Information:

Journal:	Weather and Climate Dynamics
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Title:	On the quasi-steady vorticity balance in the mature stage of hurricane Irma (2017)
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Summary:

This study is to examine the impermeability theorem for potential vorticity substance (PVS) on isentropic surfaces during the mature (i.e., quasi-steady) stage for the storm intensity of Hurricane Irma (2017). The examination is based on the vorticity budget analysis with a numerical simulation of Hurricane Irma. Results indicated that the radially outward vorticity flux due to the divergence was mostly canceled by the radially inward vorticity flux due to the diabatic heating under the vertical shear of the tangential wind above the atmospheric boundary layer as expected in the impermeability theorem. The results also indicated a minor contribution of parameterized turbulence to the vorticity balance during the mature stage.

General comments:

The authors attempted to examine the theorem based on the vorticity budget analysis with the numerical simulation. The topic has a scientific interest. Particularly, the examination of the applicability of the PVS theorem to tropical cyclones (TCs) is important for a better understanding of TC dynamics because the theorem is a basic concept to understand atmospheric dynamics. The simulation could capture the maximum intensity of Irma and maintenance of the maximum intensity. However, the budget analysis from the simulation can have some issues related to accuracy and discussions.

One critical issue is the accuracy of the budget analysis. Figure 6b (6c) shows the divergence of the total vorticity flux (η-tendency). According to Eq. (7), the η-tendency will be equal to the convergence of the total vorticity flux $\mathbf{J}$ (i.e., $-\boldsymbol{\nabla}\cdot\mathbf{J}$). I'm wondering if the pattern (or sign) of the η-tendency in Fig. 6c is opposite to that of the convergence of the total flux (which can be imagined from the divergence in Figs. 6b). If Figure 6b correctly shows the divergence of the total flux, we can expect the non-minor divergence of the vorticity flux due to the parameterized turbulence with the opposite-sign pattern to the total-flux divergence in Fig. 6b. In fact, the storm has the radius of maximum wind (RMW) at around 30 km from the center during the analysis period (Fig. 5b). According to Eq. (A6), the generation of the parameterized turbulence can be active in the inside (approximately 20-30 km from the center) of the RMW because there is strong horizontal shear of the tangential wind of the storm. The convergence of the eddy flux due to the turbulence can be negative (i.e., corresponding to a decrease in the η) within the RMW because the turbulence will work to smooth the sharp gradient of the tangential wind near the RMW. I'm concerned that it might be difficult to ignore the vorticity flux due to the parameterized turbulence if the above consideration is correct. Thus, I strongly recommend that the authors need to evaluate the eddy flux due to the parameterized turbulence in the vorticity budget. In general, to better estimate the vorticity flux due to the parameterized turbulence, we can directly use model variables which are output as external forces in the horizontal momentum equations due to parameterized turbulence.

Other issues are related to the presentation and discussion in the manuscript (Please see specific comments).

For the above reasons, I consider that the current manuscript may need major revision for the above issues. After revising, I will reconsider the recommendation.

Specific comments:

1. L46-47: Figure 1a is difficult to see the eyewall location because updrafts indicating the eyewall are not indicated. Please specify the radius of the eyewall in this sentence.

2. L49-50: Did the authors quantitatively confirm the radiative cooling for the warm-core extension? Previous studies indicated that adiabatic processes associated with subsidence in the eye can be a major contribution to the development of the warming in the eye (e.g., **??**).

3. Eq. (3): Please specify the meaning of $\hat{x}$ and $\hat{y}$.

4. L224-225: Please insert the equation numbers corresponding to the explanation in the end of the sentence.

5. L216-217: It seems that the sentence is not consistent with Fig. 5c. Figure 5c shows the turbulent flux component and the turbulent length scale squared. Please clarify it.

6. L217-218: (Related to general comments,) The η-tendency may increase in regions where the radial flux "converges", according to Eq. (7).

7. L218-220: According to Fig. 5c, the total flux has negative values at radii from 20 km to 30 km and 310-K potential temperature level, which means that the lower maximum in the advective flux is slightly weaker (not stronger) than that in the diabatic flux. Please clarify it.

8. L220: For the sentence "*A similar but reversed structure may be noted near the higher maximum (Fig. 6)*", please specify the panel number of Fig. 6.

9. Appendix A: Please specify what the authors derive in the title (ex., "Derivation of XX").

Typos:

1. L27: ”models models” $\rightarrow$ ”models”.
2. Eq. (4): The right-hand side is not closed by the parenthesis.
3. Fig. 6: The definition of the regions I and II is inconsistent with the legends of Fig. 6a. According to Fig. 6a, the result of the region I (II) is indicated by solid (dashed) line.