

Referee 1

We are grateful to the referee for devoting their time to our manuscript. The valuable comments and suggestions will help us to improve the paper.

We will here respond to comments made:

The parameters of these systems are set so that all scales behave chaotically. Though it is not totally clear how robust the results are if the parameters are perturbed.

As long as parameters are such that all scales are chaotic, we do not expect any qualitative changes with respect to the studied scenario. We tried to ensure the robustness of the results by considering two cases of "reality" (L05-2 and L05-3 systems). Furthermore, we tested as "reality" the L05-1 system with 360 variables and as "model" the L05-1 system with 180 and 90 variables. The results are consistent with the presented results. We are aware of the need to test the results on "real" systems.

The explanation of the initial decline and subsequent growth of the rate of model error growth by the notion of "drift" is a nice attempt, though it is not totally clear if this is special for the L-05 systems.

"Drift" was used by Orrell (2002) to explain the initial decline and subsequent growth of the rate of model error growth for the ECMWF system (500 hPa, Northern hemisphere for 10 d in October 1999 and total energy globally over a 15 d period in December 2000). Therefore, the results do not appear to apply only to the L05 system. We have further confirmed the behavior resulting from "drift" on the ECMWF system data in Section 5.

In the abstract, where is the claim "Generally, a system with model error (omitting phenomena) will not improve predictability." supported in the maintext? How general is it? This seems to be a very strong statement. If not, I suggest weaken this statement.

We have replaced "Generally" with "In other words" (Lines 11 – 12).

Although it maybe natural, it would be good to give a sentence of explanation about why choosing L05-2 and L05-3 as "reality"

A full explanation of why L05-2 and L05-3 systems are selected as the reality is given in Section 2.2. In addition, we have added a sentence to the introduction on lines 128 – 129 ("The omitted scale is the small scale for the L05-2 system and the small and medium scale for the L05-3 system. "). Information on why we do not use the L05-2 system as model and L05-3 system as reality is given on lines 614-615.

Is "this" in line 9 "initial error growth"? Perhaps good to be more specific.

The word "product" was added to line 9.

How about adding references to the relevant figures after "as we will see in numerical simulations" in line 270?

Reference was added.

Would you explain why geometric mean is used rather than the usual arithmetic mean in model error growth and drift terms in (11), (14), (15), (18) ?

We added an explanation on line 200: "The geometric mean is chosen because of its suitability for comparison with growth governed by the largest Lyapunov exponent. For further information, see Bednar et al. (2014) or Ding and Li (2011). "

The drift $d(\tau)$ at the beginning of line 271 is not defined yet, It does not seem to be the drift VECTOR in line 269. Please clarify.

$d(\tau)$ was changed to the absolute value of drift $d(\tau)$.

Please be consistent in terminology. For example, is "the drift $D(\tau)$ " on Page 279 the same as $d(\tau)$ in line 271? Is it the same as the "the averaged drift D " in line 283?

We improved consistency in terminology. We related $D(t)$ to eq. (18) and $d(\tau)$ to eq. (17).

Perhaps include a table summarizing the heavy notation involved.

We itemized the numbers of the equations. We hope this will help readability.

The reference list will look better if it was itemized.

We itemized the reference list.

I am not totally convinced (or understand) that the notion of the "drift" introduced really explain the model error growth as claimed. It seems that, taking time-average without an absolute value is similar to looking at the original system, when the system is ergodic. Perhaps the authors can explain more on what "explain" means other than showing another summary statistics of the system.

The difference is that it is the summation of vectors created from the difference in time evolution of different systems (but with the same initial conditions) after one time step. The model errors at successive time steps as vectors are not strongly correlated, and that therefore accumulating their absolute values is very different from accumulating them as vectors, where the absolute values sum will grow much faster than the vector valued sum, and that this slower error growth now gives a better explanation of the deviation of the trajectories.

References:

Bednář, H., Raidl, A., and Mikšovský, J.: Initial Error Growth and Predictability of Chaotic Low-dimensional Atmospheric Model, IJAC, 11, 256–264, <https://doi.org/10.1007/s11633-014-0788-3> 2014.

Ding, R., Li, J.: Comparisons of two ensemble mean methods in measuring the average error growth and the predictability, Acta Meteorol Sin, 25, 395–404, <https://doi.org/10.1007/s13351-011-0401-4>, 2011.

