Quantifying the migration rate of drainage divides from high-resolution topographic data

Chao Zhou¹, Xibin Tan², *, Yiduo Liu², Feng Shi¹,³

¹ State Key Laboratory of Earthquake Dynamics, Institute of Geology, China Earthquake Administration, Beijing 100029, China
² Key Laboratory of Mountain Hazards and Surface Processes, Institute of Mountain Hazards and Environment, Chinese Academy of Sciences, Chengdu 610299, China
³ Shanxi Taiyuan Continental Rift Dynamics National Observation and Research Station, Beijing 100029, China

*Corresponding author. E-mail address: tanxibin@imde.ac.cn

Abstract

The lateral movement of drainage divides is co-influenced by tectonics, lithology, and climate, and therefore archives a wealth of geologic and climatic information. It also has wide-ranging implications for topography, sedimentary record, and biological evolution, thus has drawn much attention in recent years. Several methods have been proposed to determine the drainage divide’s migration state (direction and rate), including geochronological approaches (e.g., ¹⁰Be) and topography-based approaches (e.g., χ-plots or Gilbert metrics). A key object in these methods is the channel head, which separates the hillslope and channel. However, due to the limited resolution of topography data, the required channel-head parameters in the calculation often cannot be determined accurately, and empirical values are used.
in the calculation, which may induce uncertainties. Here, we propose two methods to calculate the migration rate of drainage divides, based on the relatively accurate channel-head parameters derived from high-resolution topographic data. We then apply the methods to an active rift shoulder (Wutai Shan) in the Shanxi rift, and a tectonically stable area (Yingwang Shan) in the Loess Plateau, to illustrate how to calculate drainage-divide migration rates. Our results show that the Wutai Shan drainage divide is migrating northwestward at a rate between 0.21 to 0.27 mm/yr, whereas the migration rates at the Yingwang Shan are approximately zero. This study indicates that the drainage-divide stability can be determined more accurately using high-resolution topographic data. Furthermore, this study takes the cross-divide differences in the uplift rate of channel heads into account in the measurement of drainage-divide migration rate for the first time.

**Keywords**

Drainage divide; Migration rate; High-resolution topographic data; DEM; Channel head

**1. Introduction**

The evolution of the Earth’s surface is jointly controlled by tectonics, lithology, and climatic conditions (e.g., Molnar and England, 1990; Whipple, 2009; Gallen, 2018; Bernard et al., 2021; Hoskins et al., 2023), providing a basis for reconstructing the past tectonic (Pritchard et al., 2009; Kirby et al., 2012; Shi et al., 2021) or climatic
processes (Tucker et al., 1997; Hancock et al., 2002; Schildgen et al., 2022) through topography. The evolution of unglaciated terrestrial terrains is fundamentally coupled with changes in drainage systems through river’s vertical (changes in river long profile) and lateral movements (drainage divide migration and river captures) (Whipple, 2001; Clark et al., 2004; Bonnet, 2009; Willett et al., 2014). Previous studies have extensively investigated how river channel profiles respond to tectonic uplift (Whipple, 2001; Crosby and Whipple, 2006; Kirby et al., 2012), lithological difference (Duvall et al., 2004; Safran et al., 2005; Forte et al., 2016), and precipitation perturbations (Schlunegger et al., 2011; Bookhagen and Strecker, 2012). River’s long profiles have been used to study earthquake events (e.g., Burbank and Anderson, 2001; Wei et al., 2015) and the spatio-temporal variations of uplift (e.g., Whipple et al., 1999; Kirby et al., 2003; Pritchard et al., 2009; Goren et al., 2014). Recent studies show that the widespread lateral movement of river basins driven by geological and/or climatic disturbance (Yang et al., 2019; Zondervan et al., 2020; Zhou et al., 2022a; Bian et al., 2024) also interacts with the adjustment of channel profiles (Willett et al., 2014). Drainage-divide migration, one form of river lateral movement, may not only carry information on geological and/or climatic disturbance (Su et al., 2020; Zondervan et al., 2020; He et al., 2021; Shi et al., 2021; Zhou et al., 2022a; Zeng and Tan, 2023) but also influence the extraction of tectonic information from channel profiles (Goren et al., 2014; Ma et al., 2020; Jiao et al., 2021). Moreover, it has multi-facet consequences for landscape evolution (Scheingross et al., 2020; Stokes et al., 2022), sedimentary processes (Clift & Blusztajn, 2005; Willett et
al., 2018; Deng et al., 2020; Zhao et al., 2021), and biological evolution (Waters et al., 2001; Zemlak et al., 2008; Hoorn et al., 2010; Musher et al., 2022). For this reason, the stability of drainage divides has drawn more and more attention in recent years (e.g., Authemayou et al., 2018; Vacherat et al., 2018; Chen et al., 2021; Shelef and Goren, 2021; Sakashita and Endo, 2023; Bian et al., 2024).

Drainage-divide migration is essentially controlled by the cross-divide difference in erosion and topographic slope (Beeson et al., 2017; Dahlquist et al., 2018; Chen et al., 2021; Zhou et al., 2022a). The erosion rates are routinely derived from geochronological techniques, such as cosmogenic nuclides (e.g., $^{10}$Be) concentration measurements (Mandal et al., 2015; Struth et al., 2017; Sassolas-Serrayet et al., 2019), which can be used to calculate the migration rates of drainage divides (Beeson et al., 2017; Godard et al., 2019; Hu et al., 2021). However, these techniques are usually based on samples collected from a catchment outlet that is several, or even tens of, kilometers away from the drainage divide and thus may not represent the erosion rates close to the drainage divide (Sassolas-Serrayet et al., 2019; Zhou et al., 2022a). Besides, the high cost of sample processing makes it challenging to determine the drainage divide’s motion by measuring the erosion rates throughout the large landscapes. Hence, it would be ideal to find an accessible and efficient method that can be applied to the entire landscape and make full use of the $^{10}$Be-derived erosion rates.

