This manuscript introduces a method using particle swarm optimization (PSO) to estimate the three parameters of a gamma shaped raindrop size distribution (DSD) along a radial beam using two side-by-side scanning radars operating at S- and C-band. There are several concerns in the manuscript that need to be addressed before this manuscript can be considered for publication.

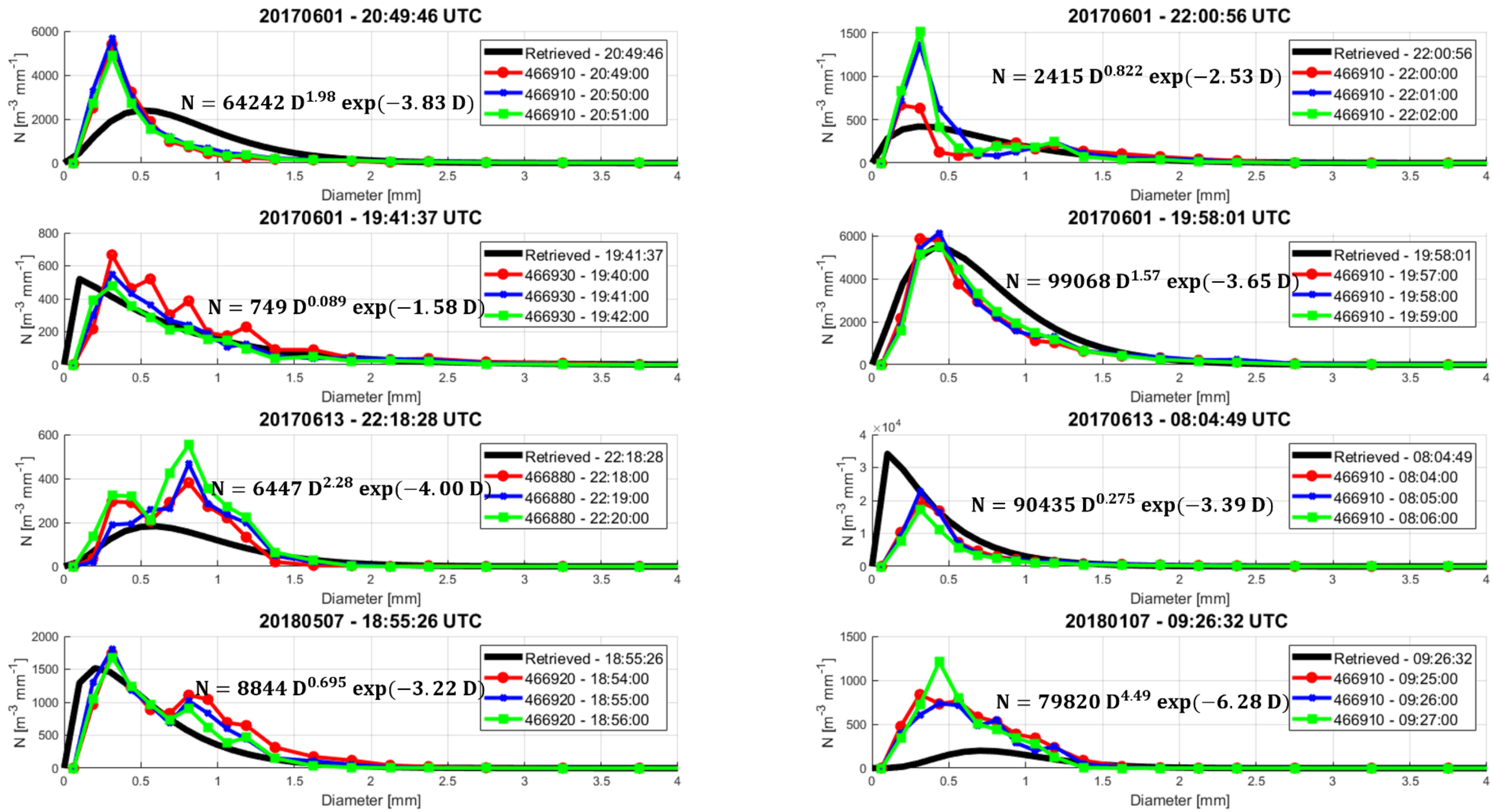
Response: Thank you for reviewing our manuscript and improving our study with your suggestions. The valid criticism you voiced concerning our original manuscript primarily hinges on the lack of a robust statistical evaluation of our retrieval results. We have reprocessed our dataset in order to collect these statistics. We would like to address your comments and questions with the following in-line responses.

