Response to the Referee 1

Thank you for your suggestions. We have responded to the questions and suggestions below. Our response is provided in red text. In addition to the revisions to the manuscript based on reviewers' suggestions, we also incorporated the momentum feedback from SDs to fluids in all SDM runs (see, e.g., Eq. (81) of Shima et al. (2020)), which was unintentionally ignored in the previous results. All the SDM plots are replaced by the new results. However, since the effect of the momentum coupling is not significant for this case, this modification does not alter the main conclusions of the present study.

Our main emphasis in this paper is on the study of the numerical convergence characteristics of SDM and SN14 for stratocumulus. While we do examine the differences between these two schemes, it's essential to recognize that this examination is not the primary focus of our research. We aim to convey this distinction to ensure a clear understanding of our research priorities.

1. Two ideas are proposed for explaining the differences between the SDM and the SN14 schemes: the numerical diffusion, and the droplet sedimentation. I was wondering if the differences could also be caused by the differences in the representation of collisions (stochastic SDM vs deterministic SM14)? Are the precipitation formation and evaporation rates and locations similar between the two schemes? Do the ql and qr co-vary in a similar way between the two schemes? I'm guessing the precipitation is more "continuous" when simulated by SN14? I realise that the precipitation rates reported in this study are very small, but I was wondering if they could still be affecting the simulations?

   Reply: The differences in the representation of collisions will not affect the results of this case. We have tried to turn off the coalescence process in both schemes, but the results didn't change much. The precipitation is very little in both SDM and SN14 simulations (Figs. 1 and 9), so collisions occur so infrequently that they have little effect on the results.

   The domain averaged precipitation formation and evaporation rates are similar between the two schemes, but their spatial distribution is quite different. The time evolution of the vertical cross sections of q_r of SDM and SN14 simulations are provided as a video supplement (see the video supplement section). As you expected, the spatial distribution of rain water in SN14 is continuous, whereas that of SDM is sparse and discrete.

   It is difficult to tell whether the q_r varies in a similar way between the two schemes, because the q_r is too small. But for q_l variations, our answer is yes. Please check the time evolution of vertical profiles from https://doi.org/10.5281/zenodo.8389677.

The small precipitation has only a small impact on the simulation. Except for precipitation, the SN14 results agree well with DYCOMS MIP. SDM also exhibited similarly low precipitation. Therefore, we can conclude that the difference in buoyancy production and total TKE are not caused by their low precipitation, but rather from the fundamental differences between the two schemes. Dziekan et al. (2019) studied the same stratocumulus case with their SDM model, which also produced little surface precipitation, and this was the biggest difference from the DYCOMS MIP. In their next study, they found that the precipitation of stratocumulus clouds was greatly increased if GCCN was included in SDM simulations (Dziekan et al., 2021).

2. Figure 3 vs 11. I didn't fully understand from the discussion why the buoyancy production and w variance are so different in the cloud layer between the SDM and SN14 runs? Is it only related to the differences in CC? Can the SDM method match those profiles in simulations with larger ql? Similarly, the integrated tke still looks different for SN14 and SDM without sedimentation on Figure 15? Could the authors comment on why that is the case?

   Reply: Due to the smaller CC in SDM, the free atmosphere slowly erodes the stratocumulus topped boundary layer, which slowly results in the smaller buoyancy production and *w* variance in SDM. This is the main mechanism we propose in this study. If we look at the buoyancy production profiles of SDM, SN14 and SDM_no_sed during the 1-1.5h, in which period the stratocumulus structure including the CC are much closer, the buoyancy production profiles are also much closer than that during the last 4 hours (Fig. A1). This supports our picture that the smaller CC of SDM eventually caused the big difference of the buoyancy production in later stages of the simulations.

   However, we have also realized that the difference in buoyancy production and *w* variance may not be fully explained by the difference in CC. To gain a deeper understanding, we decomposed the buoyancy production formula and compared the time evolution of each term of the formula in the two schemes (Movie 1 in the supplement).

$$B = \frac{g}{\langle \theta_v \rangle} \cdot \langle w' \theta_v' \rangle = \frac{g}{\langle \theta_v \rangle} \sigma(w) \sigma(\theta_v) R(w, \theta_v) \qquad (1)$$

   $R(w, \theta_v)$ is the correlation coefficient of w and θ_v, and g is the gravitational acceleration. From the time evolution of the vertical profiles, we found that each term of the equation (1) for SN14 is greater than that in SDM in the cloud layer even during the 1-1.5h. The larger CC results in stronger radiative cooling of the cloud tops and hence lower θ_v. This explains the larger $\frac{g}{\langle\theta_v\rangle}$ in SN14 and SDM_no_sed. Time evolution of $\sigma(w)$ and $\sigma(\theta_v)$ (Movie 1) and the scatter plots of w and θ_v within the cloud layer (Movie 2) illustrate a more spread of vertical wind speed and temperature distributions in SN14 even during the 1-1.5h.

