Reply to referee comment #1

We thank Referee #1 for reviewing the manuscript and the valuable comments and suggestions which we address below. The responses to the referee comments are given in blue italic letters.

General comments:

The paper is well written, clear, and of interest. I recommend publication with a few comments.

Specific comments:

- Eq. (1): When measuring the total intensity I with a polarization camera, I think it is preferable to use $I = (I_0 + I_{45} + I_{90} + I_{135})/2$ rather than $I = I_0 + I_{90}$. Ideally of course, it should be that $I_0 + I_{90} = I_{45} + I_{135}$. Nevertheless, I think it is preferable to take the information of all four pixels into account.

  Thank you for your comment. We do indeed compute the I-component of the Stokes vector from our measurements with $I = (I_0 + I_{45} + I_{90} + I_{135})/2$ using all four measured intensities. So, it was misleading to give the general definition of the Stokes vector in Eq. (1). We changed the equation to $I = (I_0 + I_{45} + I_{90} + I_{135})/2$ to be consistent.

- Section 4.5.2 Laboratory polarization calibration: I think this section could be improved. I am not sure I fully understand the calibration procedure. Some additional step-by-step explanations with equations and/or a figure explaining the three reference systems and how they relate would help me. To be a bit more specific:
  - In Step 1 (Line 326-331): Did you compute Transfer matrix Eq. (8) by means of Eq. (9) with the camera being in the camera reference frame? Is the result a transfer matrix from laboratory frame (linear polarizer) to camera frame that contains a rotation matrix that still needs to be determined? Or are camera and linear polarizer in the same laboratory reference system?

    We added more details about the different reference systems and also tried to describe the first step in more detail to make that clearer. In addition, we added a reference with sketches which visualize the different reference systems. We computed the transfer matrices in this first step in the laboratory reference frame by solving equation 9 in a least-squares sense similarly to Rodriguez et al. (2022). With the resulting transfer matrix Stokes vectors in the laboratory reference frame can be computed from measured intensities. The procedure of the laboratory polarization calibration in section 4.5.2 is independent of the theoretical polarization calibration model (equation 8) in section 4.5.1. Equation 8 gives Stokes vectors in the camera reference system. The transfer matrices obtained in the first step of the laboratory polarization calibration give results in the laboratory reference system and are rotated to the camera reference system in the second step of the laboratory polarization calibration.

  - Line 332-345: I assume the problem that is being solved here is finding the rotation induced by the window. So if the Stokes vector is rotated beforehand, does Eq. (9) become something like this,

$$I_n - d_n = A \cdot R \cdot S_n,$$

with R being the rotation matrix we are looking for? If so, why could you not simply fit a misalignment factor dphi similar to Eq. (13) in Lane et al. (2022) (This misalignment factor is also merely a rotation of angle dphi). I understand the sentence spanned from line 333-335, but couldn't you still optimize for the rotation by rotating the linear polarizer? How are the EURECA measurements polarized with respect to the camera reference system (or the scattering plane)? I think it is worth giving more details.

In contrast to Lane et al. (2022) the polarizer in our setup was not mounted on a manual but on a motorized rotation stage. This means that the relative orientation of the polarizer for the different measurements with different rotation angles are very accurate and we did not have to account for misalignments due to manual arrangement. But, what we needed to determine was the absolute orientation of the 0 degree direction of the linear polarizer in camera coordinates. In principle, it would also have been possible to rotate the incoming Stokes vector first into the camera reference system and then determine the transfer matrices as you propose with your equation above. However, we did not have a "ground truth" for the 0 degree direction which we could have used to directly optimize for such a misalignment factor due to window in front of the cameras. Because of that, we used the known property, that U=0 in the scattering plane for single scattering with our measurements from the EUREC4A campaign as described in the paper. The measurements are at first raw data, from which we computed Stokes vectors in the laboratory reference frame with the transfer matrices of step 1. Then we applied two rotation matrices to the Stokes vectors, one for the transformation from laboratory to camera reference system, which had to be optimized, and the second one for transforming from the camera reference system to the scattering plane, which was known from the geometrical calibration. By minimizing U in the scattering plane we could then find the rotation from laboratory to camera reference frame.
We added also more details to this section to make our methods more comprehensible.

The entire part about the laboratory polarization calibration reads now:

"The Stokes vector as well as the transfer matrix are always defined relative to a reference plane. In connection with the polarization calibration, we distinguish three different reference systems. The laboratory reference system is defined by the plane containing the 0◦-axis of the linear polarizer between the large integrating sphere and the instrument and the normal of this polarizer. Moreover, the reference plane for the camera reference system for each camera is given by the x-z-plane of the camera coordinate system with the x-axis parallel to the 0◦-direction of the polarizers on the sensor and the z-axis normal to the focal plane array of the camera. Finally, the Stokes vectors can be rotated from the camera reference system into the scattering plane. The scattering plane is the plane containing the vector of the incoming solar radiation and the viewing direction of ach pixel. Sketches visualizing the different reference systems can for example be found in Eshelman et al. (2019). The transformation from the camera coordinate system to the scattering plane is known from the geometric calibration and varies between different observation geometries with different vectors of the incoming solar radiation. Thus, with the laboratory polarization calibration, we aim for computing the transfer matrices in the camera reference system.