The advancement of the digital elevation model (DEM) has promoted the development of geomorphic analysis, making it possible to determine the drainage
divide’s transient motion through topography analysis. For example, Willett et al. (2014) applied the $\chi$ method to map the dynamic state of river basins. Forte and Whipple (2018) proposed the cross-divide comparison of “Gilbert metrics” (including channel heads’ relief, slope, and elevation) to determine a drainage divide’s migration direction. Others adopted the comparison of slope angle or relief of the hillslopes across a drainage divide to deduce its stability (Scherler and Schwanghart, 2020; Ye et al., 2022; Zhou et al., 2022b). These geomorphic techniques, so far, could only determine the migration direction of drainage divides. Braun (2018) provided an equation that considers both alluvial and fluvial areas to calculate the migration velocity of an escarpment (also a drainage divide). Zhou et al. (2022a) developed a technique to calculate the migration rate through the high base-level $\chi$ values on both sides of a drainage divide. These new approaches require channel-head parameters to calculate the migration rate. However, the location of the channel heads sometimes cannot be accurately identified because of the limitation in the resolution of DEMs in natural cases. For this reason, empirical values of channel-head parameters are used in these studies, which may induce uncertainties.

This study aims to establish an approach to derive the migration rate of drainage divides, at a high precision and low cost, based on topographic analysis. We choose a tectonically active area (i.e., the Wutai Shan in the Shanxi Rift) and a tectonically inactive area (i.e., the Yingwang Shan in the Loess Plateau) to demonstrate how to quantify drainage-divide migration rates (Fig. 1). We use the aerial photography acquired by unmanned aerial vehicles (UAVs) and the Structure from Motion (SfM)
technology to obtain the high-resolution DEM data of these two areas (0.67 m and
0.84 m spatial resolution in the Wutai Shan and the Yingwang Shan, respectively).
Benefiting from the high-resolution data, the location of channel heads can be
identified more accurately. We then develop two methods to calculate the drainage-
divide migration rates. One is based on the measured channel-head parameters, and
the other is based on an improved method of Zhou et al (2022a). Combining with the
geological and low-temperature thermochronology studies of the Wutai Shan
(Middleton et al., 2017; Clinkscales et al., 2020), we also quantify the cross-divide
difference in uplift rates to improve the precision of drainage-divide migration rate.

2. Methods

2.1 Channel-head-point method

According to the detachment-limited stream power model (Howard and Kerby,
1983; Howard, 1994), the channel’s erosion rate \( E \) can be expressed as:
\[
E = KA^mS^n
\]
where \( K \) is the erosion coefficient, \( A \) is the upstream drainage area, \( S \) is the gradient of
the river channel, and \( m \) and \( n \) are empirical constants.

Because of thresholds such as erosion threshold (the shear stress of overland flow
must exceed the threshold of the cohesion of bed material to generate river incision)
(Howard and Kerby, 1983; Perron et al., 2008) or landslide threshold (landslides
occur when the threshold of soil or rock strength is exceeded in high relief region)
(Burbank et al., 1996; Tucker et al., 1998), river channels (following Eq. 1) emerge at a certain distance from the drainage divide. The region between the channel head and the drainage divide is referred to as the hillslope area, where the erosion is controlled by landslide, collapse, and diffusion processes (Stoke and Dietrich, 2006; Stark, 2010; Braun et al., 2018; Dahlquist et al., 2018). The channel-head point is the highest and the closest point to the drainage divide on a river channel (Clubb et al., 2014).

Therefore, the erosion rate at channel-head points \(E_{ch}\) can be described as:

\[
E_{ch} = K A_{cr}^m S_{ch}^n \tag{2}
\]

where \(E_{ch}\) is the erosion rate at channel-head points, \(A_{cr}\) is the critical upstream drainage area of a channel-head point (Duvall et al., 2004; Wobus et al., 2006), and \(S_{ch}\) is the channel-head gradient measured along the channel near the channel-head point. Eq. 2 indicates that the side of a drainage divide with a higher \(A_{cr}\) or \(S_{ch}\) can have a higher erosion rate than the other side, and is more likely to pirate the opposite drainage basin. Besides, a high erosion coefficient can amplify the drainage basin’s erosion rate.

Drainage-divide migration is essentially controlled by the cross-divide difference in erosion rates and topographic slope (Beeson et al., 2017; Dahlquist et al., 2018; Chen et al., 2021; Zhou et al., 2022a; Stokes et al., 2022). Furthermore, the differential uplift should also be considered when using the cross-divide erosion rates at the channel heads to calculate the erosion difference across the divide, especially in the case of tectonic tilting uplift (Zhou et al., 2022a). The drainage-divide migration rate \(D_{mr}\) can be obtained according to the cross-divide difference in erosion rate and
uplift rate and the slopes across the divide (Zhou et al., 2022a):

\[ D_{mr} = \frac{\Delta E_{ch} - \Delta U_{ch}}{\tan \alpha + \tan \beta} \]  

(3)

where \( \Delta E_{ch} \) is the difference in erosion rate between the two sides (annotated as \( \alpha \) and \( \beta \)) of the drainage divide (\( \Delta E_{ch} = E_{cha} - E_{chb} \)). The choice of \( \alpha \) or \( \beta \) is arbitrary, and the positive direction of the migration rate is assigned from the \( \alpha \) to the \( \beta \) side whereas the negative is the opposite. \( \Delta U_{ch} \) is the cross-divide difference in uplift rate (\( \Delta U_{ch} = U_{cha} - U_{chb} \)), and \( \tan \alpha \) and \( \tan \beta \) are the gradients on each side of the drainage divide.

