The comments from my review have been addressed to a satisfactory degree, and I think the present manuscript is acceptable for publication. As stated in my previous review, this manuscript should be a significant contribution, as it should invite researchers to interpret cautiously apparent trends or similar features in reanalysis estimates, which have become an important source of information in climate research.

Below is a list of minor points which the authors might want to consider before publication:

l. 27-30: this short paragraph is largely redundant with the last two sentences on l. 68-70 and could be dropped.

l. 68: "… Results and conclusions are discussed …"

l. 89: perhaps replace "deviations" with "uncertainties" or "errors"

l. 90-91: "… knowledge of the initial state elements and resulting uncertainties is required. A bracket …"

l. 108: "… Goodwin Sin (1984) for a fuller discussion". Although …"

l. 113: "… with uncertainty **P**(t,-) will be …"

l. 141: one could write more generally "… then the elements of $\mathbf{B}_c$ would be zero except in its second row."

l. 142: Replace **A** with $\mathbf{A}_c$

l. 143: Replace **A** with $\mathbf{A}_c$ and **B** with $\mathbf{B}_c$

l. 158: "Equation (8) is discretized at time intervals .. time-step:"

Equation after l. 175: Please number the equation and explain shortly in the text why this equation is an approximation.

l. 185: reference to figure 3a seems to be in order here: "… and is forced by (Fig. 3a) …"

l. 195: "…, as fully unknown, i.e., $\varepsilon(t) \neq 0$."

Table 1 is useful but I think it could be improved. For example, write "observational noise standard deviation is …" (two occurrences), write "$x_0 = (1,0,2,0,0,0)^T$" (add transpose), the expression "$0.5q(t) - \epsilon(t)$" (two occurrences) is ambiguous (e.g., do the authors mean "$0.5q(t) + \epsilon(t)$"?),

and the expression "$\boldsymbol{x}_0 = \boldsymbol{e}_1$" should be replaced with "$\boldsymbol{x}_0 = (1,0,0,0,0,0)^T$ except in section 3.1.2" ($\boldsymbol{e}_1$ is not defined and different initial conditions are assumed in section 3.1.2).

l. 204-205, "Noise with standard deviation 0.01 is added to the observations": This value conflicts with the value of 0.1 in Table 1. Please remove the contradiction. For completeness, please also specify the distribution (normal one?) assumed for the observational noise.

Caption of figure 6: "(a) Correct value … $x_3(t)$ (blue line) and the estimated value from the KF (red line) … $x_6(t)$ (blue line) … from the KF (red line) with error bar …"

l. 247-248, "… and occur in the two different sets of periodic time intervals": Please specify in the text what these intervals are.

l. 262: "… via the dynamical equations (4). Bias errors …"

l. 266: Replace "commonly" with "sometimes"?

Fig. 8 is not explicitly referred to in the text. Please refer to this figure in the text, or drop it.

l. 284: "…. $x_5(t)$, its uncertainty … $\boldsymbol{P}(t)^{-1}$ gives …"

l. 308-315: Could you provide a numerical illustration, based on one or two of the examples in the manuscript (mass-spring oscillator and Rossby wave model), that the state variables computed from the free-running model equations with the adjusted initial conditions and adjusted control from the RTS are identical (within roundoff errors) to those computed from the RTS equation (19)?

l. 371: "… but model equations with adjusted initial conditions and adjusted parameters and thus …"

l. 322-323: "…. the variation of the Coriolis parameter, f, with the latitude coordinate, y."

I think equation (22) should be written as

$$\frac{\partial \nabla'^2 \psi}{\partial t'} + \frac{\beta L}{f}\frac{\partial \psi'}{\partial x'} = 0$$

Equation (29): on the right-hand side, the first term represents the contribution from the western boundary layer and the second the contribution from the interior. It is surprising that the interior contribution does not involve beta … (cf. comment from 1st review).

Equation (30): shouldn't the last element of $\boldsymbol{x}(t)$ be ψ_s, not 1?

l. 367: "… given by $\exp(-i\sigma_j \Delta t)$, and …"

l. 389: Please define b.

l. 406: "… Observations cease after T_f, mimicking …"

Figure 10: the stream function minima in the interior suggests the calculations are for a subpolar gyre. This could perhaps be clarified in the text and/or the figure caption.

l. 424: Perhaps extend the last sentence as follows: "… in the interpretation of results and in the development of observing strategies."

l. 431: "…, the control vector of $\boldsymbol{q}(t)$ in this interval, …"

l. 463-465: "… concern (ECCO … Lagrange multipliers; Stammer et al. 2002). For …"

l. 496: "Use of Kalman filters and the simple analogues … produce …"

l. 521: "… constituents (Heimbach et al. 2019). The Lagrange …"

l. 523: "… practiced in numerical weather prediction; (ii) …"

l. 536: $\boldsymbol{\Gamma}$ is the distribution matrix for the stochastic forcing in the state-transition equation.

l. 473: Should "0" be in bold, e.g., "$\mathbf{G}(t) = \mathbf{0}$"?

l. 581: "… to time T_f so that …"