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A radiative-convective model computing precipitations with the maximum entropy production hypothesis.

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Abstract.

What do we need to compute the pertinent variables for climate? Some highly detailed models exist, called Earth System Models, where all the relevant components of climate are present: the atmosphere, the ocean, the vegetation and the ice sheets. As many as possible phenomena are represented, and for accuracy, there are two ways of doing it. The first is to solve dynamics equations with a grid size as small as possible. This method induces high economic and computational costs. The second method is to compute the sub-grid processes with smart parameterizations adapted to the grid size. This method induces a massive amount of parameterizations. Some simpler models exist, e.g. 1D radiative-convective model, but like the other models, they use parameterizations. For example, to compute the material energy fluxes provoked by temperature gradients, one may use a Fourier law, saying that energy fluxes are locally proportional to temperature gradients. While this law has a well-defined parameter value at the microscopic scale, the parameter needs to be better defined for the climate scale. More than that, the button for this parameter can be turned to make the model closer to observations. This process is called tuning and exists in all accurate climate models. This article uses a new method to compute temperatures and energy fluxes, where tuning is impossible. We hope this method is more physical and universal as we have less range to tell the model to give the desired result beforehand. Therefore, it could be used for climates where few are known, such as paleoclimate or climates of other planets. The method used is based on a thermodynamic hypothesis, the maximum entropy production. For simplicity, we restrict the model to be 1D vertical for a tropical atmosphere. With conservation laws, the problem is an optimization problem under constraints. It is solved with an algorithm making a gradient descent from an initial condition. The result is the maximum of the objective function, the entropy production, where the constraints are satisfied. As constraints, energy conservation and mass conservation already give a suitable temperature profile. This article adds a new constraint on the water cycle. The water vapour is allowed to disappear, leading to precipitations, but it is not allowed to be created. The result for this new set of equations (or constraints) shows precipitations nil everywhere and positive at the top of the troposphere. It is like "cumulonimbus" precipitations. It seems coherent to what happens in the tropics, where the Intertropical Convergence Zone

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Preprint. Discussion started: 23 October 2023

leads to deep convection. Moreover, the computed order of magnitude is correct. Fundamentally, although the water cycle is often described as a complex and multidisciplinary problem, the correct order of magnitude of precipitations can be computed with almost only the knowledge of radiative transfer.

1 Introduction

Historically, climate models have evolved from elementary conceptual models to energy balance models (EBMs), then to radiative-convective models (RCMs), and after that General Circulation Models (GCMs), and finally, the state-of-the-art Earth System Models (ESMs) (Paul N Edwards, 2011); with a constant increase in complexity and calculation rate.

Researchers generally consider GCMs and ESMs "the best" models because they account for a large amount of phenomena. Indeed, if some specific part of the model does not fit well enough with observations, it is always possible to spend time to add more complexity and make it fit better. We hope that if we put enough work into it, GCMs or ESMs end up being very close to observations. Furthermore, these models cover the entire earth, accurately describing the position of oceans and continents, the orography, the cryosphere, and the vegetation... with a resolution now below a hundred kilometres. Thus, they might be beneficial to answer specific questions, like, taking an example out of many, how crop yield would evolve in a particular area within this century. Today's ESMs predict very robust temperature changes for increasing levels of CO_2 (see AR6 IPCC Fig. 4.19) for most regions of the globe. On the contrary, they do not predict robust changes in precipitations (AR6 IPCC Fig. 4.24). Individual models show opposite signs of precipitation changes in some regions (fig 4.42d of AR6 IPCC). Even when looking for global mean changes when increasing CO_2, temperatures show much less uncertainty than precipitations (AR6 IPCC fig4.2a-b). We think the reason for such uncertainty lies in the difficulty of parameterizing the equations for water fluxes.

Indeed, the atmospheric part of GCMs or ESMs is based on the Navier-Stokes equations, whose length scales range from $L = 10^3\ \mathrm{km}$ to less than $\eta = 10\ \mathrm{mm}$, is the viscous or the Kolmogorov scale of the atmosphere. The number of modes required to model every scale is $N = (L/\eta)^3 \approx 10^{24}$, by far unreachable by today's computer (we would need 10^8 times the full storage capacity of today's supercomputer to store one time-step). To deal with this problem, climate computers integrate only large scales with the Navier-Stokes equation, and sub-grid processes are parameterized, differently for every model, leading to different results. Although, the physics of every model is the same. How much the parameterizations affect our ability to truly predict climate is a deep and open question (Hourdin et al., 2017; Dommenget and Rezny, 2018). Today's ESMs have hundreds of adjustable parameters. In this study, we build a radiative-convective model with zero adjustable parameters.

To do so, the unknown variables are determined with a variational problem, the maximum of entropy production (MEP). The idea is to express the variational problem (entropy production and constraints) as a function of the unknown variables, like energy fluxes or water vapour fluxes, so they will adjust themselves to maximize entropy production. Therefore, we do not parameterize them. One could argue that MEP is just another way to parameterize, but it is very different from the usual data assimilation techniques (see, for example, Lopez (2007) for precipitations and clouds) because it does not use any data about the variable of interest to predict it (like temperatures or precipitations). A MEP model was first used for climate by Paltridge (1975) to predict meridional fluxes and showed good agreement with observations but had some parameterizations.

