



Dimensionless argument: a narrow grain size range near 2 mm plays a special role in river sediment transport and morphodynamics

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Abstract. The grain size 2 mm is the conventional border between sand and gravel. This size is used extensively, and generally without much physical justification, to discriminate between such features as sedimentary deposit type (clast-supported versus matrix-supported), river type (gravel-bed versus sand-bed) and sediment transport relation (gravel versus sand). Here we inquire as to whether this 2 mm boundary is simply a social construct upon which the research community has decided to agree via repetition, convergence and rearticulation, or whether there is some underlying physics. We use dimensionless arguments to show the following for typical conditions on Earth, i.e., natural clasts (e.g. granitic or limestone) in 20°C water. As grain size ranges from 1 to 5 mm (a narrow band including 2 mm), sediment suspension becomes vanishingly small at normal flood conditions in alluvial rivers. We refer to this range as pea gravel. We further show that

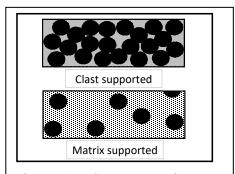


Figure 1. Clast supported versus matrix supported deposits. Clasts are > 2 mm and matrix is < 2 mm.

bedload movement of a clast in the pea gravel range, e.g. with a size of 4 mm moving over a bed of 0.4 mm particles has an enhanced relative mobility as compared to a clast with a size of 40 mm moving over a bed of the same 4 mm particles. With this in mind, we use 2 mm here as shorthand for the narrow pea gravel range of 1-5 mm, over which transport behaviour is distinct from both coarser and finer material. The use of viscosity allows delineation of a generalized dimensionless bed grain size discriminator between "sand-like" and "gravel-like" rivers that is applicable to sediment transport on Titan (ice clasts in flowing methane/ethane liquid at reduced gravity) and Mars (mafic clasts in flowing water at reduced gravity) as well as Earth.





1 Introduction

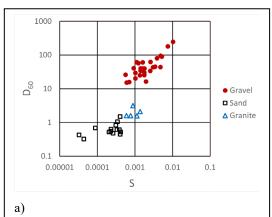
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In rivers, the grain size 2 mm is the conventional divider between sand and gravel. This size has been repeatedly used, explicitly or implicitly, as a discriminator of alluvial rivers and their deposits. For example, conglomerate deposits are often classified as clast-supported (pebble-supported) or matrix-supported, depending on whether clasts with a size in excess of 2 mm are in contact with each other (clast-supported), or whether the clasts are "floating" in a finer deposit (sand or silt: matrix-supported: e.g. Tucker, 2003; Frings, 2011). This classification is illustrated in Figure 1.

Gravel-bed rivers (characteristic bed material size > 2 mm) and sand-bed rivers (characteristic bed material size < 2 mm) have often been treated separately. Communities of researchers have even formed around the distinction. For example, the Gravel-bed Rivers conference series has existed for nearly 40 years since early work summarized in Hey et al. (1985). Note that they specifically name their volume "Gravel-bed Rivers". This distinction has continued through at



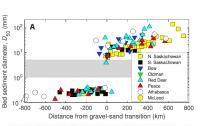


Figure 2. a) Bed surface size D_{60} versus bed slope for Japanese streams (adapted from Fujita et al., 1998): b) bed surface size D_{50} versus distance from gravel-sand transition for Canadian streams (Lamb and Venditti, 2016).

least eight successive conference proceedings (Laronne and Tsutsumi, 2018). Likewise, a large literature has been devoted exclusively to sand-bed rivers: e.g. Wright and Parker (2004), Peng et al. (2022) and Venditti and Bradley (2022). Relations for hydraulic geometry have also been derived separately for gravel-bed rivers (e.g., Parker et al., 2007; Khosravi et al., 2022) and sand-bed rivers (e.g., Xu, 2004; Wilkerson and Parker, 2011).

Many sediment transport relations have been developed exclusively for sand-bed (characteristic bed grain size < 2 mm) or gravel-bed rivers (characteristic bed grain size > 2 mm). For example, the sediment transport relation of Engelund and Hansen (1967) was developed exclusively for sand, based on the flume data of Guy et al. (1966). It was verified by Brownlie (1981), again exclusively for experiments and field data pertaining to the sand-bed case. The bedload transport relation of Meyer-Peter and Muller (1948) was originally developed exclusively using experimental results pertaining to gravel. The bedload transport relation for gravel mixtures due to Parker (1990) specifies that sand should be removed from the grain size distribution of the bed before the transport rate is calculated. The transport relation for sediment mixtures of Wilcock and Crowe (2003) changes the mobility of gravel (grain size D > 2mm) depending upon the content of sand (D < 2 mm) in the bed surface material. (The bedload transport relation of Ashida and



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Michiue, 1972, however, does include data ranging from 0.3 mm to 7 mm. i.e., across the 2 mm size.) Experimental studies of sediment transport using size mixtures often have the size 2 mm built into experimental design; such that sediment fractions finer and coarser than this divider are allowed to interact with each other. Such studies include Hill et al. (2016) and Dingle et al (2021).

