

This manuscript discusses the Rietkerk model for vegetation patterns, based on a system of nonlinear equations for biomass, soil water and surface water. Various stable and unstable solutions are found, with a special focus on the existence of mixed states, which appear at the transition between stable and unstable solutions.

In my opinion, the manuscript is interesting, providing numerical solutions which allow to understand the essential features of the system. However, part of the discussion is not clear, some of the figures are not consistent with the text, and sometimes notation is confusing, so the manuscript should be improved before being considered for publication.

My detailed comments on the text are below:

1. Page 2, before Eq. (1). In the system of equations, all quantities should be defined ( $c$ ,  $g_{\max}$ ,  $k_1$ , etc.).
2. Page 3, paragraph 1: “how they can have an influence on the dynamics of the system”. Is it correct to say that a solution can have an “influence” on the dynamics of the system? Because a solution is an expression of the dynamics itself, it does not *influence* the dynamics.
3. Page 3, after Eq. (10): “Fig. 01 shows these solutions as a function of rainfall  $R$ ”. The description in the text seems to correspond to the leftmost panel, with a vegetated solution for  $R > 1$ . But what do the other panels represent?

Also, the second and third panels show yellow lines (solutions without vegetations) with biomass larger than zero. How this should be interpreted?

4. Page 4, Eq. (18). It is confusing to use letters that have been used before ( $c$ ,  $d$ ). Please use other names for these variables.
5. Page 5, paragraph 2: “ $\delta = \cos(\kappa x)$ ”. This may be confusing, as  $\delta$  has other meanings elsewhere, please change.
6. Page 5, after Eq. (23): “This iterative procedure leads to the construction of the full branch of solutions.” Could there be other nonlinear solutions, which cannot be obtained via this method? (Because they are not in the same attraction basin of the Newton algorithm, for instance.)

7. Page 6, paragraph 4: “displayed on Fig.05.” I do not think I understand this figure. The matrix has dimension 3, so for a given value of  $R$  and  $\kappa$ , there should be 3 eigenvalues. Why are there so many points in the lower left panel of Fig. 5?

What does the black line represent? Is it there only to mark the 0 position on the horizontal axis? This should be explained. Also, it should be explained what are the vertical and horizontal axis (imaginary and real part?).

8. Page 6, paragraph 4: “ $3 \cdot 10^{-10}$ ”. Since eigenvalues represent frequency, do they have units? Other quantities in the system of equations do have units ( $R$ ,  $B$ , etc.).

If not, how is frequency normalized?

Also, please change the notation to use a centered dot ( $\cdot$ ).

9. Page 7, line 203: “we expect mixed-state modes to influence the dynamics of nearby trajectories, depending on the value of the positive eigenvalue.”. This should be clarified. How can one mode “influence” the dynamics of other orbits, and what is the role of the positive eigenvalue in this.
10. Page 8, line 219: “the basin of attraction of the latter is narrow”. Or it could be that the basin of attraction does not include regions close to the homogeneous zero state. Maybe starting from other points in phase space leads to  $n = 4$ , and the basin of attraction may turn out to be not small.
11. Page 8, line 220:  $s = 4$  should be  $n = 4$ ?
12. Page 8, line 224: “unstable mixed-state solutions are also able to influence system transient trajectories.” It is not clear that Fig. 8, which is used to explain this, really settles this. It shows that starting from an unstable mixed-state leads to an interesting dynamics. On the other hand, one should always expect that starting from an unstable solution leads to a transient trajectory, until the system is attracted to a stable solution.  
  
So it is not clear that the numerical example actually shows that mixed-state solutions “influence” trajectories. This should be better explained, and maybe rewording the statement is enough. (I am not sure that “influence” is the right word here.)
13. Page 9, table 1. Should it be “ $9 \cdot 10^{-9}$ ” in the third column?
14. Page 9, caption of table 1: “for two unstable states”. It should have some reference to Fig. 08, as curves for a given value of  $n$  are not the same for different conditions.
15. Page 9, line 244: “unstable modes on neighbouring dynamics is well-known in complex systems (Lucarini and Bódai, 2017)”. If I understand correctly, the reference provided shows that what they call an edge state can, for instance, connect to coexistent attractors via noise induced transitions. But the authors here do not discuss the role of noise to connect two stable modes (not in Fig. 8 at least), so, again, I am not sure the authors have been clear on their claims regarding this statement.
16. Page 9, line 251: “infinite-size systems”. Periodic boundary conditions may represent infinite-size systems as well, or finite systems as long as one is far from the boundaries. But changing to other boundary conditions, either for finite or infinite systems, would be interesting.
17. Page 9, line 254: “The ones with odd  $n$  number appear”. But, in the analysis presented in this manuscript, both even and odd values of  $n$  appear. What are the authors actually saying here?

18. Page 9, line 257: “reasonably robust”. It seems, from the analysis presented, that as long as periodic boundary conditions are preserved, the same results would be obtained, essentially. More modes may appear, dynamics may be more complex due to this fact, but the essential features would be the same. The result discussed by the authors by doubling the size of the systems does not seem surprising.
19. Fig. 6. At this point, the “bis” notation appears for the first point, so please explain the meaning of this in the caption, or refer to the main text to understand.
20. Fig. 8: “a pseudo phase space”. Why do the authors use this expression, if they have used “summary phase space” for the same axes in previous figures?

There are also some formal issues which should be addressed:

1. Please clarify units. In page 1, for instance, units are marked as *mm*. Does the difference between italics and roman font mean something? Or it should be mm? In general, units should be in roman font, so the authors should fix this in the whole text, but the additional mix of fonts in some variables is confusing.
2. In various places (page 1, for instance), products between units are marked with a dot (.). This should be either  $\text{g} \cdot \text{m}^{-2}$  or (better)  $\text{g m}^{-2}$ .
3. Please fix typos
  - Spaces around parenthesis, periods, commas, spaces between words in several places in the text.
  - “A this point”
  - “Fig.06,.”
  - “a an other”
  - “biommass”
  - “wit ha”
  - “pentagone”
  - “righth”
4. Check consistency of words:
  - “Those zero mode”
  - “We need to discretization”
5. Please change words where necessary:
  - “about your system”: “the” sounds better
  - “such was”

- “solution changes loses stability”
  - “the branching of yet another branch”
  - “which which”
  - “are the indeed”
6. “*3loc*”. What do the authors mean with “loc”?
  7. Math variables should be in italic font.
  8. Some references have incomplete bibliographic information (Meron 2015, Rietkerk 2021).
  9. Fig. 5. It is hard to see that the black symbol is actually a pentagon. Please change to another, simpler shape.
  10. Some decimal numbers are written with a comma, instead of a point (*e.g.* 1,13).