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

The MEP is only a hypothesis and lacks rigorous mathematical proof, but it seems very general and is used in a broad range of domains, like crystal growth, transfer of electric charge, biological evolution, and many others (Martyushev and Seleznev, 2006). First, the MEP hypothesis must not be mistaken with the second law of thermodynamics, which only states that entropy production is positive ($\sigma \geq 0$) but not necessarily maximized. Next, when equilibrium thermodynamics and entropy are reinterpreted with the formalism of information theory, it is possible to obtain the main results of equilibrium thermodynamics (Jaynes, 1957) using the maximum entropy principle (MaxEnt). Then, a possible way to understand the maximum entropy production hypothesis is to see it as the non-equilibrium or time derivative "equivalent" of MaxEnt. Following this idea, Dewar (2003) and Dewar (2005) tried to prove the MEP hypothesis using MaxEnt. But, to quote Martyushev (2021), "Dewar's argument not only involves a number of nonobvious fundamental assumptions but also is nonrigorous and erroneous in a number of points.". The MEP hypothesis has also been related to other variational approaches in fluid dynamics or climate (Ozawa et al., 2003), like the *maximum dissipation rate* when temperatures are fixed (Malkus, 1956), or the *maximum generation of available potential energy* when in a steady-state (Lorenz, 1960). Here, we do not try at all to demonstrate the MEP hypothesis, but we prefer modelling the climate and the water cycle using it, and if results happen to be close to observations, we let as an open problem for theoreticians the explanation of why it works in our particular case (see Martyushev (2021) for a recent general review on MEP hypothesis).

In climate, it has been used to predict oceanic or atmospheric horizontal heat fluxes (Grassl, 1981; Gerard et al., 1990; Lorenz et al., 2001; Paltridge et al., 2007; Herbert et al., 2011), as well as vertical heat fluxes (Ozawa and Ohmura, 1997; Pujol and Fort, 2002; Wang et al., 2008; Herbert et al., 2013b), or both horizontal and vertical (Pascale et al., 2012); where always the entropy production of heat transfers due to atmospheric turbulence is maximized under some constraints. A limitation of using the MEP hypothesis (and variational formulations in general) is that all the variables of the variational problem need to be solved at once, making it difficult to add new phenomena. Another limitation is that to get meaningful results, we have to put on the variational problem constraints that are physically relevant and represent the main processes of the atmosphere, (Goody, 2007; Dewar, 2009). This is far to be obvious and only sometimes easily solvable. Moreover, as the driver for heat transfers is the input of radiative energy in the system, the radiative code must be accurate to have results close to observations. Otherwise, we are limited to qualitative (Lorenz et al., 2001) but not quantitative (Goody, 2007) agreement with data. In previous studies, MEP has generally been used for straightforward cases or with additional parameters. For 1D-vertical models, a grey atmosphere was used, and gravity was not taken into account. Recently, a new radiative-convective model was created with a more realistic radiative code (Herbert et al., 2013b). Moreover, geopotential and latent heat were added (Labarre et al., 2019). This model is not based on a convective adjustment like Manabe and Wetherald (1967), and the dynamical part (i.e., non-radiative) is treated without any adjustable parameter. In this study, the same model was used, and a new constraint on the water cycle was added, leading to a prediction of precipitations.

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

2 The radiative code

The radiative code used here is the one of Herbert et al. (2013b) (see their supplemental material for details) and is based on the Net-Exchange formulation (Dufresne et al. (2005)). It is more advanced than the grey atmosphere models used in previous studies (Ozawa and Ohmura, 1997), thus leading to results closer to observations. Let us consider an atmosphere divided into $N+1$ layers on the vertical axis. The net radiative energy budget $\mathcal{R}_i$ (i.e. the input of energy thanks to radiation) writes :

$$\mathcal{R}_i(T,q,\mathrm{O}_3,\mathrm{CO}_2,\alpha) = \mathrm{SW}_i(q,\mathrm{O}_3,\alpha) + \mathrm{LW}_i(T,q,\mathrm{CO}_2) \tag{1}$$

where SW_i and LW_i are the solar and infrared net energy budgets. $q = m_{water}/m_{air}$, O_3 and CO_2 are prescribed vertical profiles of specific humidity, ozone and carbon dioxide concentrations, given by page 3 of McClatchey (1972), corresponding to a standard atmosphere. To take into account the water vapour feedback with temperature, the relative humidity profile $h = q/q_s(T)$ is fixed for the computation of $\mathcal{R}_i$, so that q varies with T. α is the albedo of the surface. T is the temperature profile. The parameters h, O_3, CO_2, and α are fixed, so the energy budget $\mathcal{R}_i$ is a function of the temperatures only. Note that $T = \{T_j\}_{j=0,\dots,N}$, where T_j is the temperature in box j, hence $\mathcal{R}_i(T)$ is a functional of the temperatures, i.e. a non-local function of T. That will be important for the variational problem.