1. While the abstract states that the retrieval method "is able to accurately retrieve the gamma distribution parameters without the constraints required in previous methods" (line 9), the manuscript does not present any retrieved parameters, nor does it estimate the accuracy of any retrieved parameters. Thus, it is not possible to evaluate the accuracy of the proposed method because the necessary results are not presented in this manuscript.

Response: Thank you for highlighting the omission in our manuscript regarding the presentation of retrieved parameters and their accuracy.

To address this, we will update the manuscript to explicitly include following revisions:

a.) providing the retrieved gamma distribution parameter values for each example in the plot. The updated figures are shown as below. These figures replace the subjective examples in our original manuscript. In these new figures, the disdrometer observations from three continuous minutes are presented using red, blue and green lines, respectively. The retrieved DSD using the radar data of the closet moment is delineated with a black curve.



20170601 - 20:49:46 UTC
Retrieved - 20:49:46
466910 - 20:49:00
466910 - 20:50:00
466910 - 20:51:00
N = 64242 D^1.98 exp(−3.83 D)
20170601 - 22:00:56 UTC
Retrieved - 22:00:56
466910 - 22:00:00
466910 - 22:01:00
466910 - 22:02:00
N = 2415 D^0.822 exp(−2.53 D)
20170601 - 19:41:37 UTC
Retrieved - 19:41:37
466930 - 19:40:00
466930 - 19:41:00
466930 - 19:42:00
N = 749 D^0.089 exp(−1.58 D)
20170601 - 19:58:01 UTC
Retrieved - 19:58:01
466910 - 19:57:00
466910 - 19:58:00
466910 - 19:59:00
N = 99068 D^1.57 exp(−3.65 D)
20170613 - 22:18:28 UTC
Retrieved - 22:18:28
466880 - 22:18:00
466880 - 22:19:00
466880 - 22:20:00
N = 6447 D^2.28 exp(−4.00 D)
20170613 - 08:04:49 UTC
Retrieved - 08:04:49
466910 - 08:04:00
466910 - 08:05:00
466910 - 08:06:00
N = 90435 D^0.275 exp(−3.39 D)
20180507 - 18:55:26 UTC
Retrieved - 18:55:26
466920 - 18:54:00
466920 - 18:55:00
466920 - 18:56:00
N = 8844 D^0.695 exp(−3.22 D)
20180107 - 09:26:32 UTC
Retrieved - 09:26:32
466910 - 09:25:00
466910 - 09:26:00
466910 - 09:27:00
N = 79820 D^4.49 exp(−6.28 D)
Diameter [mm]
N [m^-3 mm^-1]

b.) Following the reviewer's suggestion, we quantitatively evaluated the performance of the proposed approach and look forward to including the results in the revised manuscript. In the quantitative evaluation, the rainfall rates were first estimated using three different approaches:

i.) using the retrieved DSD parameters following equation $R = \frac{\pi}{6}\int_0^{D_{max}} D^3 N(D) v(D) dD$ (Bringi 20002, Zhang 2001, etc);

ii) using the S-band radar reflectivity (*Z*) following the WSR-88D *R*-*Z* relationship, $Z = 300\ R^{1.4}$ (Ulbrich and Lee 1999);

iii) using the DSD observed by the Parsivel disdrometer following equation $R = \frac{6\ \pi \times 10^4}{\Delta t}\sum_{j=1}^{M} \frac{D_j^3}{S_j^{2DVD}}$ (Raupach and Berne, 2015).