Assuming erosion coefficient \( (K) \) is the same on both sides of a drainage divide, Eqs. 2 and 3 allow us to derive the equation of drainage divide’s migration rate according to the parameters at the channel-head points:

\[ D_{mr} = \frac{K \left[ \left( \frac{A_{cr}^m \varepsilon_{ch}^n}{A_{cr}^m \varepsilon_{ch}^b} \right)_{\alpha} - \left( \frac{A_{cr}^m \varepsilon_{ch}^n}{A_{cr}^m \varepsilon_{ch}^b} \right)_{\beta} \right] - \Delta U_{ch}}{\tan \alpha + \tan \beta} \]  

(4)

If the exact value of \( K \) is unknown, the drainage divide’s unilateral erosion rate can be used as a substitution:

\[ D_{mr} = \frac{E_{\alpha} \left[ 1 - \left( \frac{A_{cr}^m \varepsilon_{ch}^n}{A_{cr}^m \varepsilon_{ch}^b} \right)_{\beta} \right] - \Delta U_{ch}}{\tan \alpha + \tan \beta} \]  

(5)

or:

\[ D_{mr} = \frac{E_{\beta} \left[ \left( \frac{A_{cr}^m \varepsilon_{ch}^n}{A_{cr}^m \varepsilon_{ch}^b} \right)_{\alpha} - 1 \right] - \Delta U_{ch}}{\tan \alpha + \tan \beta} \]  

(6)

\( E_{\alpha} \) and \( E_{\beta} \) are the erosion rates of the \( \alpha \) and the \( \beta \) side of the drainage divide, respectively, which can be derived through cosmogenic nuclides \( (^{10}\text{Be}) \) concentration measurements (Beeson et al., 2017; Godard et al., 2019; Hu et al., 2021). The regional average erosion rate \( \bar{E} = \frac{E_{\alpha} + E_{\beta}}{2} \) can also be used to calculate the migration rate:
Based on Eqs. 4-7, the migration rate of drainage divides can be estimated using channel-head parameters combined with one of the erosion-related parameters, erosion coefficient (K), erosion rate at one side of a drainage divide (Eα or Eβ), or regional average erosion rate (E).

\[ D_{mr} = \frac{2E}{\tan \alpha + \tan \beta} \left[ \left( \frac{\Delta \chi}{\Delta U_{ch}} \right)_{\alpha} - \left( \frac{\Delta \chi}{\Delta U_{ch}} \right)_{\beta} \right] \]  

(7)

2.2 Channel-head-segment method

A channel-head segment is the channel segment just below the channel head (Zhou et al., 2022a). Zhou et al. (2022a) developed a method based on the cross-divide χ contrast of channel-head segments to calculate the migration rate of drainage divides. The essence of the method is the cross-divide comparison of the channel-head segments’ normalized channel steepness (ksn) values. ksn is a widely used index (Whipple et al., 1999; Wobus et al., 2006; Hilley and Arrowsmith, 2008; Kirby and Whipple, 2012) that is quantitatively related to E and K \[ k_{sn} = \left( \frac{E}{K} \right)^{\frac{1}{n}} \]. χ is an integral function \[ \chi = \int_{x_0}^{x} \left( \frac{A_0}{A(x)} \right)^{\frac{m}{n}} dx \] of a channel’s upstream area (A) to horizontal distance (x) (Royden et al., 2000; Perron and Royden, 2012), and A₀ is an arbitrary scaling area to make the integrand dimensionless.

In the method of Zhou et al. (2022a), the location of channel heads cannot be accurately identified, because it is limited by the resolution of DEM. Therefore, an empirical value of \( A_{cr} = 10^5 \text{ m}^2 \) was used in the calculation. Benefiting from the high-resolution DEM in this study, we improve the method in Zhou et al. (2022a) and use...
the real location of channel heads to calculate the migration rate. When the regional
erosion coefficient \((K)\) is known and unchanged in the vicinity of the drainage divide,
the drainage-divide migration rate can be estimated by the following equation:

\[
D_{mr} = \frac{K[k_{sn(\alpha)}n - k_{sn(\beta)}n]}{\tan \alpha + \tan \beta} \Delta U_{ch} = \frac{K\left\{\left(\frac{(z_{ch}-z_b)_{\alpha}}{x_{\alpha}}\right)^n - \left(\frac{(z_{ch}-z_b)_{\beta}}{x_{\beta}}\right)^n\right\}}{\tan \alpha + \tan \beta} \Delta U_{ch}
\] (8)

where \(z_{ch}\) and \(z_b\) are the elevations of the channel heads and catchment outlets,
respectively. The detailed derivation of Eq. 8 is in Supplementary Materials. The
drainage divide’s unilateral erosion rate \((E_{\alpha} \text{ or } E_{\beta})\) can also be used as a substitution
for the \(K\) value:

\[
D_{mr} = \frac{E_{\alpha}\left\{1 - \left(\frac{x_{\alpha}}{x_{\beta}}\right)^n \left(\frac{(z_{ch}-z_b)_{\alpha}}{z_{ch}-z_b}_{\beta}\right)^n\right\}}{\tan \alpha + \tan \beta} \Delta U_{ch}
\] (9)

or:

\[
D_{mr} = \frac{E_{\beta}\left\{\left(\frac{x_{\alpha}}{x_{\beta}}\right)^n \left(\frac{(z_{ch}-z_b)_{\alpha}}{z_{ch}-z_b}_{\beta}\right)^n - 1\right\}}{\tan \alpha + \tan \beta} \Delta U_{ch}
\] (10)

Alternatively, one can use the regional average erosion rate \((\bar{E})\) to calculate the
migration rate:

\[
D_{mr} = \frac{2\bar{E}\left\{\left(\frac{x_{\alpha}}{x_{\beta}}\right)^n \left(\frac{(z_{ch}-z_b)_{\alpha}}{z_{ch}-z_b}_{\beta}\right)^n\right\}}{\tan \alpha + \tan \beta} \Delta U_{ch}
\] (11)

Based on Eqs. 8-11, the drainage-divide migration rate can be estimated using the \(\chi\)
values of high-base-level channel segments combined with one of the erosion-related
parameters, erosion coefficient \((K)\), erosion rate at one side of a drainage divide \((E_{\alpha} \text{ or } E_{\beta})\), or regional average erosion rate \((\bar{E})\).