Usually, when computing the radiative energy budget, one uses a local description of the radiative energy fluxes. In the NEF framework (Dufresne et al., 2005), the description of energy transfer is global: each radiative energy input in the layer i is broken down into the individual contributions of all different atmospheric layers. The net energy exchange rate between layer i and j is written ψ_{ij}. At this point, this formulation is strictly equivalent to the usual one, but it makes it easier to develop approximations that reduce computational time while automatically satisfying the basic laws of physics: The energy exchange rate is antisymmetric ($\psi_{ij} = -\psi_{ji}$), thus the total energy is conserved ($\sum_{i,j}\psi_{ij} = 0$) and also the second law of thermodynamics is satisfied with keeping ψ_{ij} the same sign than $T_j - T_i$. Yet, because the resolution of the variational problem is strongly sensitive to the constraints imposed, it is of utmost importance that the laws of physics are rigorously satisfied.

To approximate the radiative transfer, the infrared spectrum is divided into 22 narrow bands, and the absorption coefficient for water vapour and carbon dioxide is calculated with the Goody (1952) statistical model and the data from Rodgers and Walshaw (1966). For spatial integration, the diffusive approximation is made with the diffusion factor $\mu = 1.66$. In the visible domain, the absorption by water vapour and ozone is computed with the parameterization of Lacis and Hansen (1974).

3 The maximization of entropy production

3.1 Energy conservation

The atmosphere is divided into $N+1$ layers on the vertical axis. Each layer i has a temperature T_i, a variable of the variational problem. Between layer i and $i+1$, there is a non-radiative energy flux F_i, whose nature is not explicit yet (with no additional

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

constraints it may be interpreted as conduction). When considering a steady state, the total energy balance reads :

$$\mathcal{R}_i = F_{i+1} - F_i \tag{2}$$

Where F_0 and F_{n+1} are taken equal to zero as if no energy (other than radiative) goes to the space or comes from the ground. The entropy production associated with these energy fluxes writes:

$$\sigma = -\sum_{i=1}^{n} F_i \left(\frac{1}{T_{i-1}} - \frac{1}{T_i} \right) \overset{(2)}{=} \sum_{i=0}^{n} -\frac{\mathcal{R}_i(T)}{T_i} \tag{3}$$

At stationary state, the total input of radiative energy must be equal to zero:

$$\sum_{i=0}^{n} \mathcal{R}_i(T) = 0 \tag{4}$$

The entropy production (eq. 3) is maximized under the constraint of energy conservation (eq. 4), leading to the following variational problem:

$$\max_{(T_0,\ldots,T_n)} \left\{ \sum_{i=0}^{n} -\frac{\mathcal{R}_i(T)}{T_i} \middle| \sum_{i=0}^{n} \mathcal{R}_i(T) = 0 \right\} \tag{ENERGY}$$

It is the exact same problem as eq. 24 of Herbert et al. (2013b).

3.2 Air convection

So far, the internal energy of the atmosphere or the transport of air masses has not been expressed. It might give more physical results to put in the variational problem a constraint on how the energy is transported by air masses in the atmosphere. By taking into account the sensible heat, the gravitational energy, and the latent heat, the specific energy (the energy per unit mass) writes:

$$e_i = C_p T_i + g z_i + L q_i \tag{5}$$

where C_p is the heat capacity of the air, g the standard acceleration due to gravity, z_i the elevation of layer i, L the latent heat of vaporization, and q_i the specific humidity of water vapour (m_{water}/m_{air}). The elevation z_i is expressed as a function of the temperatures below (see appendix A of Labarre et al. (2019)), and the specific humidity q_i is taken equal to its value at saturation, $q_i = q_s(T)$. Consequently, the specific energy $e_i(T)$ is like $\mathcal{R}_i(T)$ a functional of temperatures.

Now, take a mass flow rate m_i between layer $i-1$ and i. Suppose that the air is transported adiabatically and then thermalizes once in the layer i. It means that the amount of energy in the air mass took from layer $i-1$ is fully transported to layer i, thus the energy transported upward is equal to $m_i e_{i-1}$. To easily conserve the air in all boxes, the same amount of air m_i is adiabatically taken from layer i to layer $i-1$, transporting downward energy equal to $m_i e_i$. Thereby, the total energy flux between layer $i-1$ and i writes :

$$F_i = m_i(e_{i-1} - e_i) \tag{6}$$

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

Of course, for the reasoning to be consistent, we must have $m_i \geq 0$, which gives a new constraint on the energy fluxes and the temperatures. The following equation summarizes the variational problem:

$$\max_{(T_0,\ldots,T_n),(m_1,\ldots,m_n)} \left\{ \begin{array}{l} \sum_{i=0}^{n} -\frac{\mathcal{R}_i(T)}{T_i} \,|\, \sum_{i=0}^{n} \mathcal{R}_i(T) = 0 \text{ and } m_i \geq 0 \\ \\ \text{with } \mathcal{R}_i = F_{i+1} - F_i,\, F_i = m_i(e_{i-1}(T) - e_i(T)) \end{array} \right\} \tag{CONV}$$

It is the same variational problem as eq. 11 of Labarre et al. (2019) (although expressed as a function of temperature instead of energy fluxes).