The rainfall rates from i and ii were then compared with the iii, which was treated as the ground truth. In the comparison, the relative absolute error (RAE) was calculated as.

$$\epsilon = \frac{|R_d - R|}{R_d}$$

where R_d are the rainfall rate estimated from the disdrometer as presented in approach iii, and *R* are the evaluating rainfall rate from approach i or ii, respectively.

Total 167 cases were used in the analysis. The criteria of cases selection are:

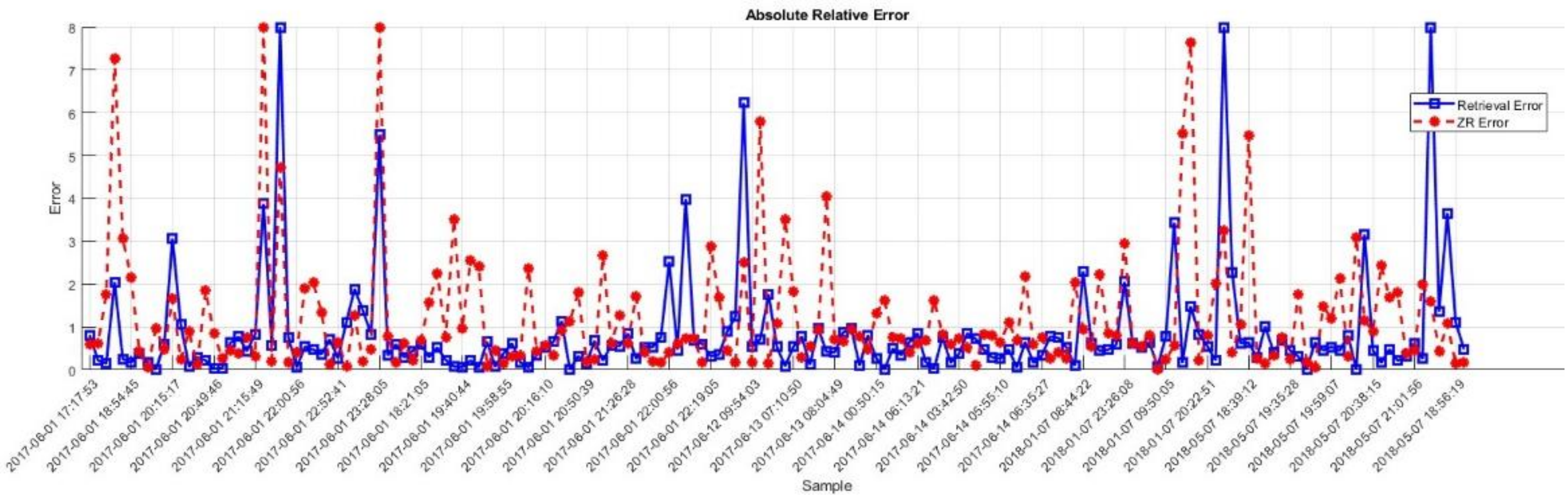
1.) time difference between S- and C- band scan is within 1 minutes

2.) only the lowest two elevation angle (0.5^{o} and 1.4^{o}) are used.

3.) reflectivity > 25 dBZ

4.) 25 km < disdrometer range < 70 km

The time series plot presented below illustrates the RAE results for two different approaches. Approach i, our proposed method, is represented by the blue line, while Approach ii, which employs the conventional *R*(*Z*) method, is indicated by the red line. The plot demonstrates that estimating rainfall rates using retrieved DSD parameters, as in our proposed approach, yields higher accuracy compared to the traditional *Z*-*R* relationship. Specifically, the median RAE for the *Z*-*R* approach stands at 0.72, which is notably reduced to 0.53 with our proposed method. This represents a significant improvement of 26.4% as observed in this study.



In the revision, the quantitative evaluation results and discussions will be added.