We use UAV-acquired aerial photography and structure from motion (SfM)
photogrammetry to derive the high-resolution DEM of the two study areas, the Wutai Shan in the Shanxi Rift, and the Yingwang Shan in the Loess Plateau, both located in North China (Fig. 1). The spatial resolution is 0.67 m in the Wutai Shan, and 0.84 m in the Yingwang Shan. Based on the high-resolution topography data, we extract the relevant parameters and calculate the drainage-divide migration rate using the two methods above for each case. Analysis of the data is based on the Matlab toolbox TAK (Forte and Whipple, 2019) and TopoToolbox (Schwanghart and Scherler, 2014).

3. Applications to natural cases

We apply the new methods to two natural examples in North China, the Wutai Shan in the Shanxi Rift and the Yingwang Shan in the Loess Plateau, to demonstrate how to calculate the drainage-divide migration rates (Fig. 1).
Figure 1. Locations and tectonic background of the two nature cases in North China. The figure is modified from Fig. 7 in Shi et al. (2021). (A) Red lines represent the main active faults. Black rectangles show the locations of the two nature cases. Red curve denotes active fault, sourced from https://www.activefault-datacenter.cn/. The topography data (ALOS DEM, 12.5 m resolution) is downloaded from the Alaska Satellite Facility (ASF) Data Search (https://search.asf.alaska.edu/). (B) The satellite image downloaded from Google Earth. White rectangles show the location of Panel A.

3.1 Wutai Shan

The Wutai Shan is a tilted fault block on the shoulder of the Shanxi Rift System located in the central North China craton (Fig. 1) (Xu et al., 1993; Su et al., 2021). The tilting uplift of the Wutai Shan is controlled by the Northern Wutai Shan fault, and there is no active fault along the south edge of the Wutai Shan horst (Fig. 2). The bedrock of the Wutai Shan area consists mainly of metamorphic and igneous basement rocks (Clinkscales et al., 2020) and there is no obvious variation in rock erodibility and precipitation in this area (Fig. S2 & S3). Zhou et al. (2022b) reveal that the Wutai Shan drainage divide is migrating northwestward due to the tilting uplift and predicts the drainage divide will move ~10 km to the northwest to achieve a steady state if all geological conditions remain. Geomorphic evidence also exhibits a northwestward migration of the drainage divide (Fig. 3). The plan and satellite views
show several barbed tributaries and a captured area around the Wutai Shan drainage
divide, which indicate that the tributaries formerly part of the northern drainage have
become part of the southern drainage (Fig. 3A&B). The $\chi$-plots analysis shows the
southern side of the drainage divide has steeper channels, higher $k_{sn}$, and lower $\chi$. The
$\chi$-plots of paired rivers illustrate obvious characteristics of shrinking-expanding and
captured-beheaded rivers (Fig. 3C).

To derive the erosion coefficient of the Wutai Shan area, we calculate the
channel steepness ($k_{sn}$) of this region, assuming $n = 1$ and $m = 0.45$ (Wobus et al.,
2006; DiBiase et al., 2010; Perron and Royden, 2012; Wang et al., 2021). We then use
the Kriging interpolation method to generate the $k_{sn}$ distribution map (Fig. 2B). In
addition, results under the assumptions of $m = 0.35$ and 0.55, respectively, are shown
in Supplementary Materials (Fig. S4). The average $k_{sn}$ value of the upthrown side near
the Northern Wutai Shan fault is $\sim 80 \text{ m}^{0.9}$ (Fig. 2D). Previous geological study shows
that the Quaternary throw rates of the Northern Wutai Shan fault are 0.8-1.6 mm/a
(Middleton et al., 2017). The low-temperature thermochronology study shows that the
time-averaged long-term throw rates in the late Cenozoic is about 0.25 mm/yr, and
there is an accelerated activity in the Wutai Shan area (Clinkscales et al., 2020).
According to these studies, we assume a $0.50 \pm 0.25$ mm/yr uplift/erosion rate in the
northern margin of the Wutai Shan (in the footwall of the Northern Wutai Shan fault).
Combining with the equation, $K = \frac{E}{k_{sn} n}$, and following the approach of previous
studies (Kirby and Whipple, 2001; Kirkpatrick et al., 2020; Ma et al., 2020), the
erosion coefficient ($K$) is calculated to be $(6.25 \pm 3.13) \times 10^{-6}$ m$^{0.1}$yr$^{-1}$ in this area.
Because there is no obvious variation in rock erodibility and precipitation in this area (Fig. S2 & S3), we use this value as the erosion coefficient ($K$) of the Wutai Shan area.

We then apply the two new methods to calculate the migration rate of the drainage divide in the Wutai Shan. We first choose three pairs of rivers (Fig. 4A) and acquire their slope-area plots (Figs. 4B, E, H) and the $\chi$-plots (Figs. 4C, F, I).

According to the location of the catchment outlets (Fig. 4A), we obtain the outlet elevations ($z_b$) from the river's long profiles. According to the breaking point of the slope-area regression line (Duvall et al., 2004) (Figs. 4B, E, H), we obtain the values of the critical upstream drainage area ($A_{cr}$). According to these values, we separate hillslope and channel areas and mark the position of the channel heads on the $\chi$-plots and the topography map (Fig. 4A). For the $\chi$-plots (Figs. 4C, F, I), we obtain the elevations of channel heads ($z_{ch}$) and $\chi$ values based on the coordinate of the channel-head points. According to the location of the channel heads on the river's long profiles, we calculate the slope of the channel heads’ tangent lines, and derive the channel-head gradient ($S_{ch}$). Topographic gradient ($\tan \alpha$ or $\tan \beta$) is calculated through the average slope of the hillslope segment near the channel head (Stokes et al., 2022) (not including the hilltop part).