3.3 Water transport and precipitation

In the formulation above, water vapour is a function of the temperature only and thus has no reason to be conserved when transported by the air masses. Infinite levels of water vapour could be created or disappear. In this study, we add a constraint on conserving water vapour that is supposed to mimic precipitations. We impose that water vapour cannot appear when transported, but it can disappear, and we call this phenomenon precipitation as if water vapour was transformed into liquid water. The flow rate m_i transports upward between layer $i-1$ and i an amount of water equal to $m_i q_{i-1}$, where q_i is the specific humidity of water vapour in the air; and it transports downward an amount of water equal to $m_i q_i$. Then the water flux between layers $i-1$ and i writes (similarly to eq. 6):

$$F_i^w = m_i(q_{i-1} - q_i) \tag{7}$$

The amount of water vapour that disappears in layer i is written :

$$P_i = F_{i+1}^w - F_i^w \tag{8}$$

Where F_0^w and F_{n+1}^w are taken equal to zero as if no water comes from underground or goes to space. The layer $i = 0$ is a surface boundary layer, supposed to be very thin, and plays the role of the surface. For $i = 1,\ldots,n$, we impose that $P_i \geq 0$, and P_i is called precipitation. On the layer $i = 0$, because $\sum_i P_i = 0$ we have:

$$P_0 = -\sum_{i=1}^{n} P_i \tag{9}$$

where $-P_0$ is the evaporation in layer $i = 0$ and is equal to the total precipitations. The specific humidity q_i is considered equal to its value at saturation $q_s(T)$ and then depends only on the temperature. The variational problem can be summarized by:

$$\max_{(T_0,\ldots,T_n),(m_1,\ldots,m_n)} \left\{ \begin{array}{l} \sum_{i=0}^{n} -\frac{\mathcal{R}_i(T)}{T_i} \,|\, \sum_{i=0}^{n} \mathcal{R}_i(T) = 0 \text{ and } m_i \geq 0,\, P_i \geq 0 \text{ for } i \geq 1 \\ \\ \text{with } \mathcal{R}_i = F_{i+1} - F_i,\, F_i = m_i(e_{i-1}(T) - e_i(T)), \\ P_i = F_{i+1}^w - F_i^w,\, F_i^w = m_i(q_{S_{i-1}}(T) - q_{S_i}(T)) \end{array} \right\} \tag{PRECIP}$$

Although it looks like a heavy equation, it is, in fact, very short when saying absolutely all the physics of the model is contained in it.

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

4 Numerical resolution

The variational problems ENERGY, CONV and PRECIP are solved using a sequential quadratic programming algorithm (Kraft, 1988; Virtanen et al., 2020). The basic principle is that it takes some initial conditions and performs a gradient descent until it finds a local maximum. Such an algorithm mathematically converges to a global maximum for a convex problem, however, for a non-convex problem, there is no guarantee that the maximum found is global. Several techniques that are not detailed here are used to get better results. One possibility is to test manually different initial conditions and see which one gives the highest entropy production. For a given problem, every result presented here is the one with the highest entropy production found. Given the fact we tested a wide amount of different initial conditions and found only a few local maxima, we are confident our results represent a global maximum. Nevertheless, it is still possible that another better result mathematically exists.

5 Results

We solve the problems with 21 boxes (1 surface boundary layer with albedo α and 20 atmospheric boxes), with prescribed standard vertical profiles of O_3, CO_2 concentrations and relative humidity took from McClatchey (1972), corresponding to a tropical atmosphere. Indeed because it is a 1D vertical model, it would not make much sense to take midlatitude or boreal profiles, as horizontal fluxes become significant there. In the radiative code, the relative humidity $h = q/q_S(T)$ is fixed so that more H_2O is present when temperature increases, which is a positive feedback. However, in the variational problem, q_i is still equal to $q_s(T)$ (eq. 5). Albedo is equal to $\alpha = 0.1$.

Results for equations ENERGY, CONV and PRECIP are shown figure 1. For comparison, standard temperatures for a tropical atmosphere (based on observations) are plotted figure 1a, taken from McClatchey (1972), as well as mean temperatures from the IPSL-CM6A-LR model between 23°S and 23°N. The comparison should remain qualitative, as our model's purpose is not to fit the observations precisely but to give a "proof of concept" that obtaining relevant results with the MEP hypothesis is possible. First, when putting more and more constraints into the variational problem, it is expected that the entropy production found decreases because the space of possibilities wanes. We get this: the entropy production is equal to $\sigma = 53.9$ $\mathrm{mW.m^{-2}.K^{-1}}$ for eq. ENERGY where only energy is conserved, $\sigma = 44.3$ $\mathrm{mW.m^{-2}.K^{-1}}$ for eq. CONV where a specific pattern of mass fluxes transports energy and $\sigma = 41.1$ $\mathrm{mW.m^{-2}.K^{-1}}$ for eq. PRECIP where water vapour is not allowed to appear.

