Two-Dimensional Numerical Simulations of Mixing under Ice Keels

Response to Reviewer #2

We would like to thank the reviewer for their thoughtful and detailed evaluation of our manuscript. We believe the manuscript has benefited greatly from your suggestions. Below, each question/suggestion from the review is listed followed by our response to it in blue. In suggestions where content was added or modified to the manuscript, the content is listed below the response *italicized*.

Reviewer comments:

Nonlinear terms in momentum balance equations (1) and (2) are different from standard expressions .Viscous terms and in equations (1) and (2) are also different from standard form. Equations (1) and (2) are different from the momentum balance equations considered in the papers of Skyllingstad et al (2003) and Hester et al (2021) given in the reference list. More detailed explanation of equations (1) and (2) is necessary for improving of understanding of the problem statin.

- The equations presented in our paper and in Hester et al. (2021) are equivalent, but presented in a different form. They derive from the vector identity $(\vec{v} \cdot \vec{\nabla})\,\vec{v} = \vec{\omega} \times \vec{v} + \vec{\nabla}(|v|^2/2)$, with $\vec{\omega} = \vec{\nabla} \times \vec{v}$ the vorticity. The left-hand side of the first equation is the more familiar version you are referring to, and the right-hand side is the formulation that Hester et al. (2021) chose for their implementation. In this formulation, the "pressure gradient" is the gradient of the familiar thermodynamic pressure, plus that of $|v|^2/2$. This minor re-casting of the advection term aside, the velocity and buoyancy solutions are strictly the same. We have summarized the above in our methods:
  - *The equations are written in a computationally advantageous form by decomposing the advection term into the sum of the Lamb vector (-wq, uq) and a gradient term incorporated into the pressure gradient.*

Diabatic mixing is caused by salt diffusion in conditions of internal waves excited by the interaction of the ice keel with water flow leading to adiabatic stirring. Coefficient of salt diffusion is set to m2/s in numerical simulations. This value is much larger the molecular salt diffusion m2/s. The large value of is chosen to dissipate eddies smaller than the resolution of the grid (line 96). Further increasing influence diabatic mixing according to formula (12). Please give more physical reasons for the choice of numerical value of.

- The choice of our salt and momentum diffusivities originate completely from numerical stability. That is, in no way could we fully resolve the large separation of scales between the scales of molecular diffusion (mm) to the domain of interest (10-100m) within a reasonable computational time and with reasonable computational cost. So, it is a common practice in modeling to increase the value of viscosity and diffusivity in order to "shrink" the range of scales to something that we could fully resolve in a reasonable time. We thank the reviewer for pointing this out and have added a comment in our methods that summarizes this rationale when introducing our diffusivities:

  - *We choose such large values for nu and mu to dissipate eddies smaller than the resolution of the grid, similar to the choice of (Zhang et al., 2022), which keeps our numerical cost tractable. These values may have quantitative consequences on mixing but should preserve our qualitative conclusions, as we discuss in Section 5.2.*

Kinematic viscosity m2/s is also larger molecular kinematic viscosity of m2/s. Is it turbulent eddy viscosity? Please explain physical sense of.

- Please see our previous response regarding large diffusivities.

Authors ignore thermal effects assuming water temperature equals -2 C. The water temperature is assumed depending on salinity (lines 91-92). Temperature at ice-water interface should be equal to the freezing point, and outside of the interface temperature is equal the freezing point or higher. Adiabatic mixing and diabatic stirring lead to increasing of water salinity and decreasing of the freezing point at ice-water interface. Decreasing of the freezing point influences ice melt leading to decreasing of water salinity and density near the interface. How strong this effect is in long term perspective?

- Thank you for bringing this to our attention. Yes, by ignoring the melting of the keel we ignore the stabilizing buoyancy flux from the meltwater, which can hinder turbulence and reduce mixing. It should be noted that the rate of melting depends on the keel's ability to stir or mix away the fresh meltwater and pull up heat fluxes from below. That is, the relationship between mixing and melting is one of negative feedback. This complicates the situation and doesn't allow us to qualitatively determine the effect that melting would have in our simulations/regimes in the long term without future study. If our keel speed were physically variable, then melting could reduce drag and allow it to travel faster/further (McPhee, 1983; McPhee, 2012), which is significant in the long term. We have added a paragraph in Section 5.2 explaining the limiting effects of our constant-temperature domains:
  - *Fifth, fixing the temperature of our domain at the freezing point of seawater necessarily suppresses melting of the keel. Aside from structurally changing the keel, melting would produce a stabilizing buoyancy flux of freshwater immediately below the ice that could hinder turbulence and, consequently, mixing. The rate of melting responds to the keel's ability to draw up heat fluxes from below and to mix or stir away the fresh meltwater immediately below the ice (Skyllingstad et al., 2003). For instance, regimes like Vortex Shedding would likely see high melting rates because of their large mixing rates and mixing depths; however, the effects of the stabilizing buoyancy flux from the subsequent meltwater on the regime's mixing rates are uncertain and require further work. If our ice keel speed were physically variable (i.e., influenced by drag), then melting could hydrodynamically ``decouple'' the ice floe and its*

