Response to RC3

We express my sincere gratitude for your continued feedback. We agree with some of your points, and we will make appropriate modifications based on the information you offered. In addition, We still have the following two points to reply to you:

(1) The primary point of disputation in our discussions pertains to that whether the hydrological data contains the 5.9-year oscillation. The topic of selecting a time window remains a subject of ongoing discussion, and we will test various hydrological model data according to different time windows. Furthermore, subsequent to our discussions, a consensus has been reached that the existing hydrological models lack accuracy. The wavelet analysis findings presented in Reply on RC2 also corroborate the notion that certain long-period signals, such as those of 11 years and 18.6 years, exhibit instability and inaccuracy over extended time intervals. In addition, we support that the 5.9-year oscillation origins from the dynamics of the Earth's core, and further causes the mantle and crust connection effects. However, whether it also causes the fluid layers at the Earth's surface to produce the associated effects (especially the hydrological effect) remains to be discussed. We believe that to fully confirm whether the 5.9-year oscillation is contained in the hydrological processes, we need to carry out more detailed verification combined with global in-site hydrological observations. Therefore, we highly recommend taking on this work in the future.

(2) Regarding the validation of the AR-*z* spectrum, we have shown in previous studies that the AR-*z* spectrum is not suitable for direct application to a single time series, but for multiple time series in the form of the product spectrum (See Ding & Chao, 2015, GJI; 2015, JGR). Indeed, the AR-*z* spectrum applied to a single time series yields the unstable result as you got in your tests. The AR-*z* spectrum employs a preset Q value for conducting analytical continuation, thereby enhancing certain spectral peaks that may potentially include unidentified noise. Therefore, based on the original AR-*z* formulation, we have developed a stabilized AR-*z* spectrum, taking advantage of a Monte Carlo noise-assisted bootstrap scheme, and demonstrate its effectiveness in obtaining more robust spectral estimates for a single record (Ding & Chao, 2018, JGR). This improved method can prevent the noise in a data record to alter the apparent estimates in the "empty" bins. Fig. R1 compares the Fourier spectrum, AR-*z* spectrum, and stabilized AR-*z* spectrum of a single synthetic time series. The synthetic signal and white noise are exactly the

same as the example you gave, and the *Q* value used in the analytical continuation of AR-*z* is 1e15 due to no attenuation for the input signal. From Fig. R1, the (stabilized) AR-*z* spectrum exhibit the significant advantage in suppressing the background noise relative to the Fourier spectrum. Applying to a single time series, the AR-*z* spectrum leads to some noise signals being mistakenly magnified because of its strong instability, whereas the stabilized AR-*z* spectrum overcomes this drawback well.

It is also worth noting that the AR-*z* method is more practical for the enhancement of the weak damping harmonic signals against the background noises. Ding & Chao (2015, GJI)'s Fig. 5 showed this point well, and we will not illustrate it here. Whether by multiplying the AR-*z* spectra of multiple real time series, or by repeatedly adding random white noise to a single series to obtain the AR-*z* product spectrum, the purpose is to make the noise variance for the empty bins to get reduced in principle through destructive interference, while the real signal's spectral peaks remain essentially unchanged.

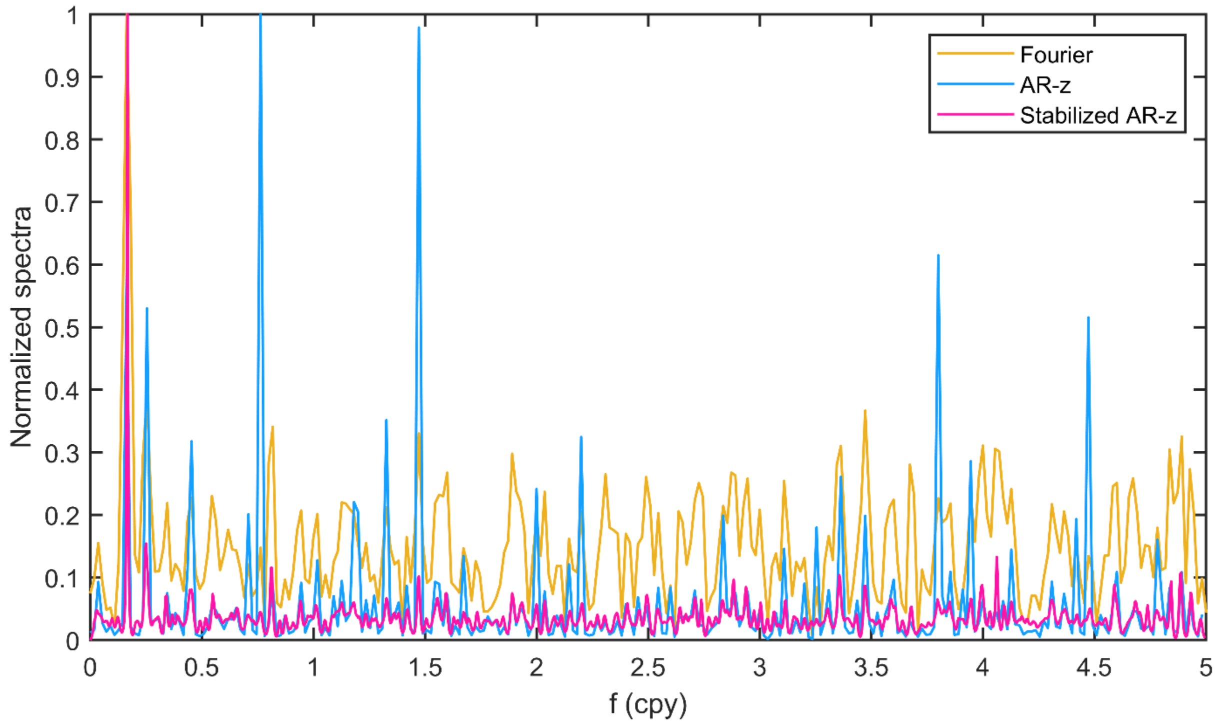

Figure R1. The normalized Fourier, AR-z, and stabilized AR-*z* spectra of a synthetic time series, which contains a 6-year signal and white noise.

References

Ding, H. and Chao, B. F.: Detecting harmonic signals in a noisy time series: The *z*-domain autoregressive (AR-z) spectrum, Geophys. J. Int., 201(3), 1287-1296, https://doi.org/10.1093/gji/ggv077, 2015.

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Ding, H. and Chao, B. F.: Application of stabilized AR-*z* spectrum in harmonic analysis for geophysics, J. Geophys. Res. Solid Earth, 123, 8249-8259, https://doi.org/10.1029/2018JB015890, 2018.