5.1 With only energy conservation (blue square points)

See Herbert et al. (2013b) for details. The only variables present in eq. ENERGY are temperatures T and energy fluxes F. However, it is possible to compute afterwards variables like energy e (5), mass fluxes m (6), or precipitations P (8), but of course, there is no reason for m or P to obey the constraint of positivity, and they do not. When looking at the entropy production equation (3), one sees two terms, an energy flux and a gradient of inverse temperature. To maximize the entropy

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

production, variables find a balance between a state where energy fluxes are very high but temperatures homogeneous (so $\sigma = 0$), and a state where temperature gradients are high but energy fluxes are equal to zero (also $\sigma = 0$). Between the two, a balance is found where entropy production is maximized. Results are plotted in figure 1a for temperature and figure 1b for energy fluxes. As expected, because the atmosphere is essentially heated by the solar radiation intercepted at $z = 0$, temperatures are higher close to the ground. The energy fluxes compensate a bit for the temperature difference by going upwards (i.e. are positive Fig. 1(b)). The specific energy (eq. 5, Fig. 1c) decreases at the bottom because sensible heat C_pT is the dominant term and increases at the top because gravity gz becomes the dominant term. "Mass fluxes" are computed with eq. 6. Because F is always positive, the sign of m is always opposite to the sign of the energy gradients ∇e. When these gradients are negative, m is positive, but higher in the atmosphere when energy gradients become positive, the computed m is then negative, which has no physical meaning regarding eq. 6. It then becomes natural to impose $m > 0$, which is the case in the following paragraphs.

5.2 With a pattern of convection (orange diamond points)

See Labarre et al. (2019) for details. Adding the constraint of convection (6) forces the energy fluxes (Fig. 1b) to be opposed to the energy gradients. Thus, to keep positive upward energy fluxes, energy (Fig. 1c) gradients must remain negative. To do so, temperatures (Fig. 1a) adapt themselves to counteract the gravity, leading to a zone in the middle of the atmosphere where energy gradients equal zero and mass fluxes (Fig. 1d) equal $+\infty$; the atmosphere is perfectly mixed in this area. Thus, precipitations (Fig. 1e) are infinitely positive or negative. Higher in the sky, geopotential becomes too strong, energy gradients become positive and energy fluxes equal 0. This region defines the stratosphere, at about $P \approx 250$ hPa and $z \approx 9$ km, where no convection occurs. By adding a convection pattern and maximizing entropy production, we see a moist adiabatic lapse rate in the middle of the atmosphere, and a stratosphere naturally emerges. This model already gives suitable results for temperatures as the profile is similar to the IPSL-CM6A-LR model (light blue triangles), or the McClatchey (1972) standard tropical atmosphere (circles). The temperature profile remains close when adding a new constraint to compute precipitations.

5.3 With a constraint on precipitation (grey triangle points)

Adding the constraint on precipitation (8), the philosophy remains the same: because the atmosphere is heated from below, energy fluxes (Fig. 1b) want to go upward, but they have to be opposed to the energy gradients (Fig. 1c), so these gradients want to be negative or to go to zero leading to infinite mass fluxes (Fig. 1d) in the middle of the atmosphere. But, saying that no water vapour can be created prevents infinite mass fluxes and nil energy gradients. Consequently, energy gradients are negative (while energy fluxes are positive) below a zone we call the tropopause; above, energy gradients become positive (while energy fluxes are equal to zero). In term of precipitation (Fig. 1e), some amount of water vapour is taken in box number 0, and continue to go up and up to maximize mass fluxes. Then it reaches the tropopause, where mass fluxes become nil because there is not enough energy to go upper, and then water vapour disappears, i.e. it precipitates. The computed precipitations are equal to 1.2 m/year, which is the correct order of magnitude of tropical precipitations. Comparison of this result with real-world or modelled tropical precipitation depends on the box size chosen for the tropical area. For example, the average precipitations in the Earth System Model IPSL-CM6-LR between -23 and +23 degrees of latitude are 1.4 m/year. For real-world data, average

