

9 December 2024

Dear Editor,

We once again thank the Editor for further considering our manuscript and providing detailed and useful feedback, which prompted additional analyses as presented in our revised manuscript.

As suggested by the Editor, we further expanded the analysis by using “free-fitting” time-dependent MAC curves. We have confirmed that free-fitting time-dependent MAC curves can better fit to the relationship between the abatement level and the carbon price from IAMs than our previous time-dependent MAC curves (referred to as “transitional” time-dependent MAC curves in our revised manuscript).

We have also confirmed that the use of time-dependent MAC curves in our emulator can improve the reproducibility of the original IAM emission scenarios, compared to the use of time-independent MAC curves. At the same time, however, we found that this occurs only under certain conditions. Namely, if we prescribe carbon price pathways to the emulator (a new experiment presented in the revised manuscript), emission scenarios are better reproduced with time-dependent MAC curves than with time-independent MAC curves. On the other hand, if we endogenously optimize carbon price pathways for given carbon budgets in our emulator (our default approach), time-dependent MAC curves are no longer superior to time-independent MAC curves. We have examined this issue in detail and further confirmed that the difference in carbon price pathways between the emulator and the IAMs plays a major role here. We think that capturing carbon price pathways is a salient point for the future development of IAM emulators. Nevertheless, a notable advance over the previous manuscript is that we now have clear cases demonstrating that time-dependent MAC curves are better at reproducing scenarios, which we believe has helped improve the clarity of our paper.

Please see the attached document for our point-by-point responses to the Editor’s comments. We believe that our revised manuscript has carefully addressed the Editor’s remaining concerns and meets the high standards of *Geoscientific Model Development*. We look forward to the Editor’s decision for publication.

Yours sincerely,



Katsumasa Tanaka and Weiwei Xiong, on behalf of the author team

--- Editor's comments

The authors put substantial into the revisions, adding to an already complex study.

However, some of the two key issues still are not fully satisfactorily addressed:

\* The authors acknowledge that a time-dependent formulation of the MACs performs better than the time-independent one. However, they argue, that when applied in the GET model, it does not make much of a difference. I don't find this convincing: Given that get has a different solution and/or optimization rationale (differences in foresight, different discount rates) between GET and the IAMs to be emulated, the MACs but not the reproduction of emissions or carbon prices should be the relevant benchmark.

[Comment] We appreciate the Editor's comment, which prompted our new additional analysis. As stated in the cover letter, we found cases in which using time-dependent formulation of MAC curves can better reproduce emissions scenarios, but only under certain conditions. In light of this finding, we have substantially revised Section 5 as copied at the end of this document, which should address the Editor's comment above.

\* I still maintain that constraining the time-dependent MAC to parameters derived from the time-independent leads to a poorer than necessary performance of the emulator. For instance, for  $t > 2050$ , the emulator seems forced to the fit derived for the full period, instead of only 2050-2100. Why? For  $t < 2050$ , the time-dependent emulator is formulated as a deviation of the time-independent fit. Fig 11 clearly shows that this formulation does not do very well in reproducing POLES and WITCH data points. The concluding comparison of time-dependent with time-independent emulation therefore seems biased against the time-dependent formulation. Have the authors tried to perform a free fit for each time step? I would expect this to result in a much better performance, and see it as a necessary basis for a meaningful discussion of time-dependent vs. time-independent.

[Comment] As stated in the cover letter, we have included the suggested formulation of the time-dependent MAC curves, which is referred to as "free-fitting" time-dependent MAC curves in the revised manuscript. Free-fitting time-dependent MAC curves indeed better capture the relationship between the abatement level and the carbon price from IAMs. Using free-fitting time-dependent MAC curves can also improve, as expected, the scenario reproducibility with prescribed carbon price pathways. For further details, please see Section 5 as copied at the end of this document.

\* In the conclusions, the authors write "The behaviors of IAMs that contain various time-dependent processes were generally well captured by the time-independent MAC curves". What finding or test

is this based upon? At least regarding short-term emissions (until ~2050) I would need to be convinced that this holds. And these shorter time-scales matter for climate change policy and for overshoots. So it seems more accurate to condition this statement on the long-term behavior. I would also ask the authors to add a caveat in the conclusions on the accuracy of the emulation for shorter timescales.

[Comment] We agree with the Editor that this statement needs to be made conditional. We have revised and expanded the text to the following. The new text also addresses the suggested point on the accuracy of the emulation for shorter timescales.

“The behaviors of IAMs that contain various time-dependent processes were generally well captured by the time-independent MAC curves in the second half of the century, although the goodness of fit varies considerably among IAMs. However, time-independent MAC curves can work only poorly on shorter timescales for many IAMs due to processes and factors that can cause inertia in IAMs, including capital stock, growth rate constraints on technology expansion, and availability of new technologies.”

Furthermore, given the new insights from the additional analyses presented in the revised manuscript, we have revised the final bullet point in the Conclusions section to the following:

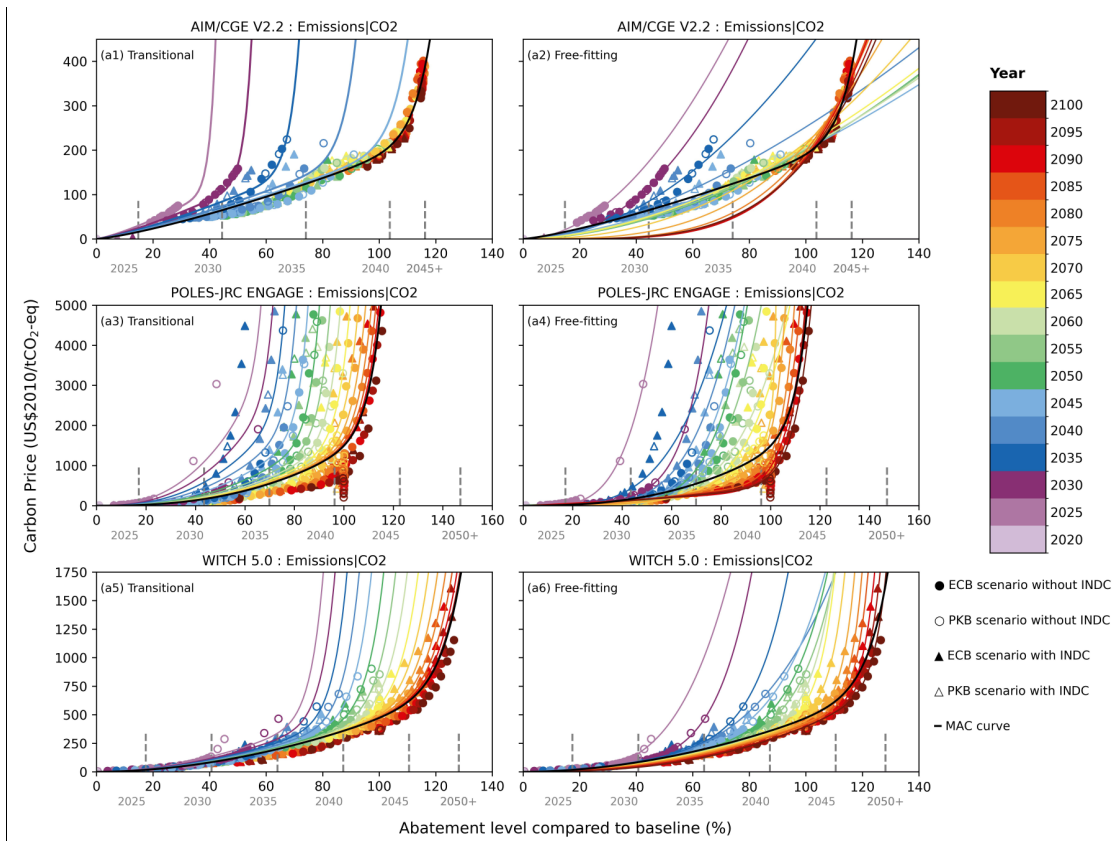
“For certain IAMs (AIM, POLES, and WITCH), time-dependent MAC curves provide a better fit to the price-quantity data generated from the original IAM than time-independent MAC curves. However, the use of time-dependent MAC curves improves the reproducibility of emission scenarios only when the equivalent carbon price pathway is prescribed to the emulator. When the carbon price pathway is endogenously optimized under the equivalent carbon budget in the emulator, it will differ from the carbon price pathway used for the IAMs. This difference in carbon prices can negate the benefit of using time-dependent MAC curves. The overall performance of the emulator is determined by a complex interplay of various factors, including the MAC curves, the upper bounds of the first and second derivative limits, and carbon price pathways. Reproducing carbon price pathways will be an important consideration for the future development of IAM emulators.”