If we assume the rock uplift rate decreases linearly from 0.5 to 0 mm/yr from northwest to southeast of the Wutai Shan horst (~40 km wide), the cross-divide uplift difference in the channel-head points ($\Delta U_{ch}$) (the distance perpendicular to the direction of the boundary fault is ~600 m) is ~0.008 mm/yr. After determining these
parameters, we adopt the channel-head-point (Eq. 4) and channel-head-segment (Eq. 8) methods, respectively, to calculate the migration rates. The required data for calculation and the migration rates are shown in Table 1. The calculated results for $m = 0.35$ and 0.55, respectively, are shown in Supplementary Materials (Table S1).

The rivers have different characteristics on both sides of the drainage divide, as illustrated on their slope-area plots (Figs. 4B, E, H) and the $\chi$-plots (Figs. 4C, F, I).

For the first site (Fig. 4D), the migration rates calculated by the channel-head-point and channel-head-segment methods are 0.21 mm/yr and 0.26 mm/yr, respectively. For the second site (Fig. 4G), the migration rates are 0.23 mm/yr and 0.27 mm/yr, respectively. For the third site (Fig. 4J), 0.21 mm/yr and 0.22 mm/yr, respectively. The drainage divides of all three points are migrating northwestward, which is consistent with the previous result inferred by the cross-divide contrast of slopes in this area (Zhou et al., 2022b). Furthermore, the migration rates calculated by the two methods are comparable in all three sites.
Figure 2. Topography (A) and normalized channel steepness ($k_{sn}$) (B) distribution of the Wutai Shan horst and surrounding area in the Shanxi Rift System. The black dashed line shows the location of the main drainage divide. Red lines show the main active faults. The black lines show the location of profiles E-E’ and F-F’. Black rectangles show the area of Fig. 3B & 4A. Gray boxes show the area of the swath profiles in Panels C and D. Green dots denote the locations of the low-temperature thermochronology samples in Clinkscales et al. (2020). The topography data (ALOS DEM, 12.5 m resolution) is downloaded from the Alaska Satellite Facility (ASF) Data Search (https://search.asf.alaska.edu/). The $k_{sn}$ is calculated using TopoToolbox (Schwanghart and Scherler, 2014), and the interpolation uses the Kriging method on ArcGIS. (C) Topography swath profile along E-E’. See location in Panel A. (D) $k_{sn}$ swath profile along F-F’. See location in Panel B. The swath profiles are extracted using TopoToolbox (Schwanghart and Scherler, 2014). The red dashed lines show the location of the main active normal faults, and the black arrow shows the location of...
the main drainage divide. Both swath profiles are 20 km wide (10 km on each side).

Figure 3. Perspective views and χ map of the drainage divide in the Wutai Shan (see Fig. 2 for location). (A) Perspective views of a captured area and the channels mapped with $k_{sn}$. The south side of the drainage divide has steeper channels and higher $k_{sn}$ than the north side. Magenta arrows show drainage divide migration directions. The satellite image is from Google Earth. (B) $\chi$ map of this area with the outlet elevation of 1300 m. The south side of the drainage divide has lower $\chi$ values than the north side. It should be noted that the catchment outlet at the north side of the drainage basins (the 1300 m contour) is out of the map. The $\chi$-plots of the rivers in bold lines are shown in Panel C. The topography data (ALOS DEM, 12.5 m resolution) is
downloaded from the Alaska Satellite Facility (ASF) Data Search

(C) \( \chi \)-plots of the three paired rivers in Panel B. The blue
and red curves correspond to the rivers on the south and north sides, respectively. The
\( \chi \)-plot of River 1 is steeper on the south side, indicating that the river on the south side
is expanding and the river on the north side is shrinking. The \( \chi \)-plots of Rivers 2 and 3
in the captured area show obvious characteristics of the captured and beheaded rivers.
The \( \chi \)-plot is extracted using TAK (Forte and Whipple, 2019) and TopoToolbox
(Schwanghart and Scherler, 2014).
Figure 4. Analytical results of the Wutai Shan drainage divide. (A) High-resolution hill-shade map (0.67 m spatial resolution) of the Wutai Shan. The black dashed line shows the location of the main drainage divide. Colored lines show the three pairs of...
selected channels used for analysis. The black dots are the channel heads. Black
rectangles show the location of the cross-divide topography swath profiles. The black
arrows show the direction of drainage-divide migration (B, E, H) Slope-area plots of
the three pairs of selected channels. The blue and orange dots are the slope-area plots
of the north (α) and south (β) sides of the drainage divide respectively. The black dots
represent the channel heads. (C, F, I) χ-plots of the selected channels. The blue and
orange lines are the χ-plots of the north (α) and south (β) sides of the drainage divide
respectively. The black dots represent the channel heads. (D, G, J) Cross-divide
topography swath profiles with the drainage-divide migration rates. The locations of
the profiles are in Panel A. The light and dark blue arrows are the drainage-divide
migration rates calculated by the channel-head-point (Eq. 4) and channel-head-
segment (Eq. 8) methods respectively.

3.2 Yingwang Shan

The Loess Plateau is hosted by the tectonically stable Ordos Block of the North
China craton (Yin, 2010; Su et al., 2021). Over the past 2.6 million years, it has
accumulated tens to hundreds of meters of eolian sediments (Yan et al., 2014),
draping preexisting topography (Xiong et al., 2014). There is no active fault and little
to no variation in rock erodibility and precipitation within the area (Shi et al., 2020;
Zhou et al., 2022b).