Here we pose the following questions. Is the 2 mm divider a social construct based on decades of repetition, convergence and rearticulation (Butler, 1997), or does it have a physical basis? Why specifically 2 mm, and not 0.6 mm or 13 mm? And if the size 2 mm has a physical basis, can it be interpreted in a universal, dimensionless way?

2 Empirical evidence for 2 mm as a discriminator in alluvial rivers

There are two lines of evidence that 2 mm, or more specifically the relatively narrow range of 1 to 5 mm, plays a special role in terms of sediment transport and river morphodynamics. Different authors define this range somewhat differently: e.g., Church and Hassan (2023) use 1 - 10 mm, whereas here we follow the lead of Lamb and Venditti (2016) and define it to be 1-5 mm (based on the effect of viscosity described below). The most direct evidence concerns patterns of downstream fining in rivers carrying a mixture of gravel and sand. Many streams show a pattern of downstream fining such that characteristic bed surface material, e.g., median size D₅₀ of the bed surface material, gradually becomes finer downstream until a size somewhat coarser than pea gravel is reached, and then abruptly declines to the range of sand. Any subreach where the D₅₀ is in the pea gravel range is either short (~ 5 widths) or non-existent (e,g, Sambrook Smith and Ferguson, 1995). The first person to document this behaviour was Yatsu (1955), who presented numerous gravel-sand transitions in Japanese rivers. In the Kinu River, for example, characteristic grain size drops from 20 mm to about 1 mm over a short reach. Yatsu (1955) speculates that this might be due to the abrupt shattering of granitic clasts into their component crystals when grain size is abraded to about 20 mm. Kodama (1994) provides some support of this view and emphasizes the role of abrasion. Shaw and Kellerhals (1982), however, document the same gravel-sand transition in rivers where non-crystalline sediments such as limestone dominate. Thus abrasion may not play a dominant role in the formation of abrupt gravel-sand transitions. This is further supported by observations in rivers with sharp transitions which have clasts that are highly resistant to abrasion (e.g., Ferguson et al., 1996; Venditti and Church 2014; see review in Dingle et al., 2021).

Both Fujita et al. (1996) and Lamb and Venditti (2016) use large data sets to illustrate that a substantial number of river reaches have coarse gravel beds (bed surface D_{50} or $D_{60} > 5$ mm) and sand beds (bed surface D_{50} or $D_{60} < 1$ mm), but very few reaches have a characteristic size in the pea gravel range (Figure 2). Both sets of authors cast this in the context of downstream fining and gravel-sand transitions. Fujita et al. (1996) in particular note that at least in Japan, the relatively few reaches with a characteristic bed size in the pea gravel pertain to streams with heavy loads of sediment derived from weathered granite.



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Gravel-sand transitions need not be abrupt. Especially in rivers sufficiently wide to develop bedform- and planform-driven variation in local flow conditions (e.g. a bar field), the transition may rather disperse and elongated. Seal and Paola (1995) have shown how gravel-sand patchiness can drive such a transition. Frings (2011) shows this for the case the Rhine River, western Europe, where the tendency for elongation of the transition region might be affected by anthropogenic effects. Dong et al. (2016) show this for the case of the Selenga River, Siberia, Russia, where anthropogenic effects are negligible. The transitional reaches in question often do not show substantial subreaches where pea gravel is the characteristic bed material size. Instead that characteristic size is gravel in excess of 10 mm upstream, is below 1 mm downstream, and locally interleaves between coarser gravel and sand in the transitional region, with relatively few locations with a characteristic bed surface size in the pea gravel range. Venditti et al (2015), for example, describe the Fraser River, British Columbia as "an archetypical abrupt gravel-sand transition with a 'diffuse extension' composed of a sand bed with some patches of gravel." Dingle et al. (2021) provide a thorough review of the gravel-sand transition and grain size gap. This issue is considered in more detail in the Discussion below. Of relevance to the analysis here is the fact that both Lamb and Venditti (2016) and Dingle et al (2021) suggest a role for viscosity in regard to the grain size gap.

A second line of evidence derives from experiments with mixtures of gravel and sand. Prominent among them is the work of Wilcock and Crowe (2003), who show that in a unimodal mixture of sand and gravel, increasing content of material less than 2 mm in the bed results in an increased transport rate of material greater than 2 mm. Indeed, they modified the basic framework of the Parker (1990) relation to specifically account for this effect, which Lin et al. (under review) have described as Gravel Transport Augmenting Sand (GTAS). The results of Wilcock and Crowe (2003) have been verified by others (e.g. Cui et al, 2003a, 2003b; Dingle and Venditti, 2023). They break, however, the completely dimensionless format of Parker et al (1990) by introducing a parameter with dimensions. We show below how this problem can be overcome.

Dingle and Venditti (2023) performed flume experiments using a bimodal mix of pea gravel (\sim 3.8 mm) and sand (\sim 0.57 mm) to show that the addition of sand strongly mobilizes the pea gravel. They suggest that adding sand to a bed consisting of material in the grain size gap produces a hydraulic smoothing effect, resulting in mobilization of material in that gap. This same effect was noted when adding sand (\sim 0.57 mm) onto a broader unimodal gravel distribution (2-22 mm, $D_{50} = 5.5$ mm), where the pea gravel fraction became disproportionately mobile relative to the other gravel fractions. The addition of sand was found to increase gravel mobility up to a sand fraction in the bed of about 0.84, beyond which the gravel clasts tend to get buried. Church and Hassan (2023) show how a continuous, only weakly bimodal mixture of sand and gravel can devolve into a grain size distribution with an autogenically strengthened grain size gap. They note: "Our experiment shows a clear tendency for grains in the range 1–8 mm to outrun both larger and smaller grains in the condition of size-selective transport."