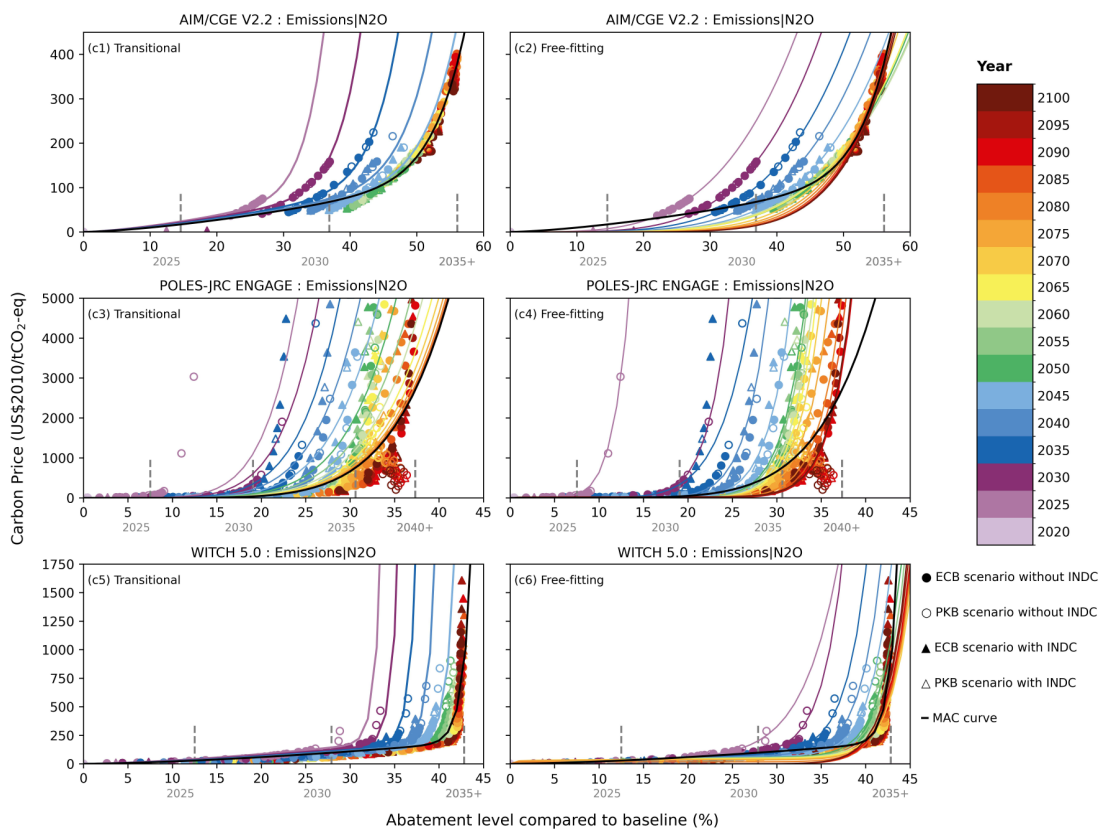
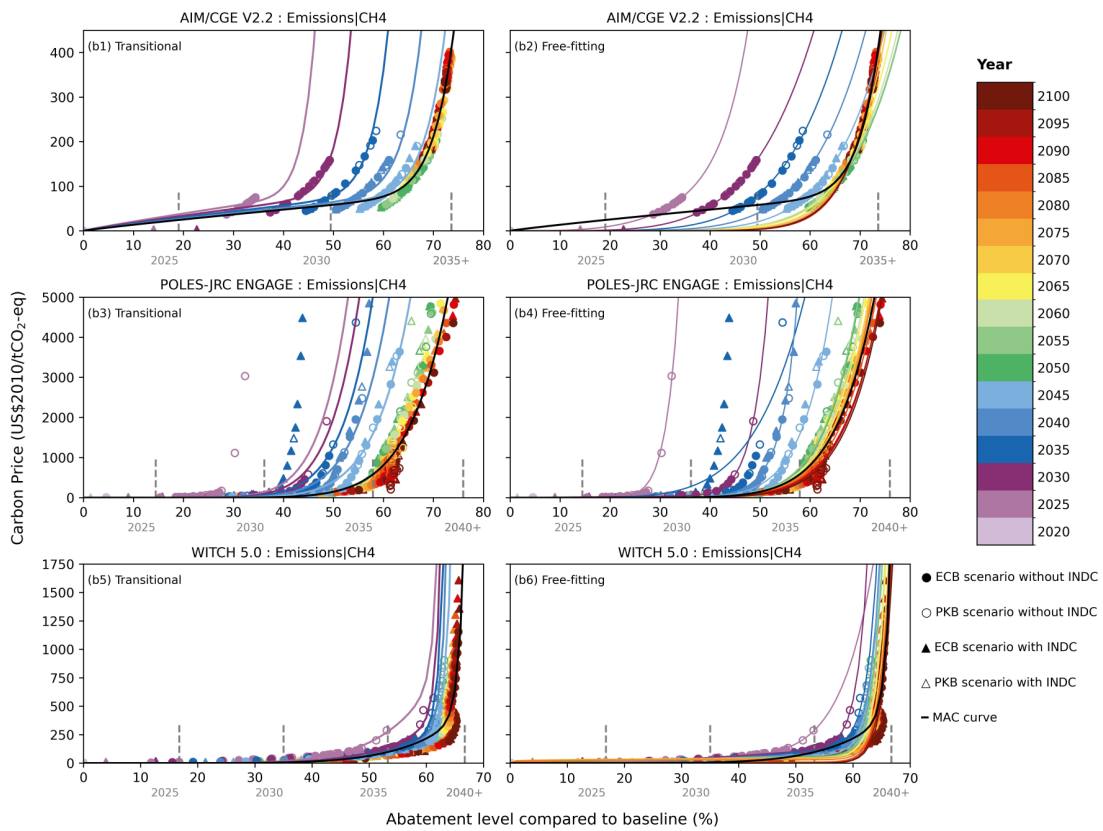
We have also checked through the entire manuscript once again. All other changes that were made in the manuscript (indicated in the manuscript with tracked changes) are very minor or editorial.