https://doi.org/10.5194/egusphere-2023-2208
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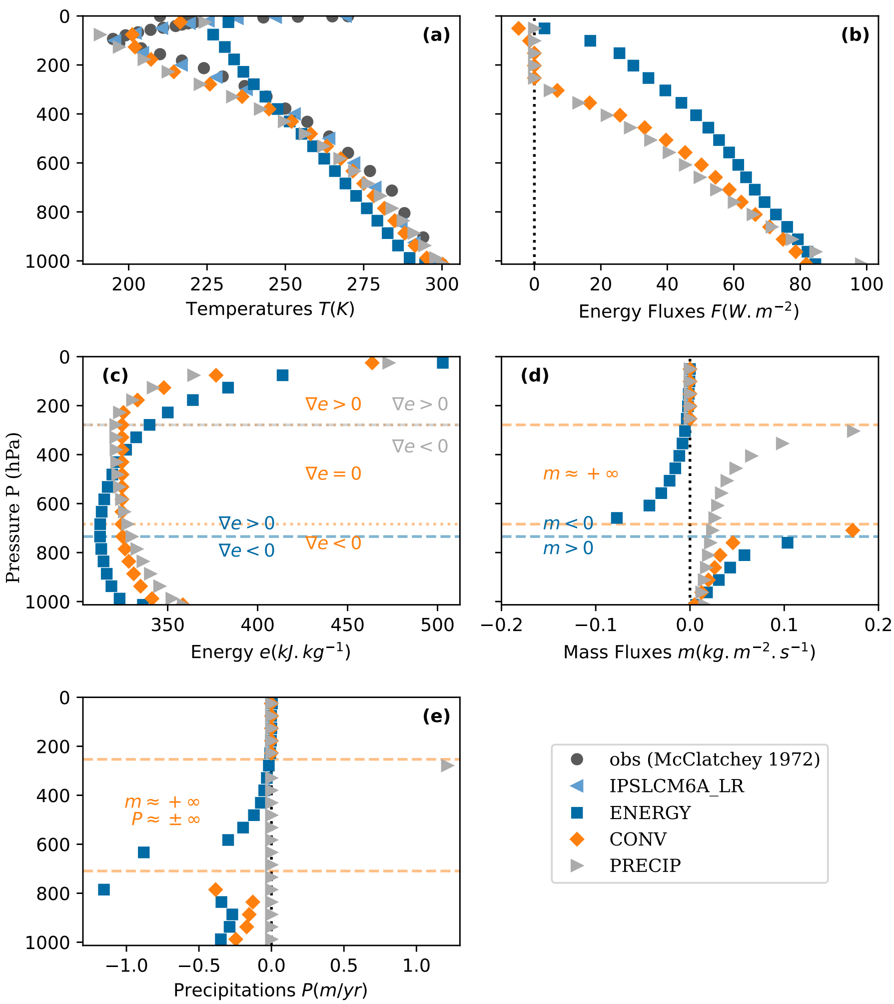

Figure 1. 3 different cases: 1) eq. ENERGY, 2) eq. CONV et 3) eq. PRECIP. 1) $\sigma = 53.917$, 2) 44.304 and 3) 41.108 $\mathrm{mW.m^{-2}.K^{-1}}$.

precipitations between 30°S and 30°N between 1980 and 1994 are 1.3 m/year (figure 8 of Xie and Arkin (1997)), and zonally averaged precipitations between 1979 and 2001 lies between 0.6 and 2 m/year (figure 5 of Adler et al. (2003)). Another local maximum of entropy production can give, for example, 2.1 m/year, though with less overall entropy production. This "uncertainty" in computed precipitations is probably due to the harsh resolution of only 20 boxes, because choosing one box or another to precipitate leads to a different value of precipitations. However, given the simple physics present in the model (a radiative code and a variational problem with a few equations), it is impressive to be able to compute precipitations with the correct order of magnitude. To the author's knowledge, it is the first time that precipitations are computed with a model using maximizing entropy production and without any data-tuned parameter. This result is of prime theoretical importance for climate scientists because it means the radiative transfer, or greenhouse gases, mainly drive atmospheric precipitations.

https://doi.org/10.5194/egusphere-2023-2208
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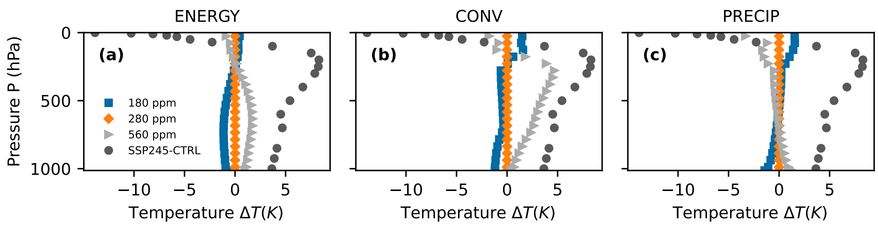

Figure 2. Differences of temperatures profiles between CO_2 at 180 ppm (or 560 ppm), and CO_2 at 280 ppm.

5.4 Doubling the CO_2 concentration

The MEP approach seems to provide a good order of magnitude for climate variables when looking at absolute values. It is interesting to test if it can capture small changes in external forcing. A classic test for climate models is to look at the climate sensitivity, which is the difference of temperature at 1.50 m between conditions where CO_2 is at pre-industrial level (280 ppm), and conditions where CO_2 is doubled (560 ppm). Here the only feedbacks present are the water vapour feedback incorporated by fixing the relative humidity in the radiative code, and the lapse rate feedback. Because box number 0 is a thin surface boundary layer, we take the temperature of layer number 1, whose middle is at 988 hPa ($\sim 220m$). The climate sensitivity is 1.1 K for eq. ENERGY, 0.7 K for eq. CONV, and 1.0 K for PRECIP (see Fig. 2). To compare, the climate sensitivity of the state-of-the-art IPSL-CM6A-LR model is 3.0 K, which is much more. It may be because they are much more feedback in this model. But the model of Manabe and Wetherald (1967) contains similar physics to the MEP model and predicts 2.9 K (see their table 5). The reason the MEP model predicts a lesser climate sensitivity needs to be investigated.