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3 The central problem

So what is so special about the pea gravel range? Consider a thought experiment. Loosely following Dingle and Venditti (2023), we consider Case 1, with 4 mm gravel (in the pea gravel range) moving over a 0.4 mm sand bed (finer than the pea gravel range), and Case 2, where we multiply all the numbers by 10, i.e., 40 mm gravel (coarser than the pea gravel range) moving over a 4 mm gravel bed (in the pea gravel range). Will the finer material of the bed increase the mobility of the coarser, overriding material in Case 2 to the same extent in as Case 1? We have a partial answer to this question. Venditti et al. (2010a,b) studied the case where both sizes are in the gravel range. They found that the mobility of coarse surface layers in gravel-bedded rivers could be enhanced by adding finer gravel as bed load. The degree of enhancement, however, is not nearly as strong as Case 1. Indeed the extra mobility of Case 2 can be explained solely in terms of the hiding functions embedded in the relations of Parker (1990) and Wilcock and Crowe (2003), as elaborated in, for example, Parker and Klingeman (1982) and Parker and Toro-Escobar (2002). These relations describe the relative mobility of different sizes in a mixture of gravel in the active (surface) layer of the bed. Relative mobility is mediated by two effects: a weight effect making coarser particles harder to move, and an exposure (hiding) effect making coarser particles easier to move. The residual of the two effects (weight versus hiding effect) renders finer gravel somewhat more mobile in a sediment mix. Evidently there is something special in regard to pea gravel moving over a sand bed. Here we explore two possibilities, one related to sediment suspension and one related to bedload, with both effects mediated by viscosity.

4 Pea gravel corresponds to the finest sizes that do not readily suspend in alluvial rivers

We revisit the relation of Garcia and Parker (1991) for the entrainment of bed sediment into suspension. Although the formulation includes relations for both uniform sediment and sediment size mixtures, we first consider the case of uniform sediment here. For equilibrium suspensions the relation takes the form

$$c_b = \frac{AZ_u^5}{1 + \frac{A}{0.3}Z_u^5} \quad , \quad Z_u = \frac{u_{*s}}{v_s} \left(D^*\right)^{0.9} \quad , \quad D^* = D\frac{\left(Rg\right)^{1/3}}{v^{2/3}} \tag{1a, b, c}$$

Here c_b is near-bed volume concentration of suspended sediment (evaluated at a point that is 5 percent of water depth above the bed), D = grain size for an equivalent sphere in terms of fall velocity, u_{*s} is bed shear velocity due to skin friction (form drag removed), v_s is particle fall velocity, g = gravitational acceleration, $R = (\rho_s - \rho)/\rho$ is the submerged specific gravity of sediment, where ρ_s = sediment density and ρ = density of the fluid in which the sediment is immersed ($R \sim 1.65$ for quartz in water), v = kinematic viscosity of fluid in which the sediment is immersed, D^* is a dimensionless grain size and A is a dimensionless constant given as

$$A = 1.3 \times 10^{-7}$$
 (1d)



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(The original formulation uses a particle Reynolds number $\mathbf{Re}_p = (RgD)^{1/2}D/\nu$, but is here recast in terms of the relation $D^* = (\mathbf{Re}_p)^{2/3}$ which is a more convenient representation, in so far as that dimensionless grain size D^* is linearly proportional to dimensioned grain size D: van Rijn, 1984).

The Garcia-Parker (1991) relation was developed solely with experimental data on suspensions of quartz sediment in water, using a total of 62 measurements with grain size D varying from 0.093 mm to 0.44 mm, so that D* varies from 2.36 to 11.1 for quartz particles immersed in 20°C water on Earth. The relation was thus not designed to be applied to the suspension of gravel. It is nevertheless illustrative to do so.

In Figure 3a, the predictions of Equations ($1a\sim d$) are shown for the sizes D=0.25 mm and 4.0 mm. Here these are nominal sizes using Earth parameters (g=9.81 m/s², R=1.65 and $v=1x10^{-6}$ m²/s). It can be seen therein that there is little difference for the predictions for near-bed concentration c_b . Yet data for the suspension of any size of gravel seems to be rare in the literature, suggesting that it is suspended only with difficulty in laboratory flumes and typical alluvial river flood flows (e.g. de Leeuw et al., 2020). That is, the Garcia-Parker (1991) relation seems to overestimate the suspension of gravel, a size range which is beyond the grain size range of the experimental data that the relation is based on.

