Edits on Equations 1-6

December 29, 2023

The equations that appear in version 3 of the manuscript:

$$\delta x(\lambda,\phi,z,t) \;=\; -\,\gamma(\lambda,\phi,z,t)\,(x(\lambda,\phi,z,t) \;-\; x_{ref}(\lambda,\phi,z,t))\,/\tau \quad , \tag{1}$$

$$\gamma(\phi,\lambda) \;=\; f(\phi,\phi_1,\phi_2)\; f(\lambda,\lambda_1,\lambda_2) \quad , \tag{2}$$

$$f(\phi,\phi_1,\phi_2) \;=` \left[1/(1+e^{-(\phi-\phi_1)/\delta_1}\right] \;\left[1/(1+e^{-(\phi-\phi_2)/\delta_2}\right] \tag{3}$$

$$f(\lambda,\lambda_1,\lambda_2) \;=` \left[1/(1+e^{-(\lambda-\lambda_1)/\delta_1}\right] \;\left[1/(1+e^{-(\lambda-\lambda_2)/\delta_2}\right] \tag{4}$$

$$f(z) \;=\; a.\; exp(bx) \tag{5}$$

$$f(t) \;=\; \left(\frac{1}{exp\left(-0.5\left(\frac{d^2}{\beta^2}\right)\right)^{2\mu}}\right) \tag{6}$$

The problems with Eqns 1-4 are fixed by writing:

$$\delta x(\lambda,\phi,z,t) \;=\; -\,\gamma(\lambda,\phi)\,g(z)\,h(t)\,(x(\lambda,\phi,z,t) \;-\; x_{ref}(\lambda,\phi,z,t))\,/\tau \quad , \tag{1a}$$

$$\gamma(\lambda,\phi) \;=\; f_1(\phi,\phi_1,\phi_2)\; f_2(\lambda,\lambda_1,\lambda_2) \quad , \tag{2a}$$

$$f_1(\phi,\phi_1,\phi_2) \;=` \left[1/(1+e^{-(\phi-\phi_1)/\delta_1}\right] \;\left[1/(1+e^{-(\phi-\phi_2)/\delta_2}\right] \tag{3a}$$

$$f_2(\lambda,\lambda_1,\lambda_2) \;=` \left[1/(1+e^{-(\lambda-\lambda_1)/\delta_1}\right] \;\left[1/(1+e^{-(\lambda-\lambda_2)/\delta_2}\right] \tag{4a}$$

Eqn 5 doesn't align with Fig. S2: if z is height above the surface (standard notation), then f goes to infinity as you go upward. Is this what you mean to write here?

$$g(z) \;=\; a\, exp(-b\, z) \tag{5a}$$

Note, the middle panel in Fig. S2 does not fit either description (the curve should go exponentially to 100% at "model level" = 1, but the figure displays a kink).

Eqn 6 doesn't align with Fig. S2 (as you go far from Jan 15, the denominator goes to zero and f goes to infinity. d appears to have units of month, but this isn't mentioned in the text. A more precise formulation would be

$$h(t) \;=\; exp\left(-d^2/\left(2\beta^2\right)^{2\mu}\right) \tag{6a}$$

where d is the time difference relative to maximum nudging time in months (e.g., $d = 0$ on Jan 15, $d = -1$ on Dec 15, etc). Outside of the nudging window, $h = 0$.

Additional issues with these equations:

- Eqns 3 and 4 don't seem to align with the mask shown in Fig. S2. Why are there two nodal points in latitude (λ_1, λ_2) and longitude (ϕ_1, ϕ_2), and what are their values? Also, f_1 and f_2 do not go to zero as you go far from the center of the patch. It seems like these equations should read as follows: "Within the nudging patch centered at λ_1, ϕ_1,

  $$f_1(\phi, \phi_1) \;=\; exp\left(-((\phi - \phi_1)/\delta_1)^2\right) \tag{3a}$$

  and

  $$f_2(\lambda, \lambda_1) \;=\; exp\left(-((\lambda - \lambda_1)/\delta_2)^2\right) \quad . \tag{4a}$$

  and outside of the patch, $f_1 = f_2 = 0$. " Note that I am assuming you used a smooth function around the center of the patch (a Gaussian). If instead, you used the exponential (as suggested by Eqn. 3), $((\phi - \phi_1)/\delta_1)^2$ would be replaced with $|\phi - \phi_1|/\delta_1$.

- In Eqns 3 and 4, δ_1 and δ_2 are not defined in the text).

- In Eqn 5, x and b are not defined in the text (also, presumably x should be z).

- The mathematical expressions on lines 133-134 appear to have been scrambled when the text was converted to the pdf.