Response to Reviewer N.2

This study combines a quantitative assessment of the main contributors in the tendency equation of isentropic slopes with a phase space analysis that is able to shed light on the joint temporal evolution of these factors. While both methods have been applied before, there combination yields novel insights into the temporal behavior of storm tracks, making this paper a valuable contribution to the literature that also fits well in the scope of Weather and Climate Dynamics. However, I think that there are still some weaknesses in the presentation of the results that I'd ask the authors to address before I can recommend the paper for publication. On the one hand, these are related to a somewhat superficial description of details in the figures, leading to a few unclear points listed below. On the other hand, at some places (in my view in particular in section 5), the general conclusions obtained from the analyses are not articulated clearly enough, as also noted by the other reviewer.

We thank the Reviewer for their thorough review of our paper. We hope to have addressed the concerns raised, which also helped us improve the manuscript. Here we provide a line-by-line response.

Specific comments

1) **Line 58: Is there a reason why you still use the old ERA-Interim reanalysis instead of ERA5, which has been out for a few years now?**

   We used ERA-Interim, as these were readily available to us, in particular the diabatic tendency of the slope. To calculate the diabatic tendency we need diabatic tendencies of potential temperature on pressure levels, which the ECMWF does not provide directly but only on model levels. The process of requesting and converting the necessary data is quite slow (in the end it took 8 months), so we used ERA-Interim in the meantime. As synoptic-scale dynamics are unlikely to change significantly between the two datasets, we do not expect significant differences in our results. Future work will be based on ERA5 and preliminary results confirm the consistency with this study.

2) **Equation 2, line 85: This is more a conceptual (and also minor) point, but is it really the material derivative of S that you're aiming at, or rather the local tendency at a specific grid point (e.g., in your composite analysis)? Of course, this would not change your analysis at all, but it would rather mean that you neglect the ADV term in equation 16 of Papritz and Spengler (2015) than the IADV term in your equation 2, right?**

   To compute the slope and its tendencies on pressure levels we made use of the Eulerian form of the tendency equation (namely, Equation 16 in Papritz and Spengler, 2015), where the advection term also includes advection of slope by the flow. As the Reviewer pointed out already, TILT and DIAB do not change between the Lagrangian and Eulerian formulations so our analysis is not affected. We do, however, agree with the Reviewer, that conceptually we are more interested in the local slope tendency and so it is more appropriate to show its Eulerian formulation. We have thus changed Equation 2 accordingly.

3) **L 90: "over the western boundary currents": This is true for the Pacific, but not for the Atlantic**

   A previous version of Figure 1 showed stippling for masking occurring more that 15% of the time, which would make the Gulf Stream also pop up, hence our comment on Line 90. We have rephrased the text as follows to make the description more consistent; "The largest amount of masking over the oceans occurs over the Kuroshio-Oyashio current, south of Greenland, and near the interface between the Atlantic and Arctic oceans (Fig. 1), while elsewhere it does not affect more than 10-15% of the time steps considered."

4) **L 105-106: As the regions of strong surface heat fluxes and high occurrence of CAOs are almost identical, it seems a bit arbitrary/suggestive to associate one with DIAB and the other one with TILT. I'd suggest to change the wording a bit to leave this more open.**

   We agree with the Reviewer that regions of strong heat fluxes and most frequent CAOs do substantially overlap. To avoid any premature speculation at this stage, we have rephrased it as follows: "Both DIAB and TILT are most intense along SST gradients associated with western boundary currents (7–8 m/km day^{-1} for DIAB, 15–17 m/km day^{-1} for TILT). The same areas of strongest DIAB and TILT also coincide with strong surface heat fluxes (Fig. 1c) and a higher occurrence of CAOs (Fig. 1e)."

5) L 118: "despite weaker slope": Is it really weaker in the Atlantic?

The original description was somewhat unclear so we have rephrased it as follows: "The four domains (Fig. 1a, Table 1) represent the upstream (GSE, KOE) and downstream (ENA, ENP) sectors of both the North Atlantic and North Pacific storm tracks. In the upstream regions, the slope features peak intensity in correspondence with strong SST gradients. Over the downstream regions, the slope is more evenly distributed spatially and maxima align with the most intense weather activity (as measured by storm track density, Fig. 1f)."

6) L 145: I'd suggest to use a different symbol for the velocity in phase space, as u already denotes the wind velocity in equation 2, which confused me in the first place.

We have changed **u** on line 145 into **c** to avoid any confusion. We have also rewritten Equation 2,

$$\mathbf{F} = (D\,c_x, D\,c_y) = \Big(-\frac{d\psi}{dy}, \frac{d\psi}{dx}\Big),$$

to include explicitly the phase velocity components c_x and c_y, which become useful for a later discussion on how we estimate the duration of a cycle.