Orrell, D.: Role of the metric in forecast error growth: how chaotic is the weather?, Tellus, 54, 350-362, <https://doi.org/10.1034/j.1600-0870.2002.01389.x>, 2002.

Referee 2

We are grateful to the referee for devoting his time to our manuscript. We will here respond to comments made to support the validity of the article for publishing:

This paper tries to explain why omitting atmospheric phenomena, which contribute little to the final value, will not improve the predictability of the resulting value. However, this paper does not provide a complete theory to show this. Although this article says that a theory explaining and describing this behavior is developed, I did not find any strict mathematical theory in this article.

The developed theory is not strictly mathematical but is based on a strictly mathematical theory describing the model error growth (Drift - Section 2.4), presented by Orrell (Orrell et al., 2001; Orrell, 2002) and on a strictly mathematical theory of classical low dimensional chaos, where one observes an exponential error growth of a tiny initial error whose exponent is given by the largest Lyapunov exponent of the system. Our extension that sees Drift produced at each time step as the error of the initial conditions is based on an experiment with Lorenz L05 systems (Appendix A) and explains and describes the model error growth in this experimental setting (Section 4). The derived results are then verified in the ECMWF systems (Section 5). Because it is not a theory in a strictly mathematical sense. We replace the term “theory” with the term “hypothesis”. We believe that our hypothesis Eq. (21) is as worthy of publication as other already commonly accepted experiment-based hypotheses such as Eqs. (2)-(6).

Line 20: ... “the instability of the system with respect to initial condition errors has grown” ..., the instability is not clear?

By instability we mean the error growth rate of the initial conditions of ECMWF systems, which is expressed by the Lambda parameter from Eq. (5). The values can be seen in Figure 15a - blue curve. More details can be found in Section 5. For better understanding, we have added "(error growth rate)" to the text.

Line 95:” the constant b in Eq. (5) which, irrespective of initial condition errors, will lead to a deviation of the model solution from reality...”. It seems that there is no a constant b in Eq. (5).

b has been replaced in the text by *beta*. We thank the referee for spotting this misprint.

Line 120: “....Including small spatiotemporal scales, i.e., improving the model's spatial and temporal resolution, therefore enhances the instability with respect to initial condition error” the exact meaning of the instability is not clear.

By instability we mean the error growth rate of the initial conditions. Brisch and Kantz (2019) and Zhang et al. (2019) associated initial error growth with scale-dependent error growth, where tiny errors grow much faster than larger ones. Lorenz (1969) gave a sketch of such error growth: a typical quantity to be predicted is a superposition of the dynamics on different scales. After a fast growth of the small-scale errors with saturation at these very same small scales, the large-scale errors continue to grow at a slower rate until even these saturate. We have added "(error growth rate)" to the text.

Line 140: “....We measure the error magnitude $e(t)$ after fixed time intervals ..”. there is not any expression for $e(t)$.

On line 140, $e(t)$ is defined as the error magnitude after fixed time intervals. The expressions for the settings are shown on lines 192, 226 and 245.

Line 160: “.... For this scheme to be meaningful, we have to ensure that the reference trajectory is on the system's attractor and that the repetition of this scheme samples the whole attractor with correct weights (the invariant measure)....”. the existence of attractors in this system is not clear.

Lorenz L05 systems are widely accepted chaotic systems with a positive largest Lyapunov exponent (which is computed and presented). For L96 system (Lorenz, 1996) the existence of attractor has been shown, and because our system can be expected to be in the same model class, we expect the existence of a chaotic attractor.

Line 195: There is no definition of. Line 230: There is no definition of. Line 245: There is no definition of.

It is probably meant that Eqs. (11), (14) and (15) are not definitions from a strictly mathematical sence, so we replace the expression "is defined" by "is calculated".

References:

Brisch, J., and Kantz, H.: Power law error growth in multi-hierarchical chaotic system-a dynamical mechanism for finite prediction horizon, *New J. Phys.*, 21, 1–7, <https://doi.org/10.1088/1367-2630/ab3b4c>, 2019.

Lorenz, E. N.: The predictability of a flow which possesses many scales of motion, *Tellus*, 21, 289–307, <https://doi.org/10.1111/j.2153-3490.1969.tb00444.x>, 1969.

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