## 5. On the time-dependency of MAC curves

### 5.1 Deriving time-dependent MAC curves: transitional and free-fitting approaches

While the time-independent assumption of MAC curves is key to simplifying our IAM emulation approach, it raises questions about what this simplification entails. Here, we test time-dependent MAC curves to better understand the implications of our time-independent approach. Of ten IAMs analyzed in our paper, we selected three IAMs (AIM, POLES, and WITCH) for such a test because, based on our visual inspection, these models provide data that appear to be suitable for the use of time-dependent MAC curves (Figure 11). As detailed below, we developed time-dependent MAC curves using two different methods.





**Figure 11. CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O abatement levels and carbon prices from three IAMs (AIM, POLES, and WITCH) and their time-independent (in black) and transitional and free-fitting time-dependent MAC curves (in chromatic colors).** Panels (a1) to (a6), (b1) to (b6), and (c1) to (c6) show the MAC curves for CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O, respectively. In each set of Panels, data from the three IAMs are presented. Time-independent MAC curves are shown in black lines. Transitional time-dependent MAC curves are in chromatic color lines on the left column; free-fitting time-dependent MAC curves are in chromatic color lines on the right column. The vertical gray bars indicate the maximum abatement levels that can be potentially achieved at each point in time every five years (gray text), as determined by the upper limits of the first and second derivatives of abatement changes, as well as the upper limit of the abatement level (Table 2). See Table 8 for the goodness of fit (coefficients of simple determination) for the time-independent and time-dependent MAC curves.

First, we introduced the time-dependency to the MAC curves in a way that smoothly extends the time-independent MAC curves and their parameterizations as originally used, referred to as “*transitional* time-dependent MAC curves” (left column of Figure 11). For AIM, the relationships between the relative abatement levels of CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O and the carbon price are adequately captured by the time-independent MAC curves from 2050 onwards. It is thus sufficient to introduce the time-dependency to the MAC curve only before 2050. Namely, we modified the time-independent functional form by introducing time-dependent terms so that the MAC curves can be shifted to the left (or shifted up) as we go back in time from 2050. Regarding the two other IAMs, we also applied the same approach to CH<sub>4</sub> from POLES and CH<sub>4</sub> and N<sub>2</sub>O from WITCH. For the remaining cases (i.e., CO<sub>2</sub> and N<sub>2</sub>O data from POLES and CO<sub>2</sub> from WITCH), on the other hand, we stretched the time-dependent MAC curve approach all the way to 2100, as it is evident that the data show a temporary shifting trend until 2100.

Hence, we extended the time-dependent MAC curve approach either to 2050 or to 2100, based on the visual inspection of the data for the relationship between the abatement level and the carbon price from each model and gas. For time-dependent MAC curves that shift until 2050, we used the following functional form for each applicable model and gas.

$$f(x_t) = \begin{cases} a \times (x_t)^b + c \times (x_t)^d, & 2050 \leq t \leq 2100 \\ a \times (x_t \times (1 + e1 \times (t0 - t)^{e2}))^b + c \times (x_t \times (1 + f1 \times (t0 - t)^{f2}))^d, & 2025 \leq t < 2050, t0 = 2050 \end{cases} \quad (6)$$

From 2050 onwards, the equation above (including the parameter values) is equivalent to the time-independent MAC curve originally used for the respective model and gas. Although the time-independent MAC curves are derived using the data for the full period since 2025, outliers in the near term have been removed (Figure 2). As a result, the time-independent MAC curves are largely representative of the data for 2050-2100. For time-dependent MAC curves till 2100, we used the following functional form.

$$f(x_t) = a \times (x_t \times (1 + e1 \times (t0 - t)^{e2}))^b + c \times (x_t \times (1 + f1 \times (t0 - t)^{f2}))^d, 2025 \leq t \leq 2100, t0 = 2100 \quad (7)$$

$x_t$  in equations (6) and (7) is the variable representing the emission abatement level in percentage relative to the assumed baseline level at each point in time  $t$ .  $a, b, c, d$  are the parameters that take the model- and gas-specific values estimated for the respective time-independent MAC curve (Table 2). To represent the time-dependency, we basically shift the MAC curves horizontally by introducing the new terms using the parameters  $e1, e2, f1, f2$ . We optimized the parameters  $e1, e2, f1, f2$  by minimizing the squared deviations from the original price-quantity data between 2025 and 2045 (for equations (6)) or between 2025 and 2095 (for equations (7)) for each model and gas (Table 7). Note that for AIM,  $e2$  and  $f2$  are assumed to be 2 for the sake of simplicity (they are optimized for POLES and WITCH), while  $e1$  and  $f1$  are optimized for all three IAMs.

**Table 7. Values of additional parameters used in the transitional time-dependent MAC curves for the three IAMs.** For the definitions of time-dependent ranges and parameters, see equations (6) and (7) and the related text.