This problem can be explained with the dimensionless fall velocity \mathbf{R}_{f}

$$\mathbf{R}_{f} = \frac{\mathbf{v}_{s}}{\sqrt{\mathsf{RgD}}} \tag{2}$$

The parameter \mathbf{R}_f is related to dimensionless grain size D* according to. e.g., the relation of Dietrich (1982), as shown in Figure 3b (solid blue line). Also plotted on Figure 3b are the experimental data of Garcia and Parker (1991), and the dimensionless sizes $D^* = 25.2$ and 126 (nominal sizes D = 1 mm and 5 mm for quartz particles in water on Earth). The regime to the left of nominal size D = 1 mm is viscous-dependent, i.e., dependent on D^* , the regime to the right of nominal size D = 5 mm is essentially viscous-independent, and the regime of nominal sizes 1 - 5 mm defines a transitional zone. The relation defined by the solid blue line in Figure 3b is in turn derived in part from the drag relation for spheres shown in Figure 3c (e.g. Haljasmaa, 2006), in which $c_D = F_D/[(1/2)\rho\pi(D/2)^2v_s^2]$ is a drag coefficient, where F_D denotes the drag force on a spherical particle and $\mathbf{Re}_{vp} = (v_sD/v)$ is a Reynolds number based on fall velocity. It is again seen that the regime to the left of nominal size D = 1 mm is viscous-dependent, the regime to the right of nominal size D = 5 mm is essentially viscous-independent (inertial regime), and the regime of nominal sizes 1 - 5 mm defines a transitional zone.





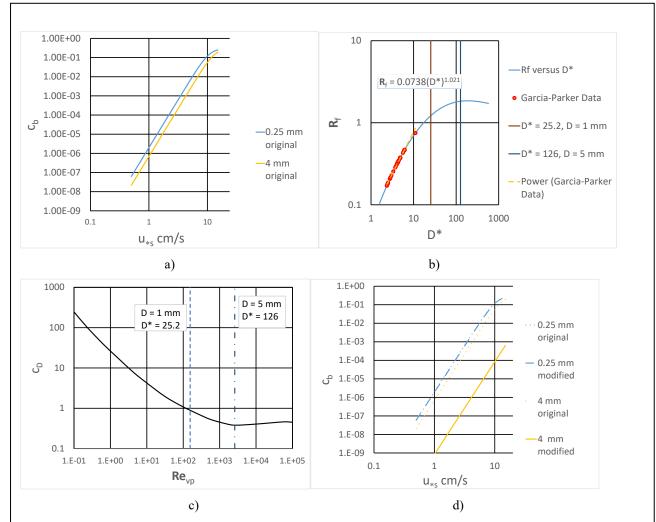


Figure 3. a) Plot of near-bed concentration of suspended load versus shear velocity due to skin friction for D = 0.25 mm and 4 mm using the original relation of Garcia and Parker (1991), i.e. Equations (1a,b,c): b) plot of dimensionless fall velocity $\mathbf{R}_f = v_s/(RgD)^{1/2}$ versus dimensionless grain size D^* using the Dietrich (1982) relation, including a power regression relation for \mathbf{R}_f versus D^* over the original range of the data of Garcia and Parker (1991): c) drag curve for spheres illustrating viscous, transitional and inertial regimes from left to right (adapted from Haljasmaa, 2006); d) version of Figure 3a using corrected relation for Garcia and Parker (Equations 4a,b,c), showing near-collapse of suspension of particles of size 4 mm.



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Figures 3b and 3c illustrate an inadequacy of the entrainment relation of Garcia and Parker (1991). The data used to derive it pertain solely to the viscous region of the relation for fall velocity, so that the relation cannot strictly be extended to coarser sizes. There is, however, a straightforward way to remedy this. Also plotted on Figure 3b (dashed line) is the following regression relation for \mathbf{R}_f versus D^* , fitted specifically over the range of the data used by Garcia and Parker:

$$\mathbf{R}_{f} = 0.0738 \left(D^{*} \right)^{1.021} \tag{3}$$

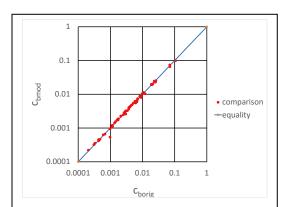


Figure 4. Plot of predicted values of near bed concentrations c_{bmod} using the modified formulation of Garcia and Parker (Equations 4a,b,c), versus those predicted from the original formulation, c_{borig} (Equations 1a,b,c). Calculation is conducted over the range of the Garcia-Parker data.

This relation can be substituted into Equations (1a, b c) to yield a revised relation in which the constant A is unmodified:

$$c_{b} = \frac{AZ_{u}^{5}}{1 + \frac{A}{0.3}Z_{u}^{5}} \quad , \quad Z_{u} = 9.95 \frac{u_{*s}}{v_{s}} R_{f}^{0.882} \quad \ (4a,b)$$

According to Equation (4b) and Figure 3b, the parameter Z_u does not increase without bound as dimensionless grain size D^* increases. When inertial effects dominate, \mathbf{R}_f becomes roughly constant, placing a bound on Z_u and preventing the oversuspension of material in the range of pea gravel and coarser material.

Figure 3d shows that when applied to the original data of Garcia and Parker (1991), i.e. Equations (1a,b,c) the modified formulation of Eqs. (4a,b) does just as well as the original formulation. Figure 3d shows that the modified formulation does not change the relation for c_b versus u_{*s} for nominal 0.25 mm quartz in water (the two lines overlap), but causes such low values

of c_b for nominal 4 mm quartz in water that they are essentially negligible. This negligibility is further reinforced by the Rousean (1939) relation for vertical profile of suspended sediment concentration. This relation contains an exponent proportional to u_*/v_s , where u_* is total bed shear velocity, and so concentration above the bed collapses as v_s/u_* becomes sufficiently large.

