Edits on Equations 1-6

December 29, 2023

The equations that appear in version 3 of the manuscript:

\[ \delta x(\lambda, \phi, z, t) = -\gamma(\lambda, \phi, z, t) \left( x(\lambda, \phi, z, t) - x_{ref}(\lambda, \phi, z, t) \right)/\tau, \quad (1) \]

\[ \gamma(\phi, \lambda) = f(\phi, \phi_1, \phi_2) f(\lambda, \lambda_1, \lambda_2), \quad (2) \]

\[ f(\phi, \phi_1, \phi_2) = \left[ \frac{1}{1 + e^{-(\phi - \phi_1)/\delta_1}} \right] \left[ \frac{1}{1 + e^{-(\phi - \phi_2)/\delta_2}} \right], \quad (3) \]

\[ f(\lambda, \lambda_1, \lambda_2) = \left[ \frac{1}{1 + e^{-(\lambda - \lambda_1)/\delta_1}} \right] \left[ \frac{1}{1 + e^{-(\lambda - \lambda_2)/\delta_2}} \right], \quad (4) \]

\[ f(z) = a \cdot \exp(bx) \quad (5) \]

\[ f(t) = \left( \frac{1}{\exp \left( -0.5 \left( \frac{a^2}{\beta^2} \right)^{2\mu} \right)} \right) \quad (6) \]

The problems with Eqns 1-4 are fixed by writing:

\[ \delta x(\lambda, \phi, z, t) = -\gamma(\lambda, \phi) g(z) h(t) \left( x(\lambda, \phi, z, t) - x_{ref}(\lambda, \phi, z, t) \right)/\tau, \quad (1a) \]

\[ \gamma(\lambda, \phi) = f_1(\phi, \phi_1, \phi_2) f_2(\lambda, \lambda_1, \lambda_2), \quad (2a) \]

\[ f_1(\phi, \phi_1, \phi_2) = \left[ \frac{1}{1 + e^{-(\phi - \phi_1)/\delta_1}} \right] \left[ \frac{1}{1 + e^{-(\phi - \phi_2)/\delta_2}} \right], \quad (3a) \]

\[ f_2(\lambda, \lambda_1, \lambda_2) = \left[ \frac{1}{1 + e^{-(\lambda - \lambda_1)/\delta_1}} \right] \left[ \frac{1}{1 + e^{-(\lambda - \lambda_2)/\delta_2}} \right], \quad (4a) \]
Eqn 5 doesn’t align with Fig. S2: if \( z \) is height above the surface (standard notation), then \( f \) goes to infinity as you go upward. Is this what you mean to write here?

\[
g(z) = a \exp(-b z) \tag{5a}
\]

Note, the middle panel in Fig. S2 does not fit either description (the curve should go exponentially to 100\% at “model level” = 1, but the figure displays a kink).

Eqn 6 doesn’t align with Fig. S2 (as you go far from Jan 15, the denominator goes to zero and \( f \) goes to infinity. \( d \) appears to have units of month, but this isn’t mentioned in the text. A more precise formulation would be

\[
h(t) = \exp\left(-d^2/\left(2\beta^2\right)^2\mu\right) \tag{6a}
\]

where \( d \) is the time difference relative to maximum nudging time in months (e.g., \( d = 0 \) on Jan 15, \( d = -1 \) on Dec 15, etc). Outside of the nudging window, \( h = 0 \).

**Additional issues with these equations:**

- Eqns 3 and 4 don’t seem to align with the mask shown in Fig. S2. Why are there two nodal points in latitude \((\lambda_1, \lambda_2)\) and longitude \((\phi_1, \phi_2)\), and what are their values? Also, \( f_1 \) and \( f_2 \) do not go to zero as you go far from the center of the patch. It seems like these equations should read as follows: ”Within the nudging patch centered at \( \lambda_1, \phi_1 \),

\[
f_1(\phi, \phi_1) = \exp\left(-((\phi - \phi_1)/\delta_1)^2\right) \tag{3a}
\]

and

\[
f_2(\lambda, \lambda_1) = \exp\left(-((\lambda - \lambda_1)/\delta_2)^2\right) \tag{4a}
\]

and outside of the patch, \( f_1 = f_2 = 0 \). ” Note that I am assuming you used a smooth function around the center of the patch (a Gaussian). If instead, you used the exponential (as suggested by Eqn. 3), \(((\phi - \phi_1)/\delta_1)^2\) would be replaced with \(|\phi - \phi_1|/\delta_1\).

- In Eqns 3 and 4, \( \delta_1 \) and \( \delta_2 \) are not defined in the text).

- In Eqn 5, \( x \) and \( b \) are not defined in the text (also, presumably \( x \) should be \( z \)).

- The mathematical expressions on lines 133-134 appear to have been scrambled when the text was converted to the pdf.