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Dear Editor, dear Reviewer,

we would like to thank you again for your time and the constructive comments. Please find our detailed replies on the comments below. We hope that we answer all your remarks.

Our replies to the reviewer's comments are highlighted in blue. To guide you better through the reply letter, comments of reviewers and the corresponding answers of the authors made in the initial review, which are still relevant for our reply, are marked in *grey*. To highlight the nature of our replies we use a traffic light system indicating agreement with the reviewer marked in green, partial agreement in yellow, and objections in red.

Reviewers' and Editors' comments:

Reviewer #2:

1. *In part, the comparisons made with other studies in the results and discussion sections are difficult to justify, or they are not explained well enough. For instance, I had trouble seeing how absolute permeability values of a single fracture of a real sandstone can or should be compared to permeabilities of fracture networks in an unspecified material (probably synthetic). The same applies to the comparison with trends of PH-derived permeability of porous media.*

We partly agree. The comparisons presented in this study are used to show that persistent homology is used for permeability assessment in a single fracture in addition to porous media and discrete fracture networks. In addition, the general trend is to be shown that persistent homology slightly overestimates the reference value. It is by no means intended to draw conclusions that a single sandstone fractures (bedding plane) behaves exactly as 3D printed discrete fracture network (DFN). Nevertheless, we elaborated the section (L286-L296):
"Of particular interest for this study are the permeabilities of fracture networks, which are displayed as dark gray diamonds in Figure 5, since they are also based on fractured instead of porous material. In general, it can be identified that permeabilities of fracture networks are distributed closer around the 1:1 line compared to porous media values (light gray crosses) in Figure 5. In addition, it is also not surprising that the results of this study have permeability values closer to those of fracture networks rather than porous rocks. This is due to mechanical aperture of the individual fractures, which form a fracture network, being of a similar order of magnitude to the single fracture investigated here. Since the most values from fracture networks are results of the analysis of fracture networks with plane fracture surfaces in the study of Suzuki et al. (2021), it is possible to estimate the influence of surface roughness as well. The rough single fracture studied here shows the same trend of permeabilities, the majority of which are overestimated slightly, as the planar fracture networks addressed. This suggests only a minor influence of the roughness on the final result of the PH analysis. However, it should be considered that typically fracture surfaces have roughnesses of H > 0.5, whereas the roughness of the used fracture is slightly lower (Hx = 0.48 and Hy = 0.42)."

- I understand the intention, but here the comparisons remain questionable in my view:
    - The authors state it is not the goal to suggest that fracture networks and single fractures behave the same. Yet, they bring up absolute permeability values for comparison to make the results seem plausible. Two "fractured media" are insufficient as

grounds for such a comparison of absolute values. Matching absolute values may be coincidental, even though the mechanical aperture seems to be similar as stated. But I would actually be surprised if in that case a non-rough network and single frac result in the same k values.

- NOTE: the stated goal of the paper is to show the applicability of PH for single fracture permeability estimation. This has been shown nicely in the paper by comparing to other methods. I'd argue that this is the value of the present study. The comparison of the results to absolute values of other studies with different study objects is unnecessary and hard to justify. I strongly suggest to delete them.

- On the other hand, it is useful to compare trends for overestimation (if any, see comment below) and state that they seem in line with other studies. This is because one compares methods, not specific samples/media.

We agree and delete the comparison of absolute values between the different samples as well as the assessment to derive the influence of roughness in this single fracture. Therefore, the following parts of the manuscript are **removed** (L279-281 and L286-305 of the manuscript after the initial review):

"Two main findings can be derived from this comparison: (1) The values determined in this study are in the same range of permeability as the data sets investigated in the previous study…"

and

"Of particular interest for this study are the permeabilities of fracture networks, which are displayed as dark gray diamonds in Figure 5, since they are also based on fractured instead of porous material. In general, it can be identified that permeabilities of fracture networks are distributed closer around the 1:1 line compared to porous media values (light gray crosses) in Figure 5. In addition, it is also not surprising that the results of this study have permeability values closer to those of fracture networks rather than porous rocks. This is due to mechanical aperture of the individual fractures, which form a fracture network, being of a similar order of magnitude to the single fracture investigated here. Since the most values from fracture networks are results of the analysis of fracture networks with plane fracture surfaces in the study of Suzuki et al. (2021), it is possible to estimate the influence of surface roughness as well. The rough single fracture studied here shows the same trend of permeabilities, the majority of which are overestimated slightly, as the planar fracture networks addressed. This suggests only a minor influence of the roughness on the final result of the PH analysis. However, it should be considered that typically fracture surfaces have roughnesses of $H > 0.5$, whereas the roughness of the used fracture is slightly lower ($H_x = 0.48$ and $H_y = 0.42$). Furthermore, the local cubic law, seems to be also valid for rough single fracture such as a bedding plane joint of a sandstone. This is overall in good agreement with many other studies that have investigated the influence of the application of local cubic law on permeability of rough fractures (Witherspoon et al., 1980; Brush and Thomson, 2003; Konzuk and Kueper, 2004; Qian et al., 2011). Witherspoon et al. (1980) investigated on artificially induced fractures in granite, basalt and marble and showed that independent of flow direction or closing of fracture, the cubic law stays valid. This general concept was proven by later studies, but with restrictions in terms of the maximum Reynolds number to be below 1 for synthetically created random single fractures (Brush and Thomson, 2003; Qian et al., 2011) and artificially induced dolomite fractures (Konzuk and Kueper, 2004). All these studies also found an overestimation of flow through a single fracture by cubic law compared to the Stokes equations. The large proportion of overestimated permeabilities by PH analysis can be due to this."