We apply the two methods to Yingwang Shan of Loess Plateau to calculate the
drainage-divide migration rate. Similar to the Wutai Shan site, we obtain the slope-area plots (Figs. 5 B, E, H), the \( \chi \)-plots (Figs. 5 C, F, I), and extract the values of \( A_{cr} \), \( S_{ch} \), \( z_b \), \( z_{ch} \), \( \chi \), \( \tan \alpha \) and \( \tan \beta \) of the rivers. The rate of soil erosion in the study area is about 500 t·km\(^{-2}\) yr\(^{-1}\) according to the distribution of silt discharge (Fu, 1989).

Combining with the assumption of the density of loess, 1.65 t·m\(^{-3}\), the present-day erosion rate in the study area is calculated to be 0.3 mm·yr\(^{-1}\). Because there is no obvious unequal uplift in this region, we assign that \( \Delta U_{ch} \) is zero. We also assume \( n = 1 \) and \( m = 0.45 \) in the calculation (Wobus et al., 2006; DiBiase et al., 2010; Perron and Royden, 2012; Wang et al., 2021). Then, we use the methods of channel-head parameters (Eq. 7) and channel segments (Eq. 11) to calculate the drainage-divide migration rates. The required data for calculation and the migration rates are shown in Table 1.

All results of the three points show that the drainage-divide migration rate here is close to zero, no matter which method is used in the calculation. The results show that the drainage divide of the study site is in topographical equilibrium, which is consistent with the inference in previous studies (Willett et al., 2014, Zhou et al., 2022b).
Figure 5. Analytical results of the Yingwang Shan in the Loess Plateau. (A) High-resolution hill-shade map (0.84 m spatial resolution). The black dotted line shows the location of the main drainage divide. Colored lines show the three pairs of selected
channels used for analysis. The black dots represent the channel heads. Black rectangles show the location of the cross-divide topography swath profiles. (B, E, H) Slope-area plots of the three pairs of selected channels. The blue and orange dots are the data of the north (α) and south (β) sides of the drainage divide respectively. The black dots represent the channel heads. (C, F, I) χ-plots of the selected channels. The blue and orange lines are the χ-plots of the north (α) and south (β) sides of the drainage divide respectively. The black dots represent the channel heads. (D, G, J) The cross-divide topography swath profiles. The locations of the swath profiles are in Panel A.
<table>
<thead>
<tr>
<th>Natural Cases</th>
<th>No.</th>
<th>$A_{cr}$ (×10^5 m²)</th>
<th>$S_{ch}$ (m)</th>
<th>$z_{ch}$ (m)</th>
<th>$z_{ch}$ (m)</th>
<th>$\chi$</th>
<th>tanα</th>
<th>tanβ</th>
<th>$\Delta U_{ch}$ (mm/yr)</th>
<th>$D_{mr}$ (mm/yr) (Channel-head-point method)</th>
<th>$D_{mr}$ (mm/yr) (Channel-head-segment method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wutai Shan</td>
<td>Fig. 4 Ia</td>
<td>1.75</td>
<td>0.16</td>
<td>1631</td>
<td>1792</td>
<td>6.4</td>
<td>0.14</td>
<td>0.66</td>
<td>~ 0.008</td>
<td>-0.21±0.10</td>
<td>-0.26±0.12</td>
</tr>
<tr>
<td></td>
<td>Fig. 4 Ib</td>
<td>0.26</td>
<td>0.63</td>
<td>1347</td>
<td>1723</td>
<td>6.6</td>
<td>0.24</td>
<td>0.70</td>
<td>~ 0.008</td>
<td>-0.23±0.11</td>
<td>-0.27±0.13</td>
</tr>
<tr>
<td></td>
<td>Fig. 4 IIa</td>
<td>0.79</td>
<td>0.23</td>
<td>1630</td>
<td>1815</td>
<td>5.4</td>
<td>0.24</td>
<td>0.70</td>
<td>~ 0.008</td>
<td>-0.21±0.10</td>
<td>-0.22±0.10</td>
</tr>
<tr>
<td></td>
<td>Fig. 4 IIb</td>
<td>0.30</td>
<td>0.67</td>
<td>1351</td>
<td>1809</td>
<td>6.1</td>
<td>0.28</td>
<td>0.65</td>
<td>~ 0.008</td>
<td>-0.21±0.10</td>
<td>-0.22±0.10</td>
</tr>
<tr>
<td></td>
<td>Fig. 4 IIIa</td>
<td>0.67</td>
<td>0.29</td>
<td>1633</td>
<td>1860</td>
<td>5.0</td>
<td>0.28</td>
<td>0.65</td>
<td>~ 0.008</td>
<td>-0.21±0.10</td>
<td>-0.22±0.10</td>
</tr>
<tr>
<td></td>
<td>Fig. 4 IIib</td>
<td>0.39</td>
<td>0.63</td>
<td>1352</td>
<td>1875</td>
<td>6.9</td>
<td>0.28</td>
<td>0.65</td>
<td>~ 0.008</td>
<td>-0.21±0.10</td>
<td>-0.22±0.10</td>
</tr>
<tr>
<td>Yingwang Shan</td>
<td>Fig. 5 Ia</td>
<td>0.54</td>
<td>0.21</td>
<td>1111</td>
<td>1224</td>
<td>5.8</td>
<td>0.21</td>
<td>0.31</td>
<td>0</td>
<td>~ 0.03</td>
<td>~ -0.01</td>
</tr>
<tr>
<td></td>
<td>Fig. 5 Ib</td>
<td>0.20</td>
<td>0.32</td>
<td>1126</td>
<td>1225</td>
<td>5.0</td>
<td>0.39</td>
<td>0.33</td>
<td>0</td>
<td>~ 0.02</td>
<td>~ -0.01</td>
</tr>
<tr>
<td></td>
<td>Fig. 5 IIa</td>
<td>0.24</td>
<td>0.36</td>
<td>1111</td>
<td>1257</td>
<td>7.4</td>
<td>0.39</td>
<td>0.33</td>
<td>0</td>
<td>~ 0.02</td>
<td>~ -0.01</td>
</tr>
<tr>
<td></td>
<td>Fig. 5 IIb</td>
<td>0.30</td>
<td>0.31</td>
<td>1117</td>
<td>1224</td>
<td>5.4</td>
<td>0.39</td>
<td>0.33</td>
<td>0</td>
<td>~ 0.02</td>
<td>~ -0.01</td>
</tr>
<tr>
<td></td>
<td>Fig. 5 IIIa</td>
<td>0.29</td>
<td>0.46</td>
<td>1089</td>
<td>1256</td>
<td>8.6</td>
<td>0.49</td>
<td>0.35</td>
<td>0</td>
<td>~ 0.02</td>
<td>~ -0.01</td>
</tr>
<tr>
<td></td>
<td>Fig. 5 IIIb</td>
<td>0.56</td>
<td>0.37</td>
<td>1096</td>
<td>1203</td>
<td>5.3</td>
<td>0.49</td>
<td>0.35</td>
<td>0</td>
<td>~ 0.02</td>
<td>~ -0.01</td>
</tr>
</tbody>
</table>
4. Discussion