Twomey activation is adopted in SN14 and it tends to make activation/deactivation processes occur more frequently (Hoffmann, 2016; Yang et al., 2023), while activation/deactivation is calculated explicitly in SDM. We speculate that the difference of CCN activation/deactivation treatment in the two schemes would be playing some role which may affect $\sigma(w)$ and $\sigma(\theta_v)$, but the mechanism is still unclear, and we will leave it for future study. Furthermore, the numerical diffusion in SN14 could lead to an unphysical artifact of liquid water and then results in an increase in the spread of θ_v. We add the discussion regarding to activation/deactivation treatment in the revised manuscript (Page 15, Line 500-504).

We must acknowledge that we have not fully comprehended the reasons behind the disparity in TKE between SN14 and SDM_no_sed. When we disabled sedimentation in SDM, aerosols, especially those near the cloud top, were sustained. Consequently, while CC increased, it also reduced the thickness of the cloud, thereby weakening TKE. We plan to conduct additional sensitivity experiments in the future to validate this hypothesis.

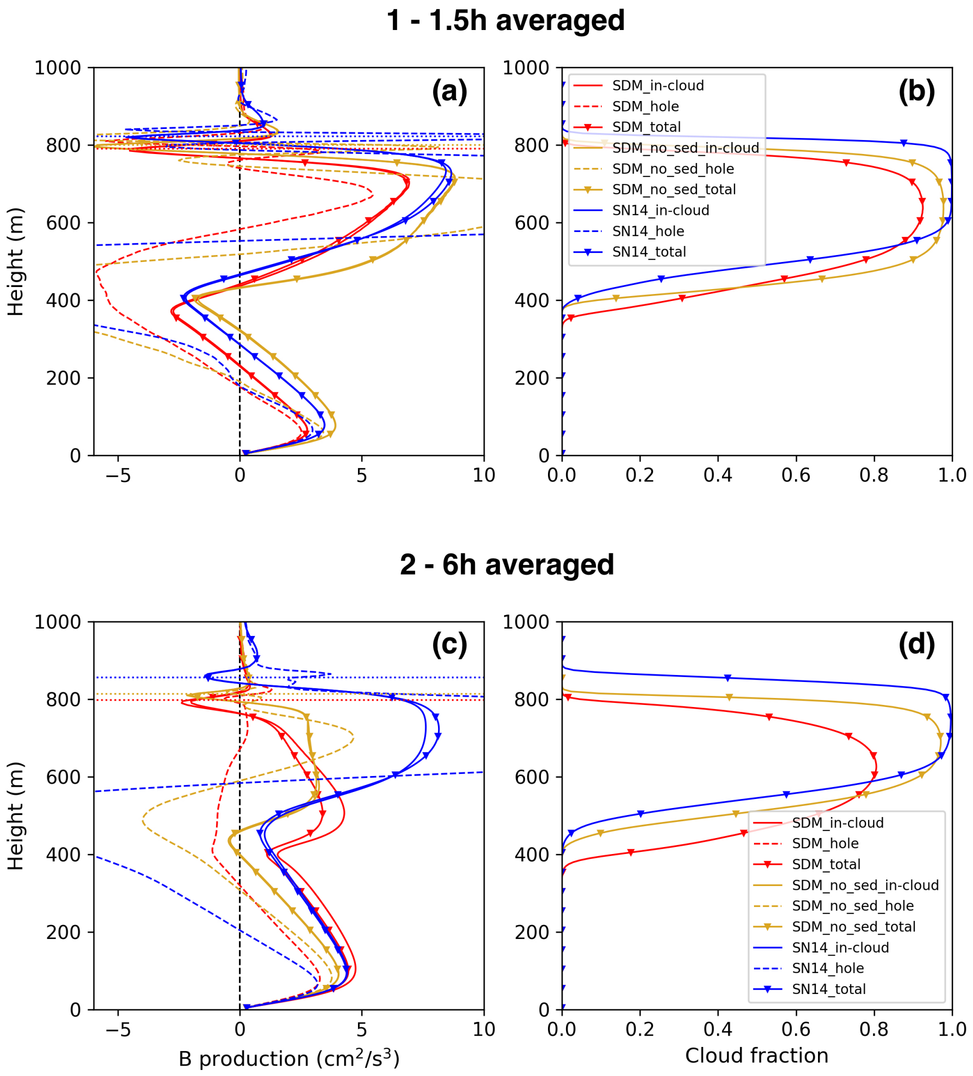

Fig. A1 Vertical profile of (a) buoyancy production and (b) cloud fraction during the 1-1.5h in SDM, SN14 and SDM_no_sed. (c) and (d) are the same profiles, but averaged during the last 4 hours. Horizontal dotted lines indicate the average inversion heights in the respective simulations during the averaging time. It is worth noting that when we calculate the covariance of w and θ_v in the cloud and in the cloud holes, we use the domain average, not the in-cloud average or in-hole average, to make them consistent to the domain average $\langle w'\theta_v'\rangle$.

3. Figure 13 - Why is the pressure so different between SDM and SN14?

   Reply: Pressure profile of SDM and SN14 with real height are similar. The inversion height (z_i) of SN14 is larger than that of SDM, so the profile with normalized height (z/z_i) looks smaller (Fig. A2). We have incorporated the explanations mentioned above in the revised manuscript, specifically on Page 14, Line 456-460, as well as in the figure caption for Figure 13.