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Since we still think comparing trends between the data of this study and the study of Suzuki et al. (2021) to proof that the methods works for different kind of cavities, we **added** the following section (L279-288 of the updated manuscript):

"The method of PH for permeability estimation in both studies provides comparable results to the respective reference method. The results of this study are closely scattered around the 1:1 line and therefore match well with the results based on fracture networks (dark grey diamonds). It appears that the results of this study can be estimated even better than most of the data points of previous data sets, especially those generated from porous media (light grey crosses). In the latter, PH tends to overestimate the permeability, which cannot be confirmed for the data in this study. However, this study indicates that the quality of the permeability estimation is not only attributable to the type of cavity (pores, single fracture or fracture networks). Based on our results, the quality of the permeability estimation by PH is also dependent on the resolution and anisotropy of the respective data set. Nevertheless, a larger number of data sets should be examined for a more precise assessment of the various influences on the quality of the permeability estimation."

2. *The authors conclusions on overestimation of permeability when using PH do not seem to convincingly match their own data. Note that is not necessarily a bad thing, because the presented permeability estimates match those from the other methods rather well.*

We partly agree. The data shows that the experimentally or numerically determined reference p are slightly exceeded for the majority of the estimated permeabilities in this study (67 % of estimated permeabilities exceed their reference value). In fact, the overestimation of permeability is rather low compared to the other permeabilities presented. Since the same trend can be seen in the study of Suzuki et al. (2021), we have included the conclusion on overestimation of permeabilities in this study.

- I remain with my concern that the conclusion on overestimation seems rather forced given the presented data. I suggest to rephrase it (see below). Interestingly, even the authors themselves state that "the overestimation of permeability is rather low compared to the other permeabilities presented". Then why is overestimation presented as one of the conclusions, rather than for instance a "good match"?

- Specifically, the 67% (8 of 12 points) seem like a clear trend, but I think the presentation of only this number is a bit misleading. This is because the 12 points only represent 6 measurements, compared to two different methods at 3 different resolutions. Looking closely, it is clear that the conclusion on over-/underestimation is also resolution and direction- dependent. In my view, this is a strong indication that a general statement in terms of over- or underestimation is not possible based on this data.

- As a hopefully constructive suggestion, I think the authors can conclude that differences to the results of other, established methods are small (=good match), and more data are needed to analyse the impact of resolution and anisotropy on the results. In this context, the results of Suzuki et al. (2021) can still be stated to give scientific context – but the presented study here does not convincingly confirm their results in my view.

We also agree on this issue. In addition to the removed parts shown in reply to comment #1 of reviewer #2 in this letter, we also **remove** additional parts regarding the overestimation of our own data (L281-285 of the manuscript after the initial review):

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"…and (2) in both studies, PH tends to slightly overestimate permeability, especially at relatively lower permeabilities < 10^{-11} m^2. In this study, 67 % of the PH results are higher than the comparing methodology. In the previous study, even 90 % of the PH results overestimate numerical simulation. However, overestimation of the results in this study is only minor or in the same order of magnitude compared to the results of Suzuki et al (2021)."

As mentioned in reply to comment #1 of reviewer #2, we still show the results of Suzuki et al. (2021) for scientific content and to proof the methodology. This should be in good agreement to the comments of reviewer #2.

Editor:

1. Dear authors,
   please note the re-review by Reviewer 2; and also that you did not address the later comments/suggestions of Reviewer 1 (e.g. from "Line 272:" onwards in their initial review). These are important points and need to be addressed before I can consider a decision for publication.

   Comment of Reviewer #1 from L272 on:
   "I don't think that you can conclude that there is only a minor influence of roughness on the PH analysis only because your permeability results are on the same range as those from Suzuki et al. (2021). First, you don't provide roughness measurements of the fracture sample, and therefore there are no quantitative parameters that allow to assess whether this fracture is rough or smooth. In fact, you call it 'rough' in the title and in line 270, and then 'relatively smooth' in line 274. I strongly recommend including a roughness quantification for this fracture in the paper. I don't know if PH allows to determine a roughness estimation, but your high-resolution scans would allow you to, for example, follow a workflow such as that by Candela et al. 2012, and determine the roughness exponent H from a power spectral analysis. There are many works that have used this approach, which would allow you to assess comparatively how rough/smooth is this fracture.

   We agree, but we answered this part of the comment already in review letter #1 (c.f. reply to comment #1 of reviewer #1) since the reviewer made two similar comments on the same issue that no roughness measurement is given.

   Second, the permeability of fractures has been shown to have fall within a very wide range of magnitudes (e.g. Walsh 1981, Kranz et al. 1979, Iwai 1976, Nara et al. 2011, and many others). That your permeability results happen to fall within the same range as those from Suzuki et al. (2021) seems coincidental, considering that you are comparing different scales and flow paths (fracture networks in a 5 cm length sample vs a single fracture of 12 by 45 cm length). I therefore don't think that you can derive meaningful conclusions from this comparison.

   We agree. Since we think that this point addresses the same issue as comment #1 of reviewer #2, please find our answer to this comment in our reply to comment #1 of reviewer #2 in this letter.

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I also don't understand why you discuss your results in the context of the Cubic Law assumption of parallel plates (L273-274), when PH analysis obviously considers the topography of the fracture surfaces."

We agree. As we removed the entire discussion about the influence of roughness on the results of PH in this part of the manuscript, we also deleted the part including discussion about Cubic Law (c.f. our reply to comment #1 of reviewer #2).