| IAM   | Gas              | Time-dependent range | Parameter               |                        |                        |                        |
|-------|------------------|----------------------|-------------------------|------------------------|------------------------|------------------------|
|       |                  |                      | $e1$                    | $e2$                   | $f1$                   | $f2$                   |
| AIM   | CO <sub>2</sub>  | Up to 2050           | $9.991 \times 10^{-4}$  | 2.000                  | $2.974 \times 10^{-3}$ | 2.000                  |
|       | CH <sub>4</sub>  | Up to 2050           | $9.684 \times 10^{-4}$  | 2.000                  | $9.610 \times 10^{-4}$ | 2.000                  |
|       | N <sub>2</sub> O | Up to 2050           | $4.099 \times 10^{-4}$  | 2.000                  | $9.593 \times 10^{-4}$ | 2.000                  |
| POLES | CO <sub>2</sub>  | Up to 2100           | $8.580 \times 10^{-8}$  | $3.794 \times 10^0$    | $4.554 \times 10^{-5}$ | $2.229 \times 10^0$    |
|       | CH <sub>4</sub>  | Up to 2050           | $6.353 \times 10^{-2}$  | $6.276 \times 10^{-1}$ | 0.000                  | 0.000                  |
|       | N <sub>2</sub> O | Up to 2100           | $1.609 \times 10^{-7}$  | $3.541 \times 10^0$    | 0.000                  | 0.000                  |
| WITCH | CO <sub>2</sub>  | Up to 2100           | $1.091 \times 10^{-10}$ | $5.038 \times 10^0$    | $1.369 \times 10^{-4}$ | $1.953 \times 10^0$    |
|       | CH <sub>4</sub>  | Up to 2050           | $6.854 \times 10^{-8}$  | $4.573 \times 10^0$    | $1.851 \times 10^{-2}$ | $4.161 \times 10^{-1}$ |
|       | N <sub>2</sub> O | Up to 2050           | $1.291 \times 10^{-4}$  | $2.390 \times 10^0$    | $6.551 \times 10^{-3}$ | $1.192 \times 10^0$    |

The transitional time-dependent MAC curves generally well captured the temporary shifting data from the three IAMs, compared to the time-independent MAC curves. The time-dependent MAC curves maintain shapes comparable to the original time-independent MAC curves and, as the time goes, converge to respective time-independent MAC curves either in 2050 or 2100.

Second, in contrast to the transitional approach discussed above, we also introduced the time-dependency to the MAC curves by optimizing the parameters in the functions of the MAC curves at each time step, referred to as “*free-fitting* time-dependent MAC curves” (right column of Figure 11). More specifically, we maintained the functional form used for the time-independent MAC curves and optimized the four parameters  $a, b, c, d$  at each time step (every five years from 2025 to 2100) for each IAM (AIM, POLES, and WITCH) and for each gas (CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O). The free-fitting approach captures the data point as closely as possible at each time step, testing the limit of the time-dependent MAC curves approach, while the transitional approach is more suited for applications as an emulator, as the underlying parameterization is simpler for

implementation. The goodness of fit in terms of the coefficient of simple determination ( $r^2$ ) is summarized for each case in Table 8.

**Table 8. Coefficients of simple determination ( $r^2$ ) of the time-independent and time-dependent MAC curves to the IAM data for the relationship between the abatement level and the carbon price.** The dark blue indicates the highest  $r^2$  value and the light blue the next highest  $r^2$  value. See Figure 11 for the MAC curves and IAM data.

| Gas              | Type of MAC curve             | IAM   |       |       |
|------------------|-------------------------------|-------|-------|-------|
|                  |                               | AIM   | POLES | WITCH |
| CO <sub>2</sub>  | Time-independent              | 0.957 | 0.466 | 0.909 |
|                  | Time-dependent (transitional) | 0.978 | 0.711 | 0.957 |
|                  | Time-dependent (free-fitting) | 0.971 | 0.812 | 0.989 |
| CH <sub>4</sub>  | Time-independent              | 0.941 | 0.739 | 0.723 |
|                  | Time-dependent (transitional) | 0.980 | 0.857 | 0.740 |
|                  | Time-dependent (free-fitting) | 0.993 | 0.937 | 0.818 |
| N <sub>2</sub> O | Time-independent              | 0.952 | 0.379 | 0.757 |
|                  | Time-dependent (transitional) | 0.981 | 0.608 | 0.790 |
|                  | Time-dependent (free-fitting) | 0.991 | 0.816 | 0.774 |

The  $r^2$  values from free-fitting time-dependent MAC curves are generally higher than those from transitional time-dependent MAC curves (seven out of the nine cases). For example, near-term data points from WITCH for CO<sub>2</sub> are better captured by the free-fitting time-dependent MAC curves than by the transitional time-dependent MAC curves (Panels (a5) and (a6) of Figure 11). On the other hand, the transitional time-dependent MAC curves are more consistent in terms of the way the MAC curves shift over time, as the underlying mathematical functions are formulated to yield such results. The free-fitting time-dependent MAC curves are less consistent because they are more strongly influenced by diverging data points from different scenario assumptions (i.e., end-of-century budget and peak budget; with and without INDC) (for example, Panels (a3) and (a4) of Figure 11).

## 5.2 Reproducing the IAM scenarios with the time-dependent emulator: methods

Now we implement the transitional and free-fitting time-dependent MAC curves to emIAM. For each carbon budget pathway of each IAM, we imposed the same remaining carbon budget to emIAM as a constraint and calculated the least-cost pathway for CO<sub>2</sub>. Our focus here is on CO<sub>2</sub> because of its greatest relevance. This approach is equivalent to Test 1 discussed in Section 4 and is the most direct and simplest way to evaluate the performance of MAC curves, among other Tests in Section 4. In this set of experiments, our emulator derives CO<sub>2</sub> emission pathways in the same way as a subset of IAMs: intertemporal optimization models using a remaining carbon budget as the constraint (Table 1).



We also performed an additional set of experiments by prescribing the carbon price pathway directly to emIAM (i.e., without endogenously optimizing it) and calculated the CO<sub>2</sub> emission pathway. This is an even more direct way to test the MAC curves than the carbon budget experiments discussed above. The prescribed carbon price pathway uniquely determines the CO<sub>2</sub> emission pathway through the MAC curve(s) without any optimization involved (the carbon budget constraints and the change rate and inertia limits for abatement are irrelevant here). Thus, any deviation from the original CO<sub>2</sub> emission pathway can be ascribed to the misfit of the MAC curve(s) to the underlying data from the IAM, while in the previous experiments, it can also be ascribed to a deviation of the endogenously optimized carbon price pathway from the original carbon price pathway of the IAM. In this set of experiments, our emulator derives CO<sub>2</sub> emission pathways in the same way as another subset of IAMs: recursive dynamic models using a carbon price pathway (exogenously computed from the remaining carbon budget) as the constraint.

We further checked the sensitivity regarding the upper limits of the first and second derivatives of abatement changes (Table 2). The same upper limits are applied to time-independent and time-dependent approaches. These limits can affect the experiments to test the MAC curves, as they define the segment of MAC curves that can be utilized at each time step (vertical gray bars in Figure 11). That is, in the near term, only a low range of MAC curves can be utilized by emIAM due to the first and second derivative limits.

In sum, we have a total of nine experimental Cases for each IAM as summarized in Table 9. The first three Cases A to C test the extent to which the CO<sub>2</sub> emission pathways of each IAM can be reproduced by emIAM under the corresponding carbon budget constraints by using the respective three different types of MAC curves and abatement limits, while also optimizing the carbon price pathways (our default setting). The next three Cases D to F are the same, except that the abatement limits are not used. The last three Cases G to I provide the corresponding tests under the carbon price constraints, instead of the carbon budget constraints. Note that we focus on the ECB scenarios without INDC, among other sets of scenarios. This set of scenarios provides the cleanest data for testing how well the MAC curves reproduce the original scenarios because these scenarios are free of constraints for net-zero emissions and INDC target levels, which cannot be captured by MAC curves.