Figure 4 verifies that over the range of the Garcia-Parker data, the modified form of Equations (4a,b,c) predicts the data as well as the original form of Equations (1a,b,c).

The same generalization, and in particular Equation (4b), carries over to the relation of Garcia and Parker (1991) for mixtures. In addition, it should carry over in a straightforward way to the relations of Wright and Parker (2004), which encompasses a much wider range of conditions, and in particular larger streams with lower slopes, than Garcia and Parker (1991). The modified form for the entrainment relation of Wright and Parker for sediment mixture (2004) thus takes the following form for equilibrium suspensions. Where D_{50} denotes the median size of the bed material, σ_{ϕ} denotes



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the standard deviation of bed material on the ϕ scale, i denotes the ith grain size range, F_i denotes the fractional content of sediment in the bed in the ith size range and D_i , $\mathbf{R}_{f,i}$, $v_{s,i}$, Z_{ui} and c_{bi} denote grain-size specific values of D, \mathbf{R}_f , v_s , Z_u and c_b ,

$$\begin{split} \frac{c_{b,i}}{F_i} &= \frac{AZ_{ui}^5}{1 + \frac{A}{0.3}Z_{ui}^5} \quad , \quad Z_{ui} = 9.95\,\lambda_m \frac{u_{*s}}{v_{s,i}} \boldsymbol{R}_{f,i}^{0.882} \bigg(\frac{D_i}{D_{50}}\bigg)^{0.2}\,S^{0.08} \quad , \quad \boldsymbol{R}_{f,i} = \frac{v_{s,i}}{\sqrt{RgD_i}} \\ \lambda_m &= 1 - 0.298\sigma_{_b} \quad , \quad A = 7.8x10^{-7} \end{split} \label{eq:continuous} \tag{5a,b,c,d,e}$$

In the above relations, S denotes streamwise slope (of the bed or energy grade line) and σ_{ϕ} denotes the standard deviation of bed sediment on the sedimentological phi scale. The above formula has not, however, been tested for gravel-sand mixtures, and likely does not accurately predict the concentration of sand over a much coarser gravel bed.

The tendency for suspension to collapse as grain size increases across the pea gravel range can be confirmed in terms of the more recent relation of de Leeuw et al. (2020). Although several relations are presented therein, the one most directly comparable with the above formulation can be expressed in the following form for uniform material:

$$c_{b} = \begin{cases} \frac{4.74 \times 10^{-0.4} \ X^{1.18}}{1 + 3 \left(4.74 \times 10^{-0.4} \ X^{1.18} \right)} &, & X > 0 \\ 0 &, & X \leq 0 \end{cases} , \quad X = \left(\frac{u_{*s}}{v_{s}} \right)^{1.5} \text{Fr} - 0.015$$
 (6a,b)

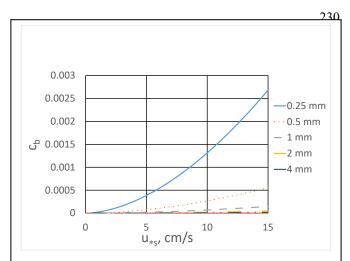


Figure 5. Near-bed concentration c_b versus shear velocity associated with skin friction from Equation (6a,b) due to de Leeuw et al. (2016). The grain sizes 0.25 mm, 0.5 mm, 1 mm, 2 mm and 4 mm are shown. Froude number **Fr** is set to 0.4.

In the above relation, the Froude number $\mathbf{Fr} = U/(gH)^{1/2}$, where U is depth-averaged flow velocity and H is depth. A plot of c_b versus u_{*s} for the grain sizes D=0.25 mm, 0.5 mm, 1 mm, 2 mm and 4 mm (quartz in water at 20°C) and Froude number $\mathbf{Fr} = 0.4$ is given in Figure 5. It is again seen that suspension tends to collapse as grain size enters the pea gravel range. Similar results are obtained for $\mathbf{Fr} = 0.2$ and 0.6.

The range of the parameter u_{s} used in Figures 3a, 3d and 5 is 0 to 15 cm/s. The higher value of these can be used as a conservative estimate for the total shear velocity u_{s} , and thus depth-averaged flow velocity U as follows. Figure 7 of Li et al. (2015) allows estimates of dimensionless Chezy friction coefficient $Cz = U/u_{s}$. The data for gravel-bed and sand-bed rivers is bracketed for the most part by the range Cz = 7 to 20. This suggests



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that the modified sediment entrainment relation should be valid for flow velocities up to the range 1-3 m/s, which are reasonable estimates for bankfull velocity in rivers (Parker, 2014; Birch et al., 2023). The implication is that material in the nominal size range ≥ 1 mm is not subject to significant suspension in typical flood flows of alluvial rivers.