2. As presented in this manuscript, the PSO method is essentially a random walk through the three DSD parameter space, and not an "optimization" method. A random walk (line 218) through the parameter space is a valid parameter estimation procedure if the neighborhood of solutions around the global solution are used to estimate an uncertainty of the retrieved parameters.

Response: We appreciate your perspective on our use of the PSO method. While we understand your viewpoint that categorizes PSO as a random walk, we would like to clarify that PSO inherently possesses mechanisms that actively drive towards a global minimum, distinguishing it from a purely random walk approach. The intent behind selecting PSO was not to showcase it as a novel optimization technique but rather to utilize a method capable of overcoming the limitations commonly encountered in gradient-based approaches, namely convergence to local minima.

In earlier iterations of our work, we employed the traditional non-linear optimization method, such as Gauss-Newton (GN) algorithm and Levenberg-Marquardt (LM) algorithm. However, this type of method tend to converge at local minimum instead of global minimum. As an alternative, we tried unsupervised learning approaches such as genetic algorithm (GA) and PSO. They both yielded results superior to those achieved with a GN or LM algorithm. The choice to transition to PSO was primarily driven by its efficiency relative to our initial genetic algorithm code. It's important to note that our primary focus was on the effectiveness of the solution method rather than the optimization routine itself. We believe that several other methods could also be successfully employed for the solver block of our algorithm.

3. It appears that the study is based on four radial samples made over two different disdrometers. The manuscript text states that nine (line 245) or ten (line 294) days of data were "used in the performance validation" (line 245), but only four reconstructed raindrop distributions are shown in Figure 9. Since no more retrieved parameters are shown in the manuscript, it must be assumed that those four radials were the only radials that were processed.

Response: We agree with regard to the noted deficiencies in our original manuscript, as noted here as well as the first comment. In order to address the absence of a statistical presentation of the data, we have reevaluated our dataset and will include the results shown in the response to comment 1 which includes the retrievals of 167 cases. More details please refer to the response to comment 1.

4. It is major limitation of this work that retrieved DSD parameters are not presented in the manuscript. The four reconstructed raindrop distributions shown in Figure 9 do not show the retrieved slope parameter (lambda) nor the shape parameter (mu). Since three of the four constructed distributions (Fig. 8 a, b, and d) do not have the number concentration approaching zero at zero diameter, this suggests that the retrieved mu value is less than zero. It is unusual for the shape parameter to be negative in scanning radar retrievals due to the large scanning radar

sample volume. Which raises concern about the validity of the retrievals. The manuscript must show retrieved parameter statistics to evaluate the performance of the retrieval method.

Response: We had originally restricted our parameters along the bounds shown in Zhang's (2001) work, which allowed μ to be as low as negative 2. We have updated the constraint on μ to be no less than zero. We believe the trade off in possibly losing a few distributions with negative μ shown in Zhang is acceptable with the gained benefit of guarding against non-sensical distributions. We thank you for that note, and all cases have been reprocessed with the updated bounds. Additionally, we have included values of μ, Λ, and N_0 in each subjective plot. Please see the updated subjective examples shown in response to your first comment.