**Table 9. Statistical validations of CO<sub>2</sub> emission pathways reproduced from emIAM against the original emission pathways from the three IAMs.** For the type of MAC curve, “Indepnd.” indicates time-independent MAC curve (default), “Depnd./Trans.” transitional time-dependent MAC curve, and “Depnd./Free” free-fitting time-dependent MAC curve. For the abatement limits, “Incl.” means that the upper limits of the first and second derivatives of abatement changes are included in emIAM (default); “Excl.” indicates otherwise. For the carbon price, “Opt.” indicates that the carbon price is endogenously optimized in emIAM (default); “Presc.” indicates that the carbon price from the original IAM is prescribed to emIAM. Dark blue indicates the highest value; light blue the next highest value.

The table shows the results for the ECB scenarios without INDC.

| Experimental case |       | A        | B                 | C               | D        | E                 | F               | G        | H                 | I               |
|-------------------|-------|----------|-------------------|-----------------|----------|-------------------|-----------------|----------|-------------------|-----------------|
| Type of MAC curve |       | Indepnd. | Depnd./<br>Trans. | Depnd./<br>Free | Indepnd. | Depnd./<br>Trans. | Depnd./<br>Free | Indepnd. | Depnd./<br>Trans. | Depnd./<br>Free |
| Abatement limits  |       | Incl.    | Incl.             | Incl.           | Excl.    | Excl.             | Excl.           | Excl.    | Excl.             | Excl.           |
| Carbon price      |       | Opt.     | Opt.              | Opt.            | Opt.     | Opt.              | Opt.            | Presc.   | Presc.            | Presc.          |
| AIM               | $r_P$ | 0.9859   | 0.9757            | 0.9821          | 0.9856   | 0.9758            | 0.9858          | 0.9784   | 0.9939            | 0.9964          |
|                   | $r_C$ | 0.9796   | 0.9648            | 0.9716          | 0.9804   | 0.9651            | 0.9779          | 0.9777   | 0.9928            | 0.9961          |
|                   | MAE   | 3.3244   | 4.4760            | 3.4676          | 3.1482   | 4.4452            | 3.1701          | 2.5589   | 1.6386            | 1.1885          |
|                   | RMSE  | 4.3878   | 5.8783            | 5.2018          | 4.2717   | 5.8526            | 4.5156          | 4.3345   | 2.5061            | 1.8183          |
| POLES             | $r_P$ | 0.9891   | 0.9862            | 0.9822          | 0.9764   | 0.9835            | 0.9823          | 0.9643   | 0.9831            | 0.9898          |
|                   | $r_C$ | 0.9891   | 0.9831            | 0.9764          | 0.9738   | 0.9815            | 0.9762          | 0.9606   | 0.9659            | 0.9892          |
|                   | MAE   | 2.0402   | 2.6271            | 2.7632          | 2.8913   | 2.7222            | 3.0789          | 4.2122   | 3.8071            | 1.7276          |
|                   | RMSE  | 2.7512   | 3.5772            | 4.1719          | 4.1323   | 3.7007            | 3.9869          | 5.4772   | 5.0704            | 2.7676          |
| WITCH             | $r_P$ | 0.9748   | 0.9725            | 0.9657          | 0.9743   | 0.9724            | 0.9698          | 0.9902   | 0.9958            | 0.9976          |
|                   | $r_C$ | 0.9625   | 0.9584            | 0.9485          | 0.9654   | 0.9602            | 0.9592          | 0.9893   | 0.9909            | 0.9972          |
|                   | MAE   | 3.7224   | 3.8778            | 4.2143          | 3.4942   | 3.7722            | 3.7820          | 1.6708   | 1.5686            | 0.6386          |
|                   | RMSE  | 4.6483   | 4.9326            | 5.4789          | 4.4011   | 4.7899            | 4.7323          | 2.2389   | 2.0672            | 1.1471          |

### 5.3 Reproducing the IAM scenarios with the time-dependent emulator: results

In the first three experiments with the carbon budget constraints including the abatement limits (Cases A to C), the statistical indicators showed that the use of the transitional and free-fitting time-dependent MAC curves did not improve the reproducibility of emission scenarios (Table 9). For all three IAMs, the scenario reproducibility was, in fact, slightly decreased with the introduction of the time-dependency to the MAC curves. In the next three experiments also with the carbon budget constraints but excluding the abatement limits (Cases D to F), the use of the time-dependent MAC curves generally only improved the scenario reproducibility for POLES. In contrast, in the last three experiments with the carbon price constraints (Cases G to I), the use of the time-dependent MAC curves unanimously improved the scenario reproducibility, with the free-fitting time-dependent MAC curves being superior to the transitional time-dependent MAC curves. To understand why the use of time-dependent MAC curves improved the scenario reproducibility only under certain conditions, we examine the results separately for the carbon budget simulations (Cases A to F) and the carbon price simulations (Cases G to I) below.

#### 5.3.1 Carbon budget simulations

In Cases A to C, both the transitional and free-fitting time-dependent approaches tend to give higher emissions in the near term and lower emissions later in the century than the time-independent approach for all three IAMs (Figure 12). This finding can be explained by the relative positions of the time-independent and time-dependent MAC curves. Because the time-dependent MAC curves are higher (i.e. higher marginal cost for a specific level of abatement) than the time-independent MAC

curves in the near term, mitigation becomes more costly, resulting in higher emissions in the near term. The results were opposite later in the century. Because the remaining carbon budget must be conserved, emissions later in the century become lower with time-dependent MAC curves to compensate for the higher emissions earlier. Now, most results from Case A show that the time-independent approach already overestimated the emissions in the near term and underestimated the emissions later. Hence, those deviations were not reduced by the adoption of the time-dependent approach (Cases B and C); it was rather increased, despite the better fit of the time-dependent MAC curves to the price-quantity data from IAMs than the time-independent MAC curves.

Our implicit hypothesis was that the time-dependent approach yields a higher scenario reproducibility than the time-independent approach; however, this hypothesis proved wrong for Cases A to C. To understand the unexpected outcome, it is important to consider the carbon price. There are two different yet associated quantities from the emulator that can be characterized as carbon price: i) value of the MAC curve and ii) shadow price. The shadow price is always higher than or equal to the value of the MAC curve, as the shadow price is not influenced by various model constraints. Although there is no definitive argument to judge which quantity should be compared to the carbon price reported by IAMs, we primarily compare the value of the MAC curve with the IAM carbon price (available in the ENGAGE Scenario Explorer) (Figure 13).