keel(s) from the upper ocean boundary layer, reducing drag and allowing the floe to travel faster; however, this is beyond the scope of this paper. The reader is referred to McPhee (2012) for more information.

Estimates of ice drift speed using wind drag coefficient are not correct in the Barents Sea regions with relatively strong semidiurnal tide and influence of Spitsbergen, Franz Josef Land and Novaya Zemlya. Semidiurnal tide is stronger in the Barents Sea than in East Arctic regions. Speed of semidiurnal tidal current may exceed 1 m/s in the region between Bear and Hopen Islands. Also, water temperature below drift ice is frequently higher than -2 C in the Barents Sea. Depending on tidal phase and wind it varies from -1C to -1.9C.

- Thank you for bringing this to our attention. It should be noted that the wind drag coefficient is only used for estimating the ice speed trend based on the wind speed trend. To factor in decadal changes in semidiurnal tides would make this side of the analysis too detailed when we made other, more consequential approximations (e.g., assuming a two-layer density model). Regarding the choice of a constant ice speed across the Arctic, we agree that this may be our largest simplification and that semidiurnal tides could alter this value depending on location. We have added a comment to bring this to the readers' attention:
    - *This is a vast simplification, as in reality U varies largely across the Arctic due to spatial and temporal variability in wind forcing and semidiurnal tides.*

All ice ridges in the Barents Sea are the first-year ridges. Shape of their keels is not no smooth as it is considered in the papers. Ridge keels are not completely consolidated, and macro porosity of their unconsolidated parts vary in the range 20-40%. Water can penetrate inside ridge keels, and boundary condition with zero normal velocity should be modified.

- Thank you for pointing this out. We agree that our boundary condition isn't entirely representative of all ice keels, especially first-year ridges. Flow through porous mediums is a complicated topic and proper treatment would require a more intricate model, which is beyond the scope of the paper. We believe that your first point about the smoothness of our keels is in fact a larger simplification than our boundary condition. Edges or irregularities in the keel can become significant turbulence generators and effectively change the entire flow behavior and hence mixing. To factor in all various forms and irregularities of a keel would require a more statistical approach and a more sophisticated numerical solving scheme, which is left for future work. We have added a couple of sentences discussing this in our limitations section with an emphasis on future work:
    - *In addition, ice keels are conglomerates of ice rubble with varying degrees of porosity. As such, our no-slip condition at the keel boundary is not necessarily realistic, particularly for young keels. Accounting for porous flow would require a more intricate model, which is left for future work.*
    - *Note that, by assuming that there are no ridges or irregularities on the keel, we ignore additional generators of small-scale turbulence, and thus of stirring. This may result in underestimating mixing.*

References

Hester, E. W., McConnochie, C. D., Cenedese, C., Couston, L.-A., & Vasil, G. (2021). Aspect ratio affects iceberg melting. Physical Review Fluids, 6(2), 023802. https://doi.org/10.1103/PhysRevFluids.6.023802

Skyllingstad, E. D., Paulson, C. A., Pegau, W. S., McPhee, M. G., & Stanton, T. (2003). Effects of keels on ice bottom turbulence exchange. Journal of Geophysical Research: Oceans, 108(12), 1–16. https://doi.org/10.1029/2002jc001488

McPhee, M. G. (1983). Turbulent heat and momentum transfer in the oceanic boundary layer under melting pack ice. Journal of Geophysical Research: Oceans, 88(C5), 2827–2835. https://doi.org/10.1029/JC088iC05p02827

McPhee, M. G. (2012). Advances in understanding ice – ocean stress during and since AIDJEX. Cold Regions Science and Technology, 76–77, 24–36. https://doi.org/10.1016/j.coldregions.2011.05.001