4.1 Location of channel heads

Willett et al. (2014) pioneered the use of cross-divide $\chi$ contrast to gauge the horizontal motion of drainage divides. According to their method, drainage divides are predicted to move toward the side with a higher $\chi$ value to achieve geomorphic equilibrium. However, in a region with spatially variable uplift rates, lithology, or precipitation, $\chi$ contrast may fail to reflect the drainage-divide migration (Willett et al., 2014; Whipple et al., 2017; Forte and Whipple, 2018; Wu et al., 2022; Zhou and Tan, 2023). In a tectonically active area, the cross-divide $\chi$ contrast can only be used in a small area where rock type, precipitation, and uplift rate are nearly uniform (Willett et al., 2014). Combining the advantages of the $\chi$ and Gilbert metrics methods, Zhou et al. (2022a) proposed to use the $\chi$ contrast with a high base level to calculate the $k_{sn}$ values at the channel heads on both sides of a drainage divide, and quantified the migration rate of drainage divides at the eastern margin of Tibet.

To reduce the cross-divide difference in uplift rate, precipitation, and rock strength, the Gilbert metrics or $\chi$-comparison method in Zhou et al. (2022a) should compare the parameters of points (slope, relief, elevation, and $k_{sn}$) on both sides of the divide as closely as possible. As the hillslope area (above the channel head) does not follow Eq. 1 (Stoke and Dietrich, 2006; Stark, 2010; Braun et al., 2018; Dahlquist et al., 2018), the channel heads are the closest point to the divide, following Eq. 1. Channel heads, therefore, are suitable for measuring the drainage-divide stability with
parameters of the upstream drainage area and channel gradient (Forte and Whipple, 2018; Zhou et al., 2022a). However, limited by the resolution of DEM, the location of the channel heads cannot always be accurately identified. The channel head parameters for calculating the migration rates are usually based on empirical values in previous studies (e.g., $A_{cr} = 10^5$ m$^2$ in Zhou et al. (2022a)), which may induce uncertainties.

In this study, we advocate the use of high-resolution DEM to determine a more accurate position and related parameters of the channel head. The use of UAVs to obtain the local DEM has become highly efficient. We advance the theory to calculate the drainage-divide migration rate based on the measured channel-head parameters. With the help of the aerial photography of UAVs and the SfM techniques, it is possible to obtain the high-resolution topography data of drainage divides (Figs. 4A & 5A) and get the required parameters (including the exact locations of the channel heads across the drainage divide) through topography analysis, which could improve the quantitative research on the drainage-divide migration. Furthermore, the method provides a new avenue to combine with catchment-wide $^{10}$Be erosion rate or low-temperature thermochronology data to calculate the migration rate, which has great potential for application in places where some variables are hard to be constrained.

4.2 Cross-divide difference in the uplift rate of the channel heads

Although the channel heads across the divide are very close on the spatial scale of
an orogenic belt, differential uplift between the channel heads ($\Delta U_{ch}$) could still exist, especially in a tilting horst, such as the Wutai Shan. The cross-divide difference in uplift rate could impact the calculation of the migration rate of drainage divides (Zhou et al., 2022a).

In this study, we quantify the influence of the cross-divide difference in rock uplift rate ($\Delta U_{ch}$) on the calculation of the migration rate of drainage divides at the Wutai Shan, benefiting from the available tectonic and chronological research (Clinkscales et al., 2020) and the newly obtained high-resolution topographic data. In the Wutai Shan horst, $\Delta U_{ch}$ across the drainage divide is ~0.008 mm/yr. We estimate the influence of $\Delta U_{ch}$ on the drainage-divide migration rate in this case study, which can reduce the error theoretically. If $\Delta U_{ch}$ is ignored, the drainage-divide migration rate would decrease by ~4% in the Wutai Shan case. Although ~4% seems to be negligible, such a ratio will increase if the mountain belt is narrower, the tilting uplift is stronger, or the divide is closer to the steady state (i.e., the migration rate is lower) (Whipple et al., 2017; Ye et al., 2022). In other words, the differential uplift may play a significant influence on the measurement of drainage-divide stability in some situations. If we consider an extreme example where the main drainage divide of a tilting mountain range (relatively narrow in width) is at a steady state, the gradient, relief, and elevation of the channel heads (collectively called “Gilbert metrics”) (Forte and Whipple, 2018) will show a systematic cross-divide difference in theory. In this case, the drainage divide would be considered unstable if $\Delta U_{ch}$ were neglected. Therefore, this study highlights that $\Delta U_{ch}$ should be taken into account, either in a
4.3 Limitations and uncertainties

This study develops the method to calculate the drainage-divide migration rate based on the measured channel-head parameters. However, uncertainties still exist because of the limitations of this technique. First, we assume the erosion coefficient \( K \) is the same on both sides of a drainage divide in the derivation of the equations. If there are differences in rock erodibility or precipitation across the divide, uncertainties should exist in the results. Second, the calculation of migration rate is based on the erosion rates at the channel area in this study. However, the occurrence of drainage-divide migration is directly driven by the differential erosion of the hillslope area across the divide, mainly via the processes including landslide, collapse, and diffusion (Stoke and Dietrich, 2006; Stark, 2010; Braun et al., 2018; Dahlquist et al., 2018).