We now ask why both the time-independent and time-dependent approaches overestimated near-term CO<sub>2</sub> emissions and underestimated long-term CO<sub>2</sub> emissions. Taking AIM as an example, the emission overestimations till mid-century are primarily caused by the difference in carbon price between the emulator and the IAM. The MAC estimates are generally lower than the corresponding carbon prices of AIM, with differences depending on the carbon budget of the scenario. The generally lower MAC estimates largely explain the emission overestimations till mid-century. Later in the century, on the other hand, the MAC estimates become higher than the AIM carbon prices, resulting in the emission underestimations. The MAC estimates from different carbon budget pathways converge after the emissions reach the lower limit defined by the maximum CO<sub>2</sub> abatement level for AIM (116.2% relative to the baseline (Table 2)). An exception is the emission overestimations in 2025, which stem from the upper limits of the first and second derivatives of abatement changes, which do not allow a rapid emission reduction required to follow the original AIM scenarios. If these assumed upper limits are dropped (Cases D to F), the 2025 emissions became substantially lower and better reproduced the original emission levels (e.g., Panels (a1) and (b1) of Figure 12). However, the impact of these abatement bounds is limited to the very near term. The emission overestimations till mid-century are better explained by the carbon price differences discussed above.

Additional descriptions of the results from the other two IAMs follow (Cases A to F). For POLES, the time-independent approach slightly underestimated the emissions in the near term. Similarly to the results from AIM, both time-dependent approaches overcorrected this negative discrepancy and resulted in emission overestimations in the near term. Later

in the century, the time-dependent approaches overcorrected the discrepancy in the opposite way and resulted in emission underestimations. When the abatement limits are removed (Cases D to F), the transitional time-dependent approach outperformed (Table 9), which was however primarily the consequence of the excessive drop in 2025 emissions of the time-independent approach (Panels (a4) and (b4) of Figure 12), with high penalty in the statistical indicators for the time-independent approach. For WITCH, the differences in the results between the time-independent and time-dependent approaches are the smallest among the three IAMs. This reflects the fact that the time-independent MAC curve largely captured the relationship between the abatement level and the carbon price in the case of WITCH, except for a limited number of near-term data points representing very high abatement levels (Panel (a5) of Figure 11). The WITCH results also exhibited the general deviation trend seen from other models: emission overestimations in the near term and emission underestimations later in the century. This general trend can be also explained by the carbon price differences. Furthermore, the comparison of the carbon prices indicates that the discount rate in WITCH may be lower than the assumed discount rate of 5% used in our emulator. As discussed earlier, in the absence of information on the discount rate used by all but a few IAMs, our emulator assumes 5% for all IAMs. The discount rate in IAM may follow the Ramsey rule, meaning that the discount rate is time-dependent, depending on the future economic growth.