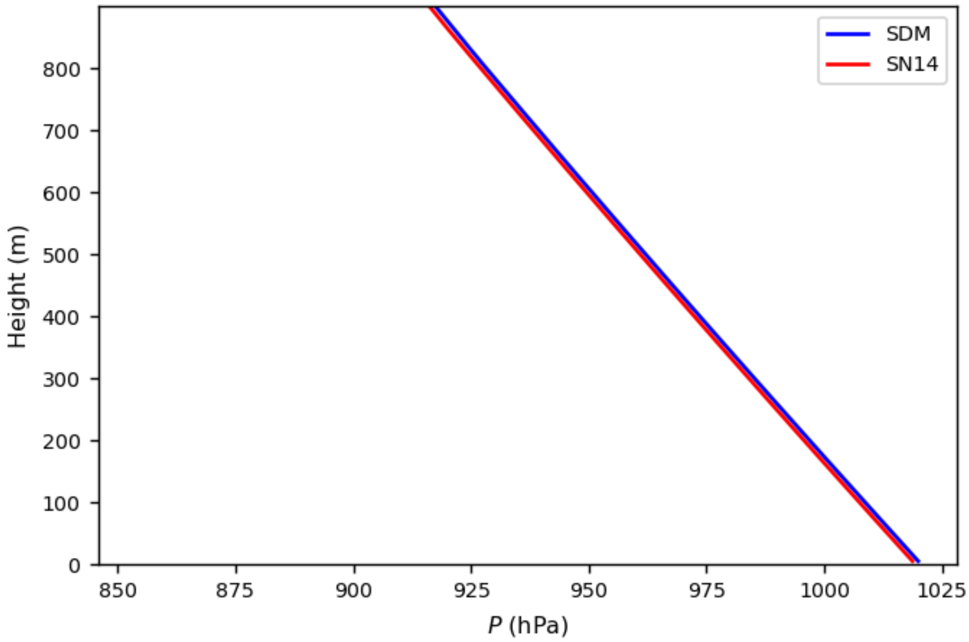

Fig. A2 Vertical profile of pressure in SN14 and SDM.

4. dz ~ 2.5 m is a very fine resolution for LES, and yet the simulations do not converge. I was wondering what recommendations the authors have related to that issue. What should be done in cases where due to computational limitations the simulations cannot be run at such high resolutions? Would using stretched grids help? Would using higher order advection schemes help? Any other suggestions?

   Reply: If the computational resources are limited but more accurate simulation results are needed, we recommend reducing the domain size with as fine a grid resolution as possible. Mesoscale circulation is important for modeling stratocumulus clouds. Therefore, if computing resources are sufficient, it is recommended to use a domain size no smaller than the one we are using (6 km x 6 km x 1.5 km). For the SDM simulations, the number of SDs can be reduced appropriately (16 SDs/cell for this case).

   Using stretched grids would be useful. As mentioned in this paper, for the simulation of stratocumulus clouds, the liquid-water related variables (e.g., LWP, CC, CF) are more dependent on the vertical resolution than the horizontal resolution. Therefore, the vertical resolution can be set finer in the boundary layer than in the free atmosphere to save computational resources. Moreover, some studies have pointed out that strong radiative cooling at the top of stratocumulus maintains a very thin inversion structure, and turbulent entrainment through this thin layer can have significant feedback effects on boundary layer and cloud properties (Mellado et al., 2018). As a results, the vertical resolution near the cloud top should be fine.

For the simulations shown in the paper, the 3rd-order upwind scheme (3UD) is used for tracer variables. We also tried the 4th-order central difference scheme (4CD) for the coarse-resolution case. Since we do not have enough resources to perform the grid resolution convergence experiments under the higher order advection scheme, we cannot prove whether changing the advection scheme is helpful.

For more advice, you can refer to this paper (Matsushima et al., 2023). The authors improved the SDM algorithm to increase the computational efficiency drastically.

5. Table 1 - Seems like most of the dts are the same. Would it improve the presentation to only show the different ones? For example in the last column just say DT_cnd = DT_coa = DT_adv and then just print one number in the column?

   Reply: It's a good idea. Thank you! The last two columns of Table 1 are modified as "(DT=DT_PHY_SF=DT_PHY_TB=DT_PHY_MP=DT_PHY_RD)/DT_DYN (s)*" and "DT_cnd=DT_coa=DT_adv (s)**", respectively.

6. Figure 3 and 11 - Would it be possible to also include a qr plot with the axis limits set to showcase the SDM and SN14 results?
   Reply: We wanted to show the big difference between our simulation results and the DYCOMS MIP, so we used this x-axis to show all the results. Since the results of our simulated q_r are very small (much less than 0.001 g/kg), it is not very meaningful to show the exact values of SDM and SN14 in the plot.

7. Caption of Fig 13 - Should be ql and not qt?
   Reply: You're right. Problem is solved. Thank you!

Video supplement.
The video supplement related to this response is available online at: https://doi.org/10.5281/zenodo.8397654.

Reference

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