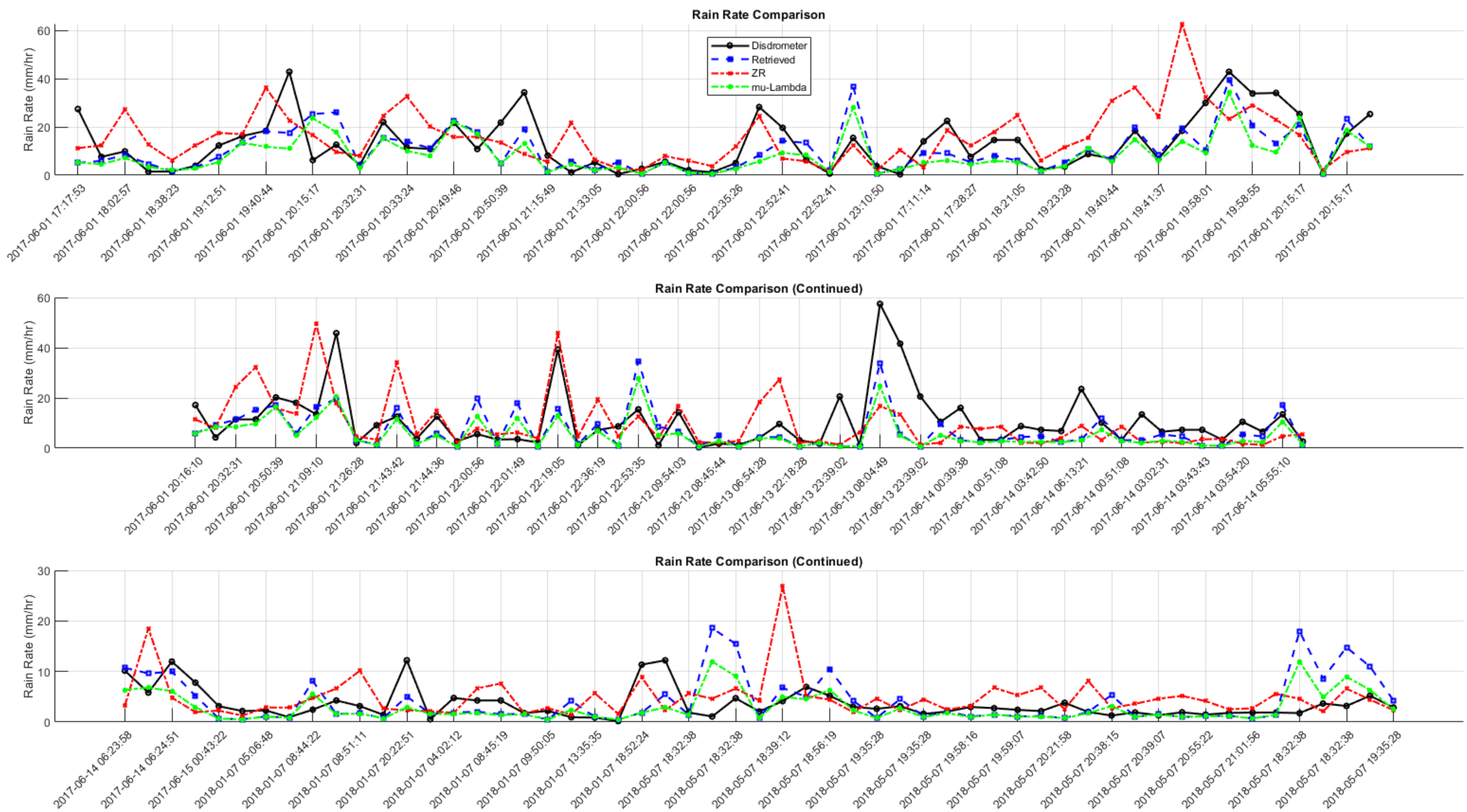
5. How much improvement in rainfall estimation does the proposed method provide compared to using one of the mu-lambda relationships proposed by Zhang et al. (2001), Brandes et al. (2002), or Cao et al. (2010)? If the new method does not produce comparable or better rainfall estimates than the mu-lambda constraints, then will this proposed method be an improvement to QPE? The mu-lambda constraints should be the baseline that the new method should aim to beat. In summary, without the manuscript showing results of the retrieved DSD parameters, it is not possible to evaluate the proposed retrieval method. The proposed method may produce results that are superior to results from imposing a mu-lambda constraint. But, as written, the manuscript does not show the evidence needed to verify the statements made in the manuscript.