The results presented above do not imply that it is physically impossible to suspend gravel. For example, Larsen and Lamb (2016) infer that gravel could be suspended by the megafloods that sculpted the Channeled Scablands, USA. Recently Lin et al. (2022) and Song et al. (2022) have modelled sediment transport in the aftermath of a breach of a high landslide dam in the Himalaya Mountains. Under such conditions, shear velocity was predicted to reach as high as 2 m/s, and mean flow velocity was predicted to reach as high as 10 m/s. Neglecting form drag for the moment, these values applied to the modified Garcia-Parker relation presented here with a grain size of 4 mm yields a near-bed concentration taking the maximum possible value of 0.3. This is consistent not only with suspension, but also with the formation of a thick grain flow that can be considered transitional to a debris flow (e.g. Hernandez-Moreira et al., 2019).

5 Pea gravel is preferentially transported as bedload over sand

The above analysis provides evidence that pea gravel represents the finest range of approximately spherical gravel that cannot easily be suspended by typical alluvial river flood flows (as opposed to megafloods or flows in steep bedrock streams). The bedload transport rate of a given size D tends to be augmented when it moves over a bed of finer material, as compared to a bed of the same size D. This effect, however, is dependent on absolute size, as well as relative size. Here we consider an example using 4 mm as a characteristic size within the pea gravel range. We argue that the bedload transport rate of a grain with size 4 mm moving over a bed of 0.4 mm sand (as opposed to a bed of the same 4 mm material) is augmented to a considerably higher extent than the size 40 mm moving over a bed of 4 mm material (as opposed to a bed of the same 40 mm material (Wilcock and Crowe, 2005; Venditti et al. 2010). Wilcock and Crowe (2003) identify 2 mm as a threshold, such that increasing content of material finer than this significantly augments the transport of material coarser than this (GTAS effect).

The problem can again be viewed in the context of viscosity. There is abundant evidence that changes in viscosity, e.g. through temperature change, can significantly affect both the transport rate and bedforms in sand-bed streams (e.g. Chen and Nordin, 1976; Southard and Boguchwal, 1990). Simons and Richardson (1961) have modified a dimensionless bedform regime diagram proposed by Liu (1957), which indicates that the effect of viscosity on bedform regime becomes negligible as particle grain size passes through the range 1.71 - 5 mm (quartz particles on Earth in water at 20°C). Indeed, the effect of viscosity is embedded in the modified Shields relation for the threshold of (significant) bedload transport presented in Garcia (2006). Where τ_c^* denotes a critical Shields number, the threshold condition can be represented as



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$$\tau_c^* = \frac{u_{*c}^2}{\text{RaD}} = 0.5[0.22 \left(D^*\right)^{-0.9} + 0.06 \cdot 10^{(-7.7(D^*)^{-0.9})}] \tag{7}$$

Here u_{*c} is the shear velocity at the threshold of (significant) motion. Viscosity enters the problem via the definition of dimensionless grain size D*. For uniform material over a bed of the same size, assuming quartz and water at 20°C, the values of τ_c * are 0.029 for 40 mm material, 0.024 for 4 mm material and 0.017 for 0.4 mm material. Clearly particles become easier to move as grain size reduces across the pea gravel range. That is, viscosity lubricates a bed that is sufficiently fine.

But the more important effect concerns how a grain of a given size moves over a bed of finer sizes. Turbulent flows over a granular bed are traditionally divided into a turbulent smooth regime, a turbulent rough regime and a transitional regime (e.g. Streeter, 1975). Julien and Bounvilay (2013) show data indicating that coarse particles moving over a hydraulically smooth bed do not consistently travel at higher velocities than those traveling over a hydraulic rough bed. They are, however, be entrained more easily. Novak and Nalluri (1975) provide convincing evidence that the threshold Shields number for motion of a given grain size is substantially reduced when that particle moves over a hydraulically smooth bed. Viscous sublayer thickness of a turbulent boundary layer can be scaled as (e.g., Garcia, 2006)

$$\delta_{v} = 11.6 \frac{v}{u_{*s}} \tag{8}$$

where δ_v = nominal thickness of the viscous sublayer. (Here "nominal viscous sublayer thickness, i.e. estimate, is used in a different sense from "nominal grain size", the latter of which corresponds to Earth-like conditions.) Now let u_{*s} = the shear velocity at the threshold of motion u_{*c} . Between Equations (7) and (8), the ratio $(\delta_v/D)_c$, i.e. the value of δ_v/D at the threshold of motion. is found to be:

$$\left(\frac{\delta_{v}}{D}\right)_{c} = \frac{11.6}{(D^{*})^{3/2}f(D^{*})} , f(D^{*}) = \sqrt{0.5[0.22(D^{*})^{-0.9} + 0.06 \cdot 10^{(-7.7(D^{*})^{-0.9})}]}$$
(9)

This relation is plotted in Figure 6. Consider first a nominal 40 mm grain moving over a bed of nominal 4 mm material. The value of $(\delta_v/D)_c$ for either grain size is no larger than 0.074, indicating a turbulent rough bed with no role for viscosity. In the case of a 4 mm grain moving over a 0.4 mm bed, $(\delta_v/D)_c$ of the bed is 2.81. This indicates that the 4 mm grain moves over a bed that is transitional to turbulent smooth, and thus is subject to the increase in mobility via lowered threshold Shields number demonstrated by Novak and Nalluri (1975).