Such discontinuous processes in the hillslope area make it challenging to constrain erosion rates over such short timescales. Over a relatively longer period (i.e., spanning multiple seismic and climatic cycles), the erosion rate at the channel head area in this study can be comparable with that at the hillslope area (Hurst et al., 2012; Godard et al., 2020).

The accuracy of the data and parameters can also impact the reliability of the results. First, we use the typical values of \( n = 1 \) and \( m/n = 0.45 \) in the two natural cases to calculate the migration rate. If the actual values largely deviate from the...
assumption, errors would be introduced into the results. For this reason, we have added the cases of $m/n = 0.35$ and $0.55$ in Supplementary Materials. Further estimation of these values (Mudd et al., 2018) could improve the accuracy of the results. Second, in the case of the Wutai Shan, we refer to the geological and low-temperature thermochronology studies and assume a $0.50 \pm 0.25$ mm/yr erosion rate at the northern margin of the Wutai Shan (i.e., the footwall of the North Wutai Shan fault). Combining with the present-day $k_{an}$, we calculate the erosion coefficient ($K$) and derive the migration rates of the drainage divide. If the present-day erosion rate deviates from the assumption, errors would be inevitable in the results. Moreover, the horizontal and vertical errors of the DEM data, as well as the calculation errors in slope, upstream area and channel steepness can also affect the reliability of the results. In the case study of the Yingwang Shan, the lush vegetation may bring errors to the DEM data based on the SfM technology. The application of airborne light detection and ranging (LiDAR) technology may help reduce this error. Future studies should take these challenges into account and overcome them.

5. Conclusions

We have developed a new method (called the "channel-head-point method") to calculate the migration rate of drainage divides based on channel-head parameters. We have also improved the previously proposed "channel-head-segment method" (Zhou et al., 2022a) to adapt the theory to areas where the parameters of channel-heads can
be accurately determined.

Using the new methods and high-resolution topographic data, we determined the exact locations of the channel heads on both sides of the drainage divide and quantified the drainage-divide migration rates in two natural cases in North China: Wutai Shan in the Shanxi Rift, and Yingwang Shan in the Loess Plateau. The migration rates of the study sites in the Wutai Shan are 0.21-0.27 mm/yr (northwestward). The rates are close to zero in the Yingwang Shan.

Based on the locations of the channel heads and the uplift gradient of the Wutai Shan, we calculated the cross-divide difference in the uplift rate at the channel heads ($\Delta U_{ch}$), which is taken into account in the calculation of the drainage-divide migration rate for the first time. If $\Delta U_{ch}$ is overlooked, the drainage-divide migration rate of the study sites in the Wutai Shan will be underestimated by ~4%. Our study highlights that $\Delta U_{ch}$ should be considered in the assessment of drainage divide stability based on the cross-divide difference in channel-head parameters.

**Data availability.** The analysis of data is based on the Matlab toolbox TAK ([Forte and Whipple, 2019](#)) and TopoToolbox ([Schwanghart and Scherler, 2014](#)). The topography data (ALOS DEM, 12.5 m resolution) is downloaded from the Alaska Satellite Facility (ASF) Data Search ([https://search.asf.alaska.edu/](https://search.asf.alaska.edu/)).

**Acknowledgements.** We would like to thank the Editor Simon Mudd, the Reviewer Thomas Bernard, and an anonymous reviewer whose suggestions have greatly improved the paper.
Financial support. This study is supported by the CAS Pioneer Hundred Talents Program (E2K2010010) and the Fundamental Research Funds for the State Key Laboratory of Earthquake Dynamics (LED2021A02).

Competing interests. The authors declare that they have no conflict of interest.

Author contributions. XT and CZ contributed to the design of the research scheme. CZ performed the geomorphic analyses. CZ, XT, and FS carried out field data collection. CZ, XT, YL, and FS contributed to the text and reviewed the paper.

References


Clark, M.K., Schoenbohm, L.M., Royden, L.H., Whipple, K.X., Burchfiel, B.C.,
https://doi.org/10.1029/2002tc001402

https://doi.org/10.1038/nature04379


https://doi.org/10.1016/j.geomorph.2005.08.023


Godard, V., Dosseto, A., Fleury, J., Bellier, O., Siame, L., 2019. Transient landscape
dynamics across the Southeastern Australian Escarpment. Earth Planet Sc Lett 506, 397-406. https://doi.org/10.1016/j.epsl.2018.11.017


Mudd, S.M., Clubb, F.J., Gailleton, B., Hurst, M.D., 2018. How concave are river


Sassolas-Serrayet, T., Cattin, R., Ferry, M., Godard, V., Simoes, M., 2019. Estimating the disequilibrium in denudation rates due to divide migration at the scale of
river basins. Earth Surface Dynamics 7, 1041-1057.

https://doi.org/10.5194/esurf-7-1041-2019


asymmetry: Analytical solution, numerical modeling, and natural examples.


Wei, Z., Arrowsmith, J.R., He, H., 2015. Evaluating fluvial terrace riser degradation using LiDAR-derived topography: An example from the northern Tian Shan,


Zhou, C., Tan, X., 2023. Quantifying the influence of asymmetric uplift, base level
elevation, and erodibility on cross-divide $\chi$ difference. Geomorphology 427,

strength and structural controls on fluvial erodibility: Implications for drainage
divide mobility in a collisional mountain belt. Earth Planet Sc Lett 538.
https://doi.org/10.1016/j.epsl.2020.116221