Response: We acknowledge the oversight of not benchmarking our initial manuscript. In addition to the comparison with the Z-R derived rain rate, we have also conducted an analysis of our algorithm incorporating the μ-Λ relationship outlined in Zhang et al. (2001):

$$\mu = -0.016\,\Lambda^2 + 1.213\Lambda - 1.957$$

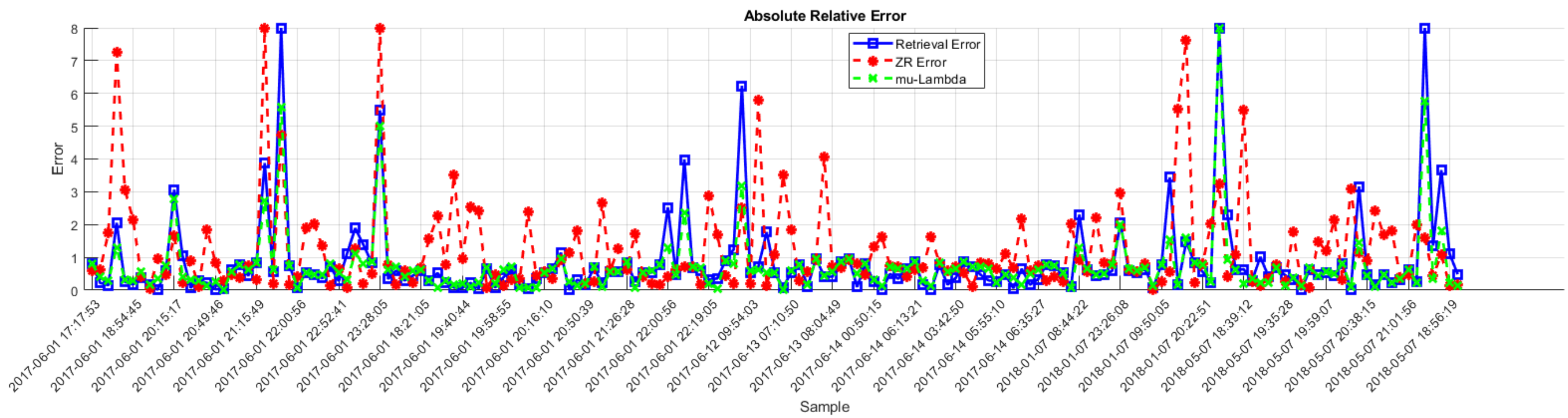
Different from the proposed method that retrieves three DSD parameters independently, this approach retrieves the DSD with above μ-Λ relationship. The PSO method is also used in this approach.

The following figure shows the rain rate derived from the proposed method's retrieval (blue), the disdrometer truth (black), *Z-R* relation (red), and the PSO retrieval with a μ-Λ constraint (green).