The tendencies shown here are corroborated by the results of Dingle and Venditti (2023), who show, for example, enhanced mobility of clasts of size 3.8 mm as sand of size 0.57 mm occupies an increasing fractional content in the bed surface layer. It should be noted, however, that were bedforms to be present, they would tend to break up the effect of viscosity, as they set an internally-generated roughness (e.g. Lapôtre et al., 2017).





6 "Sand-bed-like" versus "gravel-bed-like" rivers; generalization for Earth, Titan and Mars

For reference, we here repeat the definition of D^* given in Equation (1c):

$$D^* = D \frac{(Rg)^{1/3}}{v^{2/3}}$$
 (1c)

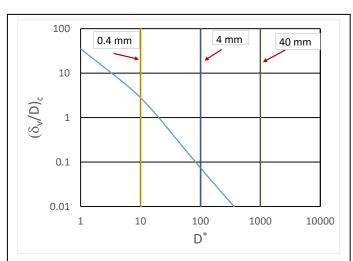


Figure 6. The ratio of nominal thickness of viscous sublayer to grain size at the threshold of motion $(\delta_v/D)_c$ as a function of dimensionless grain size D^* is denoted by the light blue, monotonically decreasing line. The three solid vertical lines denote, from left to right, $D^* = 1.01 \times 10^1$ (nominal size D = 0.4 mm), $D^* = 1.01 \times 10^2$ (nominal size D = 4 mm) and $D^* = 1.01 \times 10^3$ (nominal size D = 40 mm). The grain sizes D are nominal values for Earth-like conditions.

All specific evaluations of D* given above have been for a natural (e.g. granitic or limestone) particle (e.g. R = 1.65 for quartz) on Earth ($g = 9.81 \text{ m/s}^2$) in water at 20°C $(v = 1.00 \times 10^{-6} \text{ m}^2/\text{s})$. These same arguments apply to limestone particles with only slight modification; here we group these together as "natural particles on Earth." Based on the range 1 - 5 mm, we can loosely divide river reaches into "sand-bed-like" or "gravel-bed-like" depending on whether or not characteristic bed surface size (e.g. D₅₀) is less than or greater than 2 mm. For the above conditions, the size D = 2 mm yields a value of D^* = 51. When using this size as an approximate boundary between "sand-like" and "gravel-like" behavior, however, it must be borne mind that the relevant parameter is the dimensionless one. Even on Earth, the kinematic viscosity of water can vary from a low of 2.94x10⁻⁷ m²/s at 100°C to a high of 1.79x10⁻⁶ m²/s. In addition, Viparelli et al. (2015) have documented the mobility of sediment with submerged specific gravities R ranging from 0.5 to 3. A discriminating value $D^* = 51$ thus corresponds to a range of sizes D from as low as 0.73 mm to as high as 4.42 mm.

We are now able to cast the bedload transport relation of Wilcock and Crowe (2003) in purely dimensionless form. Generalizing from their 2 mm criterion, as the content of grains in the bed with dimensionless size $< D^* = 51$ is increased, the transport of grains with dimensionless size $> D^* = 51$ is enhanced. The formulation is now directly applicable to rivers on Mars and Titan as well as Earth.

Birch et al. (2023) have illustrated how the dimensionless number D^* transforms into dimensioned numbers for Mars, where gravitational acceleration is significantly lower, and Titan, where gravitational acceleration is even lower, the clasts are ice rather than quartz and the fluid is a mix of methane and ethane. They show that the discriminator $D^* = 51$





translates to about 2.66 mm on Mars (mafic sediment in water at 20°C), and 3.16- 4.42 mm on Titan, (ice particles in liquid methane/ethane at 84 - 96°K. Lamb and Venditti (2016) have performed a similar calculation.

We emphasize here that the nominal dimensioned size 2 mm is used as shorthand for the narrow range 1-5 mm corresponding to natural particles (e.g. granitic or limestone) on Earth in water at 20°C. The corresponding dimensionless range for D*, which we argue to be more universal, is 25-126.

340 7 Discussion

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We do not present the above analysis in the context of a specific morphodynamic model. Instead, the analysis bears on the physics underlying what Church and Hassan (2023) describe as a "clear tendency for grains in the range 1–8 mm to outrun both larger and smaller grains in the condition of size-selective transport" via bedload (rather than suspended load) transport. This tendency can in turn be related to the evolution of the pea-gravel grain size gap in the bed surface layer of a long profile of a net-depositional river, as expressed in terms of the bed material sizes D₅₀ and D₆₀ shown in Figure 2. Pea gravel is not easily suspended, but can be preferentially moved as bedload over a coarser bed (weight versus hiding effect) as well as a finer bed (hydrodynamic smoothing effect). The implication is that even if there is no grain size gap in the feed sediment, the pea gravel tends to become diluted over a long reach.

Consider the following thought experiment. We assume a long river reach undergoing deposition. The grain size distribution of the deposit contains three size ranges: a "sand" range, a "pea gravel range" and a "coarse gravel range". We further specify that the fraction of material in each of the three ranges is equal: 1/3 "sand", 1/3 "pea gravel" and 1/3 "coarse gravel". We divide the reach into upstream and downstream segments of equal length. Let the upstream deposit be 2/3 "coarse gravel" and 1/3 "pea gravel", and the downstream deposit be 2/3 "sand" and 1/3 "pea gravel". The total amount of the deposit in each size range is equal. Yet the median size D₅₀ of the deposit must abruptly drop from the "coarse gravel" size to the "sand" size halfway down the reach. No paucity of pea gravel is necessary for such behaviour. Instead, the pea gravel is diluted due to its preferential mobility as bedload compared to coarser and finer sediment. A first attempt to incorporate the above ideas into a morphodynamic model is given in An et al. (2020).

