Reply to Referee 1

First of all, thank you very much for reviewing our manuscript in detail and giving us very valuable feedback. In what follows, we respond to your comments and questions, point by point, and propose changes to the manuscript in accordance. We think that these changes will improve the quality and clarity of our manuscript.

In order to improve the readability of our replies we applied a color/type coding to discriminate our replies from the referee's comments. We have attached our replies as a pdf document since color coding is not available in the browser based text editor.

Color/type coding:
Comment by the referee.
Reply from the authors.

While I think the paper can be published as it is, I suggest a few minor points that could improve the presentation of the results:

1. The authors have used Dansgaard-Oeschger (DO) events as indicators of tipping in the AMOC. They briefly mention this in the abstract (line 5) and later provide some references in the introduction (lines 48-51). However, if the main message of the paper is to propose "robust precursor signals for a possible future AMOC collapse," I think further discussion is required to establish a clear connection between DO events and the transition from a strong to a weak AMOC. I think the paper could benefit from a new section that addresses this point.

Thank you for pointing out this. There are some pieces of evidence that DO events have associated reorganizations of AMOC. One is concurring changes of the North Atlantic temperatures and the ocean circulation indices such as Pa/Th (e.g., Henry et al., Science 2016). Recent general circulation models also support that the changes in AMOC and its meridional heat transport are key elements of DO oscillations, as briefly mentioned in the introduction (line 48-51). Thus we suppose that DO cooling transitions recorded in Greenland records reflect past AMOC tippings. In the revised manuscript, we will add explanations about the connection between DO events and AMOC changes, to thoroughly address the referee's concern.

Our results show that several DO cooling transitions are indeed preceded by statistical precursor signals. This may increase our confidence that an AMOC transition from the strong to the weak state can be captured by the critical-slowing-down-based statistical precursor signals (e.g., Boulton et al. 2014; Boers, 2021). However, we have not proposed that the same precursor signals 'must' be observed at a possible future AMOC collapse since from the mechanistic point of view the recent AMOC weakening, which is likely driven by global warming, is different from the past AMOC declines during the glacial period (as already mentioned in line 62). We will emphasize this more in the revised manuscript.

Ref. Henry, L. G., et al. "North Atlantic ocean circulation and abrupt climate change during the last glaciation." Science 353.6298 (2016): 470-474.

2. It is well established that changes in variance and autocorrelations are good indicators of critical slowing down (occurring during codimension one bifurcations). However, does this approach work as effectively for more complex tipping mechanisms, such as excitability (suggested in section 4 as a possible mechanism)?

Thank you for this remark. We consider that there exist chances to observe statistical precursor signals (SPS) in the critical slowing down indicators if the fast subsystem has a critical point (like a saddle-node bifurcation point) and if a component of the slow subsystem works like a slowly-changing parameter crossing the critical point, as shown in examples in Figs. 5b-5g. However, these are not always rigorous critical slowing downs. In the example of an excitable system (Figs. 5b-5c), the underlying system always has a weakly stable fixed point, and no true bifurcation leading to critical slowing down occurs. In fact, the actual tipping in this case would be noise-induced. However, we can effectively observe the SPS in the critical slowing down indicators in this case as well, since the system would in each cyclic iteration move from more stable to less stable conditions until it finally tips to initiate the next cycle; and this partial decrease in stability is imprinted in the CSD indicators (Fig. S29). Definitely each high-dimensional mechanism giving rise to SPS in Section 4 must be investigated in more detail. We will mention this in the revised manuscript and will suggest further theoretical work in this regard as a valuable topic for further research.

3. I would like to draw attention to the rate-induced mechanism, where an excessively rapid change in forcing can tip the system even before reaching the bifurcation point. This mechanism could arise from mechanism 3 (the Hopf bifurcation), where the system can cross the unstable limit cycle (regular threshold) and tip. It could also be relevant to mechanism 4, where the rate of forcing might push the system to cross an irregular threshold in the form of a maximal canard. Please see (Wieczorek et al. 2023) and (O'Sullivan et al. 2023), for more details:

Wieczorek, Sebastian, Chun Xie, and Peter Ashwin. "Rate-induced tipping: Thresholds, edge states, and connecting orbits." Nonlinearity 36.6 (2023): 3238.

O'Sullivan, Eoin, Kieran Mulchrone, and Sebastian Wieczorek. "Rate-induced tipping to metastable zombie fires." Proceedings of the Royal Society A 479.2275 (2023): 20220647.

Thank you for pointing out the rate-induced mechanism and providing these useful references. Indeed the rate-induced tipping is proposed as a possible mechanism of AMOC shutdown especially under a rapid increase in freshwater forcing (e.g., Alkhayuon et al. 2019; Lohmann and Ditlevsen 2021; Ritchie et al. 2023). While we have interpreted DO cooling transitions as an analogue of bifurcation-induced tipping (with slowly changing parameter), the rate-induced mechanism (with rapidly changing parameter) is definitely worth mentioning. We will mention it in the revised manuscript.