Rain Rate Comparison
Rain Rate (mm/hr)
Disdrometer
Retrieved
ZR
mu-Lambda
2017-06-01 17:17:53
2017-06-01 18:02:57
2017-06-01 18:38:23
2017-06-01 19:12:51
2017-06-01 19:40:44
2017-06-01 20:15:17
2017-06-01 20:32:31
2017-06-01 20:33:24
2017-06-01 20:49:46
2017-06-01 20:50:39
2017-06-01 21:15:49
2017-06-01 21:33:05
2017-06-01 22:00:56
2017-06-01 22:00:56
2017-06-01 22:35:26
2017-06-01 22:52:41
2017-06-01 22:52:41
2017-06-01 23:10:50
2017-06-01 17:11:14
2017-06-01 17:28:27
2017-06-01 18:21:05
2017-06-01 19:23:28
2017-06-01 19:40:44
2017-06-01 19:41:37
2017-06-01 19:58:01
2017-06-01 19:58:55
2017-06-01 20:15:17
2017-06-01 20:15:17
Rain Rate Comparison (Continued)
Rain Rate (mm/hr)
2017-06-01 20:16:10
2017-06-01 20:32:31
2017-06-01 20:50:39
2017-06-01 21:09:10
2017-06-01 21:26:28
2017-06-01 21:43:42
2017-06-01 21:44:36
2017-06-01 22:00:56
2017-06-01 22:01:49
2017-06-01 22:19:05
2017-06-01 22:36:19
2017-06-01 22:53:35
2017-06-12 09:54:03
2017-06-12 08:45:44
2017-06-13 06:54:28
2017-06-13 22:18:28
2017-06-13 23:39:02
2017-06-13 08:04:49
2017-06-13 23:39:02
2017-06-14 00:39:38
2017-06-14 00:51:08
2017-06-14 03:42:50
2017-06-14 06:13:21
2017-06-14 00:51:08
2017-06-14 03:02:31
2017-06-14 03:43:43
2017-06-14 03:54:20
2017-06-14 05:55:10
Rain Rate Comparison (Continued)
Rain Rate (mm/hr)
2017-06-14 06:23:58
2017-06-14 06:24:51
2017-06-15 00:43:22
2018-01-07 05:06:48
2018-01-07 08:44:22
2018-01-07 08:51:11
2018-01-07 20:22:51
2018-01-07 04:02:12
2018-01-07 08:45:19
2018-01-07 09:50:05
2018-01-07 13:35:35
2018-01-07 18:52:24
2018-05-07 18:32:38
2018-05-07 18:32:38
2018-05-07 18:39:12
2018-05-07 18:56:19
2018-05-07 19:35:28
2018-05-07 19:35:28
2018-05-07 19:58:16
2018-05-07 19:59:07
2018-05-07 20:21:58
2018-05-07 20:38:15
2018-05-07 20:39:07
2018-05-07 20:55:22
2018-05-07 21:01:56
2018-05-07 18:32:38
2018-05-07 18:32:38
2018-05-07 19:35:28

The accuracy of the μ-Λ constrained retrieval method is found to be comparable to that of our unconstrained retrieval technique. Evaluating performance with the RAE metric reveals that the constrained approach has a median error of 0.55, closely aligning with the unconstrained method's median error of 0.53. While a mean RAE evaluation might suggest enhanced performance with the mu-Lambda relationship, we believe using median values for assessment provides a truer reflection of performance, particularly due to the influence of outliers. To illustrate this, the attached figure displays error plots for all methods, with outliers truncated at a value of 8.


Absolute Relative Error
Retrieval Error
ZR Error
mu-Lambda
Error
Sample
0
1
2
3
4
5
6
7
8
2017-06-01 17:17:53
2017-06-01 18:54:45
2017-06-01 20:15:17
2017-06-01 20:49:46
2017-06-01 21:15:49
2017-06-01 22:00:56
2017-06-01 22:52:41
2017-06-01 23:28:05
2017-06-01 18:21:05
2017-06-01 19:40:44
2017-06-01 19:58:55
2017-06-01 20:16:10
2017-06-01 20:50:39
2017-06-01 21:26:28
2017-06-01 22:00:56
2017-06-01 22:19:05
2017-06-12 09:54:03
2017-06-13 07:10:50
2017-06-13 08:04:49
2017-06-14 00:50:15
2017-06-14 06:13:21
2017-06-14 03:42:50
2017-06-14 05:55:10
2017-06-14 06:35:27
2018-01-07 08:44:22
2018-01-07 23:26:08
2018-01-07 09:50:05
2018-01-07 20:22:51
2018-05-07 18:39:12
2018-05-07 19:35:28
2018-05-07 19:59:07
2018-05-07 20:38:15
2018-05-07 21:01:56
2018-05-07 18:56:19

It should be noted that the approach proposed by Zhang et al. (2001) incorporates Z_{DR} (Differential Reflectivity) in retrievals, a factor which we acknowledge could potentially enhance accuracy. However, we opted not to use Z_{DR} due to its requirement for regular calibration, which introduces an element of uncertainty that we preferred to avoid. We acknowledge a μ-Λ constraint might be advantageous in various contexts, however, we found it to be non-essential for our specific application. The advantage of bypassing the need to establish this relationship through extensive prior data collection is clear. We will further investigate the impacts of the μ-Λ constraint in the retrieval results in the future work.