In the analysis above, grain size D is interpreted as an equivalent diameter of a sphere. The analysis would require modification for grain shapes that deviate significantly from spherical. The fall velocity relation of Dietrich (1982) includes a correction factor for grain shape. Particles with a plate-like shape may be significantly easier to suspend than spheres. The arguments above do not rely on the assumption of grain abrasion. Abrasion may, however, play a role in the evolution of some sharp gravel-sand transitions.

The mobilization effect observed when sand is added to a gravel bed, without major change in bed slope, has been verified experimentally by Cui et al. (2003a) and Dingle and Venditti (2023). Lamb et al. (2008) indicate that the critical Shields number of sediment increases with bed slope. In the original experiments of Wilcock and Crowe (2003), where the flow was allowed to reach mobile-bed equilibrium, a higher sand content correlated with a lower bed slope. The





evolution of this lower slope may also be combined with the tendency for critical Shields stress to be slope-dependent. To date, however, this slope effect has not yet been incorporated into any sediment transport relation for mixtures.

7 Conclusions

The analysis presented here does not specifically identify the size D = 2 mm itself as special. Instead, it serves as shorthand for the dimensionless size D* = 51, and the dimensionless range D* = 25.2 - 126, corresponding to nominal size range of pea gravel ranging from 1 – 5 mm (e.g. granitic or limestone particles on Earth in water at 20°C). We show that this range corresponds to the finest sizes that cannot be significantly transported in suspension in typical floods (~ bankfull flow) of alluvial rivers. In addition, we show that pea gravel is preferentially moved as bedload, both over a coarser gravel bed and a sand bed (at least in the absence of bedforms). The physics of the problem are embodied in the dimensionless grain size D*, which contains kinematic viscosity. These conclusions have bearing on the formation of gravel-sand transitions, because they imply that even in the absence of abrasion or a grain size gap in the feed sediment, pea gravel is subject to dilution within any long depositional reach along which downstream fining is observed. That is, pea gravel intrinsically "has trouble finding a home" where it can dominate in the sediment deposit.

The formulation is directly applicable to Mars, where gravitational acceleration is lower than Earth, and Titan, where the gravitational acceleration is even lower, the particles in transport are ice rate than e.g., quartz, and the fluid is mixture of methane and ethane rather than water.

Table A1. Notation

- 385 A Constant in Equation (1a) or Equation (5e) [1]
 - c_b near-bed equilibrium volume concentration of suspended sediment [1]
 - c_{b,i} near-bed equilibrium volume concentration of suspended sediment in the ith grain size range [1]
 - c_D drag coefficient on a sphere [1]
 - D grain size [L]
- 390 D₅₀ median grain size of bed surface [L]
 - D_{60} grain size such that 60 percent of bed surface material is finer [L]
 - D_i characteristic size of the ith grain size range [L]
 - $D^* = (Rg)^{1/3}D/v^{2/3}$; dimensionless grain size [1]
 - F_i fraction of material in the surface layer in the ith grain size range [1]
- 395 **Fr** = $U/(gH)^{1/2}$, Froude number [1]
 - g acceleration of gravity [L/T-2]

flow depth [L]



Η



R = $(\rho_s - \rho)/\rho$; submerged specific gravity of sediment [1] $Re_{\rm vp}$ $=v_sD/v[1]$ 400 dimensionless fall velocity of sediment; dimensionless fall velocity of sediment in the ith grain size range [LT⁻¹] $\mathbf{R}_{\mathrm{f}}, \mathbf{R}_{\mathrm{f},\mathrm{i}}$ U depth-averaged flow velocity [LT⁻¹] shear velocity [LT-1] u_* shear velocity at threshold of motion [LT⁻¹] u_{*c} shear velocity due to skin friction [LT-1] $u_{\ast_{S}}$ 405 S streamwise slope (of bed or energy grade line) [1] fall velocity of sediment; fall velocity of sediment in the ith grain size range [LT⁻¹] $V_s, V_{s,i}$ X parameter defined by Equation (6b) [1] Z_{n} parameter defined by Equation (1b), Equation (4b) [1] $Z_{u,i}$ parameter defined by Equation (5b) [1] nominal thickness of viscous sublayer [L] 410 $\delta_{\rm v}$ parameter defined by Equation (5d) [1] $\lambda_{\rm m}$ kinematic viscosity of fluid (water or methane-ethane mixture) [L²T⁻¹] ν density of fluid (water or methane-ethane mixture) [ML⁻³] ρ density of sediment (quartz, limestone, ice particles etc.) [ML⁻³] ρ_s

Competing interests

The contact author has declared that none of the authors has any competing interests.

critical Shields number for the onset of significant sediment transport [1]

standard deviation on the ϕ scale of bed surface sediment [1]

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 Q^{ϕ}

 ${\tau_c}^*$





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