But the vertical temperature distribution within the atmosphere is more interesting. It is plotted for eqs. ENERGY, CONV, PRECIP figure 2, that is the difference of temperatures between a case with 560 ppm (or 180 ppm corresponding to the last glacial maximum) and a case with 280 ppm. The difference between 560 ppm and 280 ppm is also plotted for the IPSL-CM6A-LR model, with a temperature average took in the tropical region (between $\pm 23°$). When looking at the shape of temperature distribution and comparing it to the IPSL-CM6A-LR, the best model seems to be the problem CONV, since the temperature difference increases with height and then decreases in the stratosphere. For the PRECIP model, it just decreases in the entire atmosphere. A possible explanation is that with a constraint on precipitation, the model is too constrained and more degrees of freedom should be added. For example, the convection pattern could allow mass fluxes between layers that are not adjacent, or water vapour could be allowed to vary between zero and saturation. However, when interpreting these results, one should keep in mind that depending on the resolution method of the variational problem (or the choice of initial conditions), results may differ by about 1 Kelvin. They differ even more by choosing an arbitrary local maximum of entropy production instead of the "supposedly" global maximum. Indeed, we note that because we changed the resolution method and found a new result with higher entropy production, the climate sensitivity of the problem CONV is a bit different than in Fig. 6 of Labarre et al. (2019).

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

6 Discussion

The state-of-the-art GCMs or ESMs models and the MEP model are based on the same conservation laws. In a GCM, the conservation laws are local and lead to partial derivative equations that are true in the limit of infinitely small differentials. The momentum conservation is the Navier-Stokes equation (present only to the horizontal), the energy conservation is the thermal energy equation, and the mass conservation is $\nabla \cdot u = 0$. These equations were demonstrated for infinitely small increments. However, in GCMs and ESMs, they are integrated under a grid that needs to be smaller. Indeed, because of non-linearity small scales do have an impact on large scales. Therefore, more than the first conservation equations are needed for consistent results. Additional equations involving tunable parameters are added and are sometimes called "closure equations". In the MEP framework, the energy conservation is eq. 4, and the mass conservation is immediately imposed by the convection pattern (eq. 5) and constraint $m \geq 0$. The water conservation is imposed by $P \geq 0$ in eq. PRECIP. So with MEP, the conservation laws are defined as constraints of an optimization problem, and unknown variables are resolved simultaneously to reach the maximum of entropy production.

Everything else in our MEP model is similar to what is done in a GCM. The radiative code is based on integrating Planck's law on different wavelength bands corresponding to different constant extinction coefficients (see supplementary materials of Herbert et al. (2013a)). The air is considered an ideal gas, and the hypothesis of hydrostatic equilibrium is made ($gdz = -\rho dp$). Of course, well-known parameters like heat capacity C_p or the relation between q_s and T (see appendix B of Labarre et al. (2019)) are not variables of the optimization problem but just taken equal to well-established values used in GCMs and ESMs.

Still, there are many reasons why our MEP model could give different results than an ESM like the IPSL-CM6A-LR. Our MEP model does not have a continental surface. In fact, because there is no constraint on evaporation, the ground can be seen as an infinite water reservoir, like an ocean. Also, no clouds (i.e., liquid water) are present in the air, although they are known to have a non-negligible impact on the radiative forcing. Moreover, the model is only 1D vertical and works well only for a tropical column where vertical fluxes dominate. Finally, a reason for getting different results could be the possible lack of validity of the MEP hypothesis. That said, obtaining the same order of magnitude of precipitation as in the IPSL-CM6A-LR model is surprising, which means 1.2 $\mathrm{m.yr}^{-1}$ compared to 1.4 $\mathrm{m.yr}^{-1}$. We note that when extracting data from the IPSL-CM6A-LR model, we could have chosen a different latitude to take the mean or focused only on oceans. It would have changed the obtained precipitation but not the order of magnitude. Here we chose to take the mean precipitations between $\pm 23°$ because it is coherent with the vertical temperature and gases columns took from McClatchey (1972).

Finally, because there is no firm evidence of the validity of the MEP hypothesis, the true solution may not be the one with the highest entropy production but might correspond to a local maximum of entropy production. And a few local maxima exist. For example, there is a local maximum corresponding to precipitations equal to 2.3 $\mathrm{m.yr}^{-1}$ (instead of 1.2$\mathrm{m.yr}^{-1}$ with a global maximum). Between the two, temperatures differ by a few kelvins (see figure A1 in the appendix for comparison). Overall, results for local maxima of entropy production are usually qualitatively similar to the one with a global maximum. Consequently, the MEP hypothesis is at least an efficient tool to find an order of magnitude or a qualitative shape of the solution.

https://doi.org/10.5194/egusphere-2023-2208
Preprint. Discussion started: 23 October 2023

The MEP model could be improved by exploring several approaches. First, the specific humidity of water vapour q could be chosen not equal to saturation. Then, it is not clear if an additional constraint on precipitation should be imposed, for example, saying that precipitation can occur only if $q = q_s$, that is $(q - q_s)P = 0$, where P are precipitation. Moreover, this constraint is highly non-convex and numerically very harsh to solve. Second, convection is not allowed between every layer because air masses are compelled to move to adjacent layers. However, in the tropics, there is a phenomenon called deep convection where an air mass can go adiabatically from the bottom to the top of the troposphere. This phenomenon is not authorized with the convection pattern imposed. But it could be by changing equation 5 and adding fluxes for non-adjacent layers.