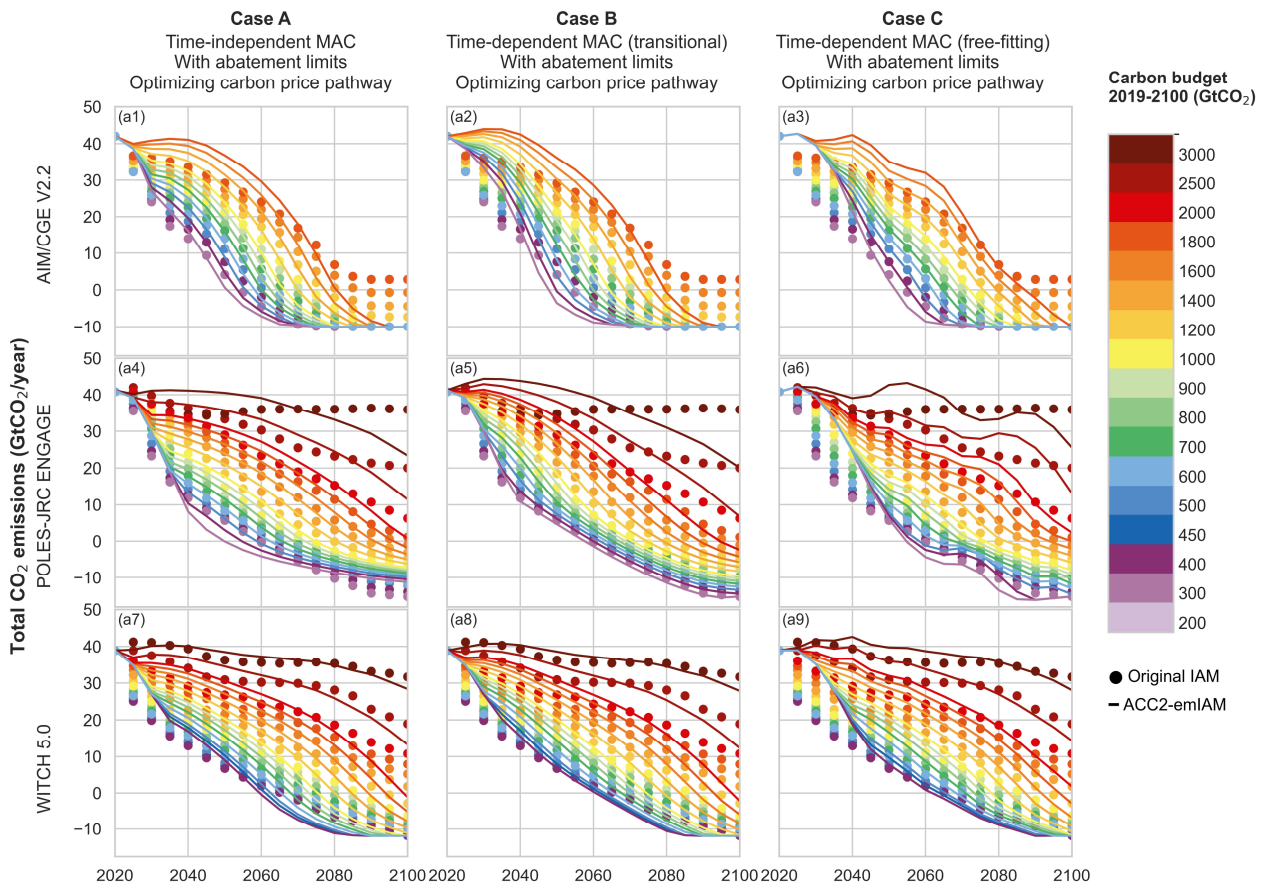
### **5.3.2 Carbon price simulations**

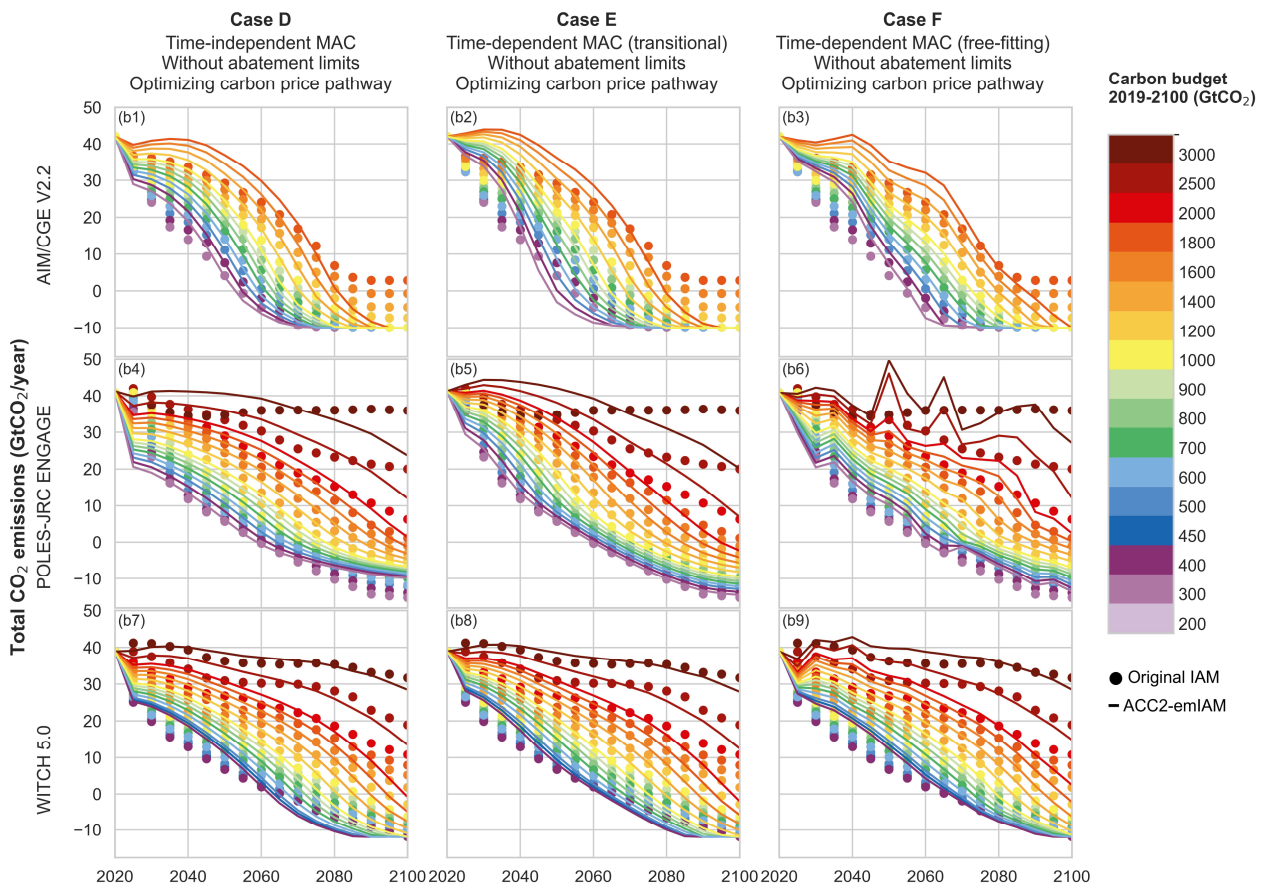
In stark contrast to the results discussed above, the results based on the experiments using prescribed carbon prices (Cases G and I) show that the use of time-dependent MAC curves can improve the reproducibility of CO<sub>2</sub> emission scenarios over the use of time-independent MAC curves (Panels (c1) to (c9) of Figure 12). In particular, near-term emission pathways up to mid-century were more closely reproduced with the use of time-dependent MAC curves, following our expectation. This is because time-dependent MAC curves capture the near-term relationship between the abatement level and the carbon price much better than time-independent MAC curves. On the other hand, near-term emissions were underestimated with the use of time-independent MAC curves because such MAC curves tended to be lower (i.e., lower carbon price for a given level of abatement) than the near-term data points, which led to an underestimation of near-term mitigation costs and thus an overestimation of abatement. The use of free-fitting time-dependent MAC curves yielded higher scenario reproducibility than the use of transitional time-dependent MAC curves.

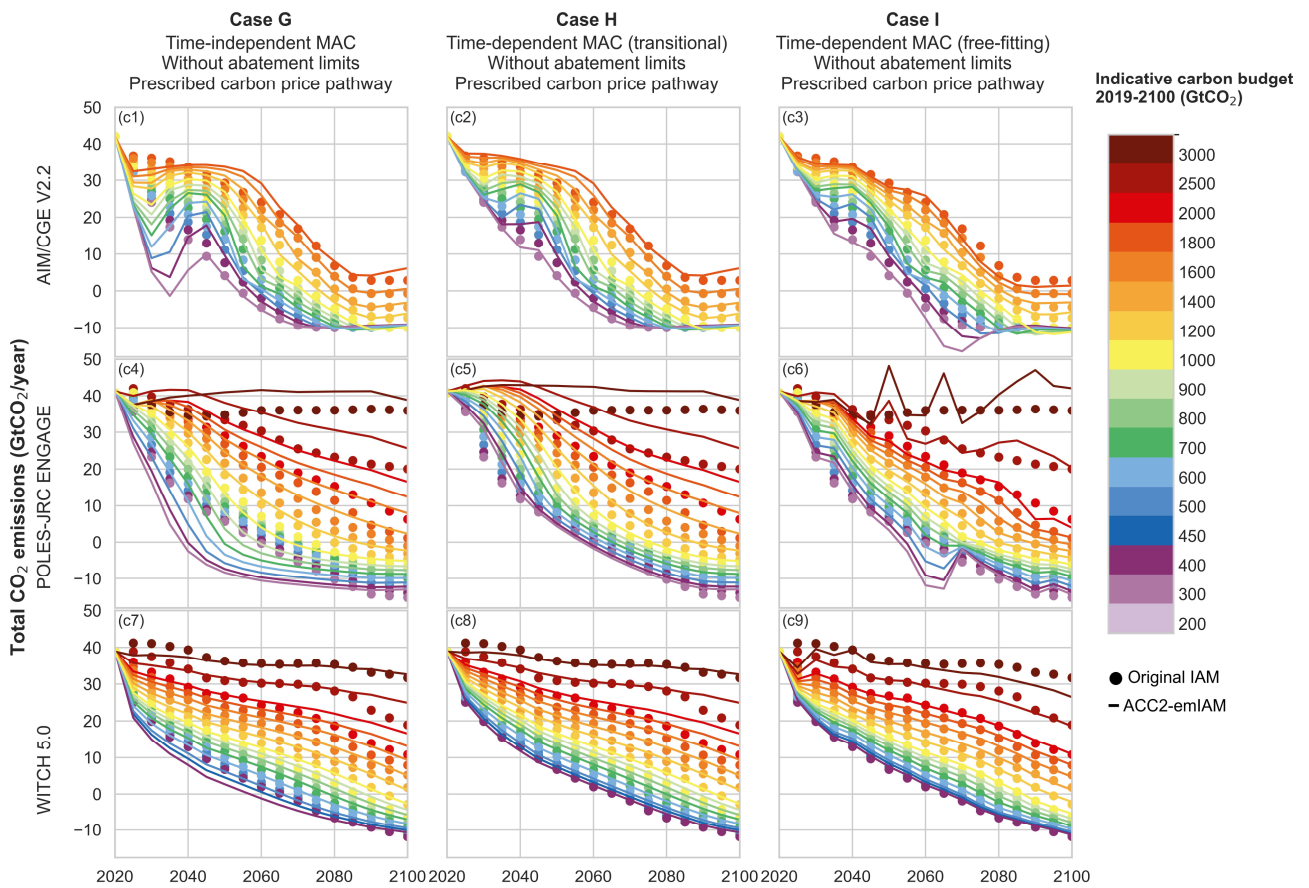
The superiority of time-dependent MAC curves over time-independent MAC curves discussed above can be confirmed by the statistical indicators in Table 9. This table also indicates that such results can only be found under the simple experimental setup with prescribed carbon prices. Under the more complex (and more applied) setup, in which carbon price pathways are endogenously optimized under given carbon budgets, the superiority of time-dependent MAC curves become