7) L 167: "slightly shorter cycles": This is only true for the western boxes; for the eastern ones, cycles are actually longer in the free troposphere.

As the difference in duration is not further commented upon, we decided to remove this remark and simply state: "We obtain an average duration for one revolution of about 5 days, with values ranging between 4.3–5.8d (see Table 2)"

8) **L 172: "increases both with DIAB and TILT": As it is larger for more negative TILT, this statement is technically not correct.**

L 174: "peaks in the lower-left quadrant": I can see this only for ENP.

In light of both comments, we have rephrased the text as follows: "The mean isentropic slope in the near-surface troposphere over the GSE region increases both with DIAB and TILT, reaching maximum values around one standard deviation above its time-mean in the lower-right quadrant of the phase space (Fig. 3a), while it increases primarily with TILT and peaks in the lower quadrant in the other regions (Fig. 3b-d)."

9) **L 189-190: I find this sentence quite unclear. If the time reference does not correspond to the typical duration, what does it measure at all?**

The progression along any of the closed trajectories in Figure 3 maps onto time in a non-trivial way. We compute the duration of a cycle by integrating phase speed (i.e., $\sqrt{{c_x}^2 + {c_y}^2}$, where c_x,c_y are the horizontal components of the phase space velocity field $\mathbf{c}$) along a closed trajectory. However, as the duration of individual events may be shorter or longer, the resulting time duration between different stages is purely indicative and does not necessarily represent the actual time it takes to transition from one stage to the next. Still, the total time duration of a cycle is informative on the time scale associated with the dynamical system, which here is 4–6 days and thus consistent with typical synoptic time scales.

In the revised manuscript, we have rephrased lines 160-161, which now read: "We estimate the average duration for one revolution by integrating the phase speed (i.e., $\sqrt{{c_x}^2 + {c_y}^2}$) along isolines of the streamfunction. For the closed trajectories shown in Fig. 3, we obtain values of about 5d, ranging between 4.3–5.8d (see Table 2)."

We have also rephrased lines 189-190 as follows: "We determined the exact location of each stage to facilitate the comparison across the four spatial domains. Consequently, time intervals between different stages are somewhat different, ranging between ≈0.7–2d. While the overall cycle duration is indicative of the timescale associated with a dynamical system, the duration of individual events may be shorter or longer. Therefore, the time reference primarily serves the purpose of conveniently tracking evolution along a trajectory, rather than indicating the typical duration of a stage."

10) **L 201: "dominated by TILT": This is not really clear from the figure; the red regions (corresponding to DIAB) are even larger.**

L 205: "intensifies further upstream": Again, not really obvious to me. There is a lot of overlap and no clear spatial shift.

L 208-209: Now I'm totally confused. There are more blue regions (corresponding to TILT) downstream, hence I'd write this sentence the other way around.

We have rephrased and expanded the first two paragraphs to provide a more accurate description of the composites. In particular, on line 201 we were referring to the increase in TILT that characterises the first stage, as defined from inspection of the phase portraits without considering the composites. On line 205, we were referring to the slight spatial shift between areas of strong TILT and strong DIAB, with the former always slightly to the west of the latter, which is arguably not easy to discern from the figures in the current resolution. We have tried to improve their resolution in the revised manuscript. On line 209, when we say that DIAB gradually spreads downstream, we meant 'downstream' as a direction, that is, DIAB is spreading towards the downstream.

The first paragraph now reads: "In the first stage, which is characterised by the intensification of TILT (see phase portraits, Fig. 3e–h), all of the four domains feature negative anomalies in Z500 and Z1000 (Fig. 5a,b and Fig. 6a,b), indicating advection of cold air from continents for KOE and GSE and from polar oceans over warmer ocean surfaces for ENP and ENA. The structure of the flow is consistent with the onset of CAOs, which are most frequent in these regions in winter (Grønås and Kvamstø, 1995; Dorman et al., 2004; Kolstad et al., 2009). The spatial distribution of strong TILT trails behind the advancing cold air front, while DIAB intensifies upstream of peaks in TILT (i.e., to their west). Strong DIAB and TILT outside of the averaging domain, such as over the Davis Strait for GSE composites or south of the Bering Strait for KOE composites (Fig. 5), likely reflect their climatological mean rather than a specific relevance to a particular stage in the phase portrait."