7 Conclusions

Since Ozawa and Ohmura (1997), many improvements have been made. Taking a more realistic radiative code (Herbert et al., 2013a) leads to a stable atmosphere: the potential temperature decreases with altitude. Adding a convection pattern (Labarre et al., 2019) gives much more realistic results in terms of temperature and reveals a stratosphere up to ≈ 250 hPa where no convection occurs. Then, imposing a constraint on water vapour conservation leads to precipitations in the correct order of magnitude of what an ESM would find. These results look great in absolute value, but when one performs a sensitivity experiment such as looking at the temperature difference when doubling CO_2, results seem less satisfying.

In the future, several approaches such as changing the convection pattern or letting relative humidity vary need to be explored. Along these lines, it might be possible to build a climate model 2D or 3D, with a representation of clouds, vegetation, and oceans, whose "closure equations" would not at all be based on "tuned with observations" parameters. Such a model could be used to model climates where little is known, such as other planets or paleoclimates.

Code availability. The code used to produce the results can be found at https://doi.org/10.5281/zenodo.7995540

Appendix A: A global and a local maximum

Author contributions. Didier Paillard is the brain having the global understanding. Karine Watrin implemented many possibilities in the resolution code. Quentin Pikeroen also worked on the code, used it to get the article's results, and wrote the manuscript.

Competing interests. The contact author has declared that none of the authors has any competing interests.

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https://doi.org/10.5194/egusphere-2023-2208
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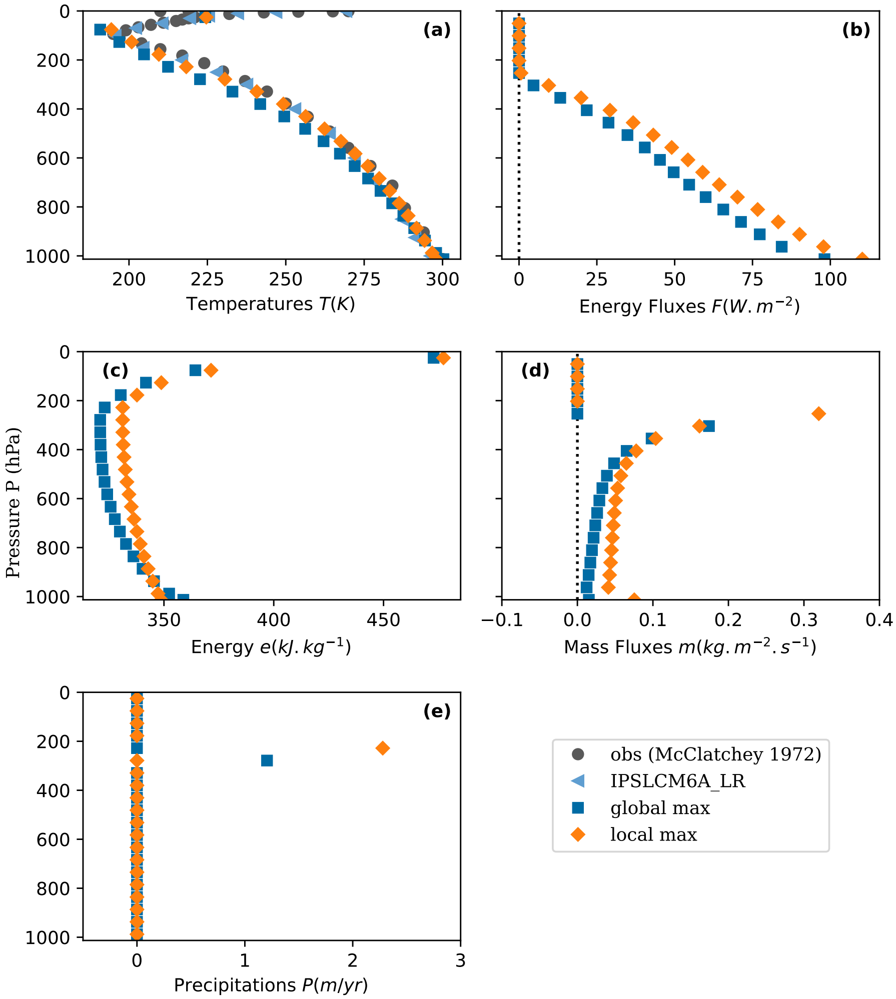

Figure A1. 2 different cases: 1) a global maximum of entropy production $\sigma = 41.108\ \mathrm{mW.m^{-2}.K^{-1}}$. 2) A local maximum of entropy production $\sigma = 40.078\ \mathrm{mW.m^{-2}.K^{-1}}$.

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