less clear. This is due to the effect of carbon price pathways – an important determinant of scenario reproducibility – which can even negate the benefit of using time-dependent MAC curves.

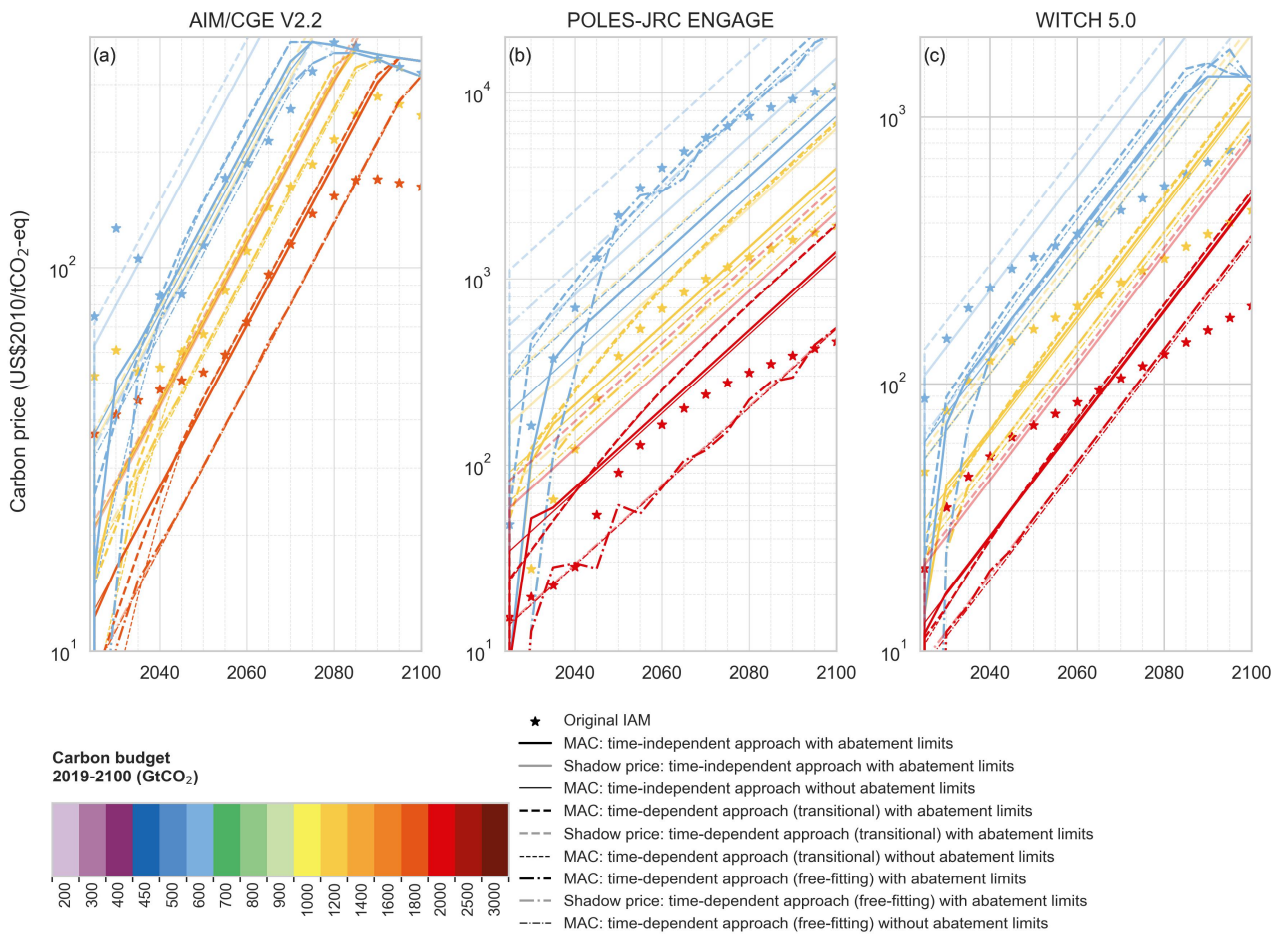
Ultimately, emission scenarios will be perfectly reproduced, if the following two conditions are met: first, the original IAM data (the relationship between the abatement level and the carbon price) are perfectly captured by the MAC curve; second, the carbon price pathways are also perfectly reproduced by the emulator. While the first condition can be adequately satisfied with the use of time-dependent MAC curves within limits set by the functional form of the MAC curve, the second condition cannot necessarily be met due to various constraints in the IAMs that cannot be captured by the emulator. For example, the AIM carbon price pathways have first peaks in the near term, followed by second peaks later in the century. Such complex terrains of carbon price pathways, which are exogenously imposed in recursive dynamic models, cannot be reproduced by our intertemporal optimization emulator. Even the carbon price pathways of the intertemporal optimization model WITCH, which shows a monotonic and exponential increase over time, differ from the carbon price pathways of the emulator. The discussion here points to the importance of investigating carbon price pathways to further improve the IAM emulator.







**Figure 12. Comparison between the reproduced CO<sub>2</sub> emissions from emIAM and the original emissions from the three IAMs for the experimental cases summarized in Table 9.** The figure shows the results for the ECB scenarios without INDC. In Panels (c1) to (c9), carbon budgets are only indicative, as the simulations were driven by carbon prices, without using carbon budgets.



**Figure 13. Carbon price pathways from the time-independent and time-dependent emulators and the three IAMs.** MAC indicates the value of the MAC curve at each period under each scenario. Shadow price indicates the change in the total policy cost (the area of the MAC curves) for an infinitesimal change in emissions from the optimal level. The carbon prices of IAMs are indicated by star symbols. Selected three carbon budget scenarios are shown for each IAM. Vertical axes are on a logarithmic scale.