The first half of the second paragraph now reads: "According to phase portraits, the second stage is characterised by the steepest slope, as TILT reaches its maximum while DIAB is still increasing. Composites for this stage (Fig. 5c,d and Fig. 6c,d) show a strengthening and, especially in the ENA region (Fig. 6c), a downstream progression of the cyclonic circulation that emerged in the previous stage. We observe again that the spatial distribution of TILT features a shift to the east with respect to that of DIAB, which is now dominant and gradually spreading westwards."

11) L 213-214: TILT also prevails in the regions of the boundary currents.

While it is true that TILT retains some of its intensity, it is still weaker than that observed in the previous stage (Fig. 5c,d) and distributed over a smaller area compared to DIAB. We have rephrased those two lines avoiding the term 'prevail', which was perhaps confusing. We have also specified that the ENP region behaves differently. "TILT has subsided compared to the previous stage, while DIAB has retained its strength. The picture is somewhat different for the ENP region, where TILT does not appear to have changed much while DIAB is visibly stronger."

12) L 215: "anticyclonic geopotential anomalies start building up": not so much in the Pacific, at least at this stage

We have added the following: "except over the KOE region, where positive anomalies have not yet formed at this stage."

13) Section 5.1: The more general conclusions from this section are not clear to me. I got a bit lost in the details, which, in addition, are not always consistent between text and figures (as noted above).

We have added a summarising paragraph that hopefully clarifies the main conclusions that can be drawn for the near-surface. A summary of both near-surface and free troposphere is provided in the concluding section as well.

"In the near-surface, we can therefore ascribe the particular phasing between DIAB and TILT to the effect of cold air advection associated with CAOs as well as cold sectors of midlatitude weather systems. The propagation of the cold front associated with these events contributes to a local steepening of the slope. Steep slope prompts an almost instantaneous response in TILT, whereby isentropic surfaces are flattened as cold air masses sweep in over the ocean surface. The thermal contrast between the cold air masses and the warm ocean surface eventually triggers surface heat fluxes that force the isentropic surfaces back to their initial position, thus diabatically restoring the near-surface slope."

14) **L 225: "particularly in correspondence with the upper-level anomalies": Again not that obvious; e.g., in 7a DIAB is clearly shifted towards the lower-level anomaly.**

We have rephrased that sentence to provide a more accurate description. We meant to highlight the spatial link between peaks in DIAB and the upper-level anomalies to which they seem more closely connected. DIAB shown here is integrated across the free troposphere, not the near-surface.

"DIAB is most active in close proximity to these cyclonic anomalies, particularly on the southern and western flanks of the upper-level anomalies, with the exception of the ENA region (Fig. 7a), where DIAB is somewhat weaker and peaks to the north of the upper-level anomaly. TILT, on the other hand, remains negligible across the four regions."

15) **Section 5.2: Again, the general conclusions could come out more clearly. Most of the corresponding statements are quite generic (diabatic processes are important for midlatitude waves, L 246; primary importance of latent heat release in the diabatic restoration, L 256; role of moist diabatic processes in the evolution of cyclones, L 263) and already quite well known from previous studies (also from your own group...). What are really the new aspects**

from this analysis?

Also in response to the first Reviewer, we rephrased a few paragraphs to make our conclusions more evident and to highlight the new aspects that we show in this study, namely the particular phasing between the diabatic and tilting tendencies and the physical mechanisms behind it. Our results are consistent with previous work by Papritz and Spengler (2015) and Weijenborg and Spengler (2020), adding to a growing body of literature on the crucial role of moist diabatic processes in the representation of storm tracks (Willison et al., 2023; Schemm, 2023; Fuchs et al., 2023).

16) **L 277: "DIAB takes place ahead of storm activity, both in time and space": I'm not sure that this conclusion is justified. For instance, over the North Atlantic (Figs. 7a,c; 8a,c), both cyclonic anomalies and DIAB develop in parallel, and spatially quite well aligned.**

We were primarily referring to the upper-level anomalies (Z500), where DIAB is more markedly ahead/downstream. It was somewhat misleading so we have rephrased it as follows: "Composites across the phase space confirm that DIAB is tightly linked to the development and further evolution of storm activity, both in time and space (Figs. 7,8)"

17) **Figure 5: Should the caption read "below -15... for TILT"? Also, 15 seems to be quite high for DIAB when compared to Fig. 3; is this really correct? Indicate in the caption what the numbers on the sides of the plots indicate (mean DIAB/TILT).**

A previous version of the manuscript had "above 15... in magnitude" which would have implied below -15 for TILT. We have changed the caption accordingly and also included the meaning of DIAB and TILT to the left and right, respectively, of each panel.

The values for DIAB might seem quite high, especially with respect to climatology. However, these peaks occur over a limited spatial extent compared to the entire domain over which the spatial average is computed.