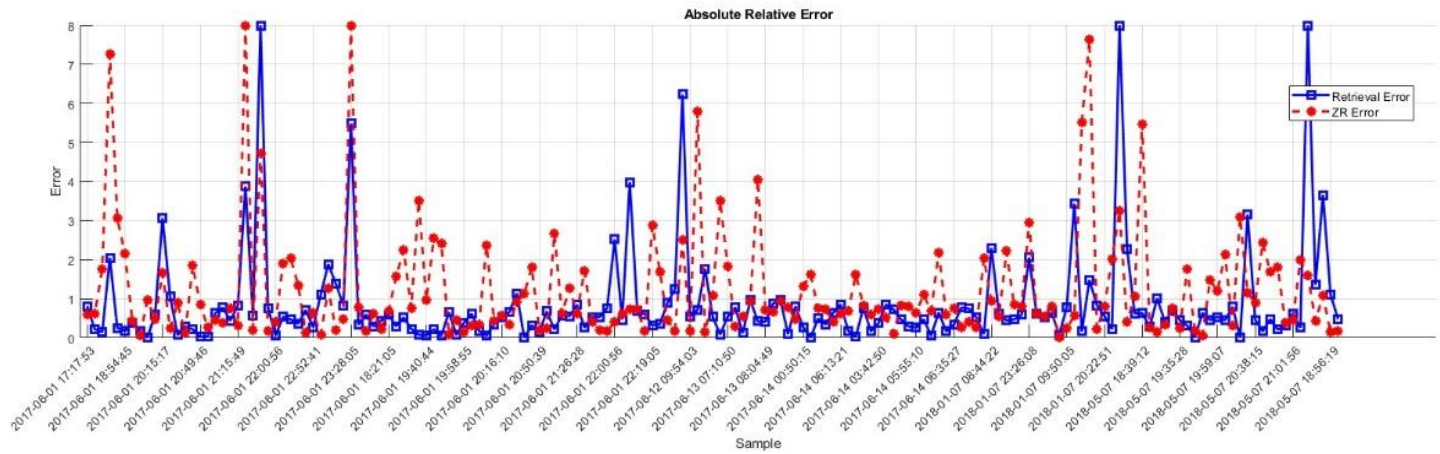

Figure 5 – Retrieval Error Evaluation– The retrieval algorithm's performance evaluated as RAE is shown in blue while the RAE associated with the Z-R derived rainrate is shown in red. Outliers are truncated at 8 in order to maintain a more useful vertical scale in the plot.

In the revision, the quantitative evaluation results and discussions will be added.

5) Another implicit assumption (that should, at least be made explicit) is the fact that the T-matrix calculations are perfect and do not carry uncertainties. This requires at least some more details such as the refractive index model used. A better approach would have been to estimate the uncertainties in Z and Kdp given by the choice of refractive index (I believe that the reference temperature of 10 degrees C might not be correct at different altitudes). Furthermore, the assumption of null canting angle is quite extreme. Also when comparing with the disdrometer one might want to take into account the limited resolution and maximum observable size. While the integrals of equations 3,4, and 5 go from 0 up to infinity, the parsivel do not (nor natural raindrops), are the integrals truncated at a certain minimum and maximum value? What about the size resolution when computing the integrals?

Response: The dielectric constant of water is calculated using the formula proposed by Cole and Cole (1941), and the refractive index is then calculated as the square root of the obtained dielectric constant.

Regarding the temperature assumption for the radar volume, we selected 10 degrees Celsius based on the average ground temperature in Taiwan during June, which is around 25 degrees Celsius, and considering the radar volume's position beneath the melting layer. We acknowledge that temperature varies with altitude, but due to the beam's coverage of multiple altitudes, we determined that a single temperature estimate would be a practical and reasonable approximation for our purposes. We did ensure through T-matrix simulations that the difference in radar parameters is negligible when compared across the temperature range of 0 to 25 degrees.

On the topic of the canting angle, we are grateful for your attention to detail. The T-matrix formulations of the parameters we used contain terms relating to the canting angle that depend on the sine of the cant. This represents a very small value, however a brief note explaining the anticipated error addition in such an assumption should be included in the revised manuscript.

Regarding the measurement capabilities of the Parsivel disdrometer, it categorizes drop sizes into 32 bins, ranging from 0.06 to 24.5 mm. For our numerical integrations, we conducted them with a resolution of 0.1 mm, spanning from 0.1 mm to 8 mm. While a narrower range could have been considered, we chose the upper limit of 8 mm as it effectively captures the spectrum of naturally occurring raindrop sizes. We agree that this aspect of the methodology merits a brief discussion in our updated manuscript to provide clarity on our decision-making process and we will update the integrals to state that integration occurs at these D_{min} and D_{max} values rather than the infinite upper limit.

Reference:

Cole, Kenneth Steward, and Robert Hugh Cole 1941: Dispersion and absorption in dielectrics I. alternating current characteristics. Journal of Chemical Physics. 9 (4) 341-351.