Review of ‘Increasing numerical stability of mountain valley glacier simulations: implementation and testing of free-surface stabilization in Elmer/Ice

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1 Summary

In this manuscript the authors describe a technique for approximating the Backward Euler method for time stepping in a coupled Stokes-free surface system for glacier evolution. The so-called free surface stabilization algorithm only involves the inclusion of an additional term in the Stokes equations, and so is cheap to implement and seems to increase the size of allowable time steps (though probably not to the same degree that a proper implicit solver would). The main difference between this paper and a previous one by the same author on this subject is its implementation in Elmer/Ice and its application to a realistic geometry.

I think the paper is a nice contribution and the method described is potentially useful. Unfortunately, I think that the work suffers from a lack of specificity that hampers a careful reader from really understanding the performance characteristics of the method. Towards the purpose of improving clarity and providing a more sober view of what can be expected of this method, I have included some comments below.

2 Line-by-line comments

L29 I don’t think models ‘suffer’ from time-step restrictions, but they are subject to them. They are not necessarily ‘parabolic’ either - when flow is dominated by bedrock slopes, the equations have a more hyperbolic character.

L32 Here and elsewhere, the word ‘stability’ is used without precision. How is this concept characterized here? Is it just the lack of visibily detectable wiggles? Is it when a simulation blows up?

L120 This comparison is a bit contrived because it relies on one particular paradigm for solving free-surface Stokes, namely that the nonlinear coupling is managed via Picard iteration between the velocity and thickness solves. There are alternatives: solving simultaneously with Picard, using Newton's method (although these are admittedly both easier to implement in a terrain-following coordinate system). These alternatives don't necessarily involve solving Stokes more than once in the way that is described here. It's important to be specific!

Eq. 3.1 This might be an appropriate place (although there are others) to mention the very important condition for all of your equations, namely that they are only valid when $z_s > z_b$! Also, how do you deal with this constraint (presuming that you do, because the 'ice-free' region in the Midtre Lovenbreen experiment expands).

L132 These 'appropriate function spaces' are never stated explicitly. Presumably the Taylor-Hood element is used here?

Eq. 11 How is the transport equation discretized in space? A finite element method? If so, which function space? If it's solved nodally, then how is the spatial derivative in surface elevation calculated? This is an advection equation, so often requires a stabilization scheme, e.g. upwinding. Is that done here? Does whatever representation of the surface elevation satisfy an inf-sup condition?

L150 Is this supposed to be referencing Eq. 12? If so should it be that the first term on the *right* side of Eq. 12 is zero?

Eq. 13 Maybe worthwhile to say that you're using a forward Euler discretization of the time-derivative in Eq. 12. Also, the superscript on Ω seems to be messed up.

Eq. 14 Does the u that appears in the 'new' part of the weak form need a superscript too?

Eq. 15 I'm not sure that including the equation adds anything here. It might be clearer to just write that 'in the case of the SIA, the FSSA coincides precisely with evaluating the pressure at the end of the time integration. In the case of the Stokes' equations, this is an approximation, etc.'

Eq. 18 It's worth noting that with this strictly non-negative mass balance and no way for mass to enter or leave the system, that this glacier will grow without bound. This is unfortunate because it would be interesting to see the result of applying this method to grow a glacier to steady state.

L210 I think it's unfortunate that the solution is only compared to other model results run with a smaller time step. A more robust and complementary approach would be to evaluate this method against a manufactured solution. This would also potentially provide insight into the ambiguous results later about whether the FSSA is more or less accurate than without.

L216 Again, how is stability defined? Is the LST computed by using bigger and bigger time steps until the solver produces NaNs?

L240 Could this be explained in a way that relates more closely to theory? In principle, so long as the CFL condition is satisfied, the forward Euler and backward Euler (which the FSSA approximates) have the same order of numerical accuracy. Why would the accuracy deviate between how the time derivative is discretized?

Fig. 3 I really struggle to distinguish between the lines. Can these be made thicker, or the plot larger or something to make this more easily seen?

L302 What is the 'derivative of the viscosity'? Do you mean the whole Jacobian? If so, then including a relaxation parameter is pretty standard.

L324 I don't understand the notion of higher or lower accuracy for advancing or retreating glaciers. This needs to be justified or removed.

L329 Did these instabilities only appear in the absence of the FSSA or with it too? Later text seems to indicate the former, but it's not clear here. What does it mean for an instability to be 'specific to the setup'? That would seem like a very bad property to not know whether a simulation is going to be stable or not a priori. I can't see how 'other' instabilities are being suppressed here - this doesn't seem to be shown.

Table 2 How do we know its 20 and not, say 17.8 or something?

L355 I don't understand what a 'more viscous behavior' means.

Appendix A I don't think it's all that relevant as to how bedrock was generated (there are many methods of doing this, e.g. Gaussian random fields, random fourier features, etc.), but nonetheless this section is quite opaque. It might be better just to reference something rather than include this sort of insufficient description.