For that, we defined the polarizer angles φ for the incoming Stokes vectors S_n relative to the 0◦-axis of the linear polarizer between the large integrating sphere and the instrument and computed the transfer matrices first in the laboratory reference frame with the normalized super-pixel method described above. Therefore, we combined the laboratory measurements for different tilt angles into one laboratory reference system and solved equation 9 in a least-squares sense similarly to Rodriguez et al. (2022) for the transfer matrices using the measured intensities and dark signal as well as the incoming Stokes vectors computed from the polarizer angles φ. We only included illuminated pixels with viewing directions within ±20◦ perpendicular to the polarizer where the polarizer can be considered perfect. In addition, we excluded pixels with dirt or reflections on the window. With the resulting transfer matrices, Stokes vectors in the laboratory reference frame can be computed from measured intensities.
In a second step, we transformed the obtained transfer matrices from the laboratory reference system into the camera reference system. The direct determination of the rotation from the laboratory to the camera reference frame through the identification of the polarizer orientation visible in the measurements was not possible due to the angle dependent shift introduced by the window, which is relevant at small distances. However, for single scattering, the U component of the Stokes vector is zero in the scattering plane due to symmetries. We used this fact to find the rotation from the laboratory to the camera reference frame using measurements taken during the EUREC4A campaign (Stevens et al., 2021) by minimizing U along the scattering plane. Contributions from multiple scattering can in principle cause deviations of U from zero. To minimize the influence of multiple scattering, we chose measurements from EUREC4A without clouds and minimum amount of aerosol. We applied the computed transfer matrices to measurement data from the EUREC4A campaign to compute Stokes vectors in the laboratory reference frame. Then, we rotated the obtained Stokes vectors with a single rotation matrix first from the laboratory into the camera reference system and next for every pixel from the camera reference system into the scattering plane. Since the transformation from the camera reference system to the scattering plane is known we could optimize for the rotation from the laboratory to the camera reference system by minimizing the absolute value of U along the scattering plane. With that, we obtained transfer matrices in the camera reference system for every measured pixel by applying this rotation matrix to the transfer matrices in the laboratory reference system obtained during the first step."

- The statements in Line 279 "a single matrix …. To all pixels" and Line 294 "the camera lens has only little influence" citing Lane et al. are slight oversimplifications. Lane et al. used a 105m lens set to f/22 (fairly straight rays) to show that the super-pixels on the sensor are generally consistent. When they compare the lenses, they merely focus on the central pixels. However, and presumably particularly important for wide-angle lenses, lenses can show an effect called polarization aberration of lenses. This is nicely explained in the reference [1], section 1.7.2, page 22 ff. (also note the effect of high numerical aperture wavefronts described in section 1.7.3). The effect is particularly high at the edges (see Fig. 1.38 in [1]), which might explain your larger differences in the corners (mentioned in line 482). My suggestion would be: it is fair to assume one transfer matrix for all pixels, as the superpixels should generally be consistent across the entire sensor. However, this will probably not fully correct the entire lens (as you already concluded yourselves in line 486). I do not see a need to change any data / results. But it is worth to correct the statements and to mention the potential effect of polarization aberration of lenses.

Thank you very much for pointing that out. We reworded both lines and added more details to make the differences between the setup of specMACS and Lane et al. (2022) clearer and avoid oversimplifications. Lines 279 and 294 read now:

"Lane et al. (2022) calibrated the monochromatic version of the polarization resolving cameras from the same manufacturer. They focused on the central pixels of the sensor and found that the transfer matrices are consistent across this sensor region and a single matrix can be applied to all pixels. In addition, the deviation between the measured matrices and the ideal matrix was small for the central pixel region with small incident angles which they considered."

And

"According to Lane et al. (2022), the choice of the camera lens has only little influence on the transfer matrices for the central pixel region of the camera where the incident angles of the rays are small. Thus, we assume that our theoretical model of the transfer matrices is a good approximation. However, lenses can introduce polarization aberrations especially for larger incident angles towards the corner regions (Chipman et al., 2018). This effect is not included in the theoretical polarization calibration model. Because of that, we validated the theoretical model with a laboratory polarization calibration."

Technical corrections:

- Eq. (1): It should be $I_{right} - I_{left}$ and not $I_{left} - I_{right}$, see [1], page 64, Eq. (3.1)

  Thank you very much for noting that. We corrected the equation.

- Line 137, 142: altitude instead of attitude

  We do indeed mean the attitude of the aircraft here. The BAHAMAS data provides aircraft position (latitude, longitude, height) and attitude (roll, pitch, and yaw angles). We added more details to clarify that:
  "Precise information about aircraft position (latitude, longitude, and altitude of the aircraft) and attitude (roll, pitch, and yaw angles) is available from the Basic HALO Measurement and Data System (BAHAMAS)."

[1]: Chipman, Russell, Wai Sze Tiffany Lam, and Garam Young. Polarized light and optical systems. CRC press, 2018