

Dear referee,

We express our sincere gratitude for the comments and valuable suggestions you've provided. In response to your feedback, we've made the following changes in the revised manuscript, especially for the paragraph starting in Line 185.

Comment #1. For the comments of "There are some vague statements about Cui et al. lacking "detailed analyses" of K_c and τ (in Line 187) and lacking a discussion of "practical applications" of their work (in Line 191), but a concise problem statement is lacking. So the novelty of the work is not clear to readers. I recommend revising this paragraph with a series of direct statements about specifically what was missing from Cui's work and how the present work addresses those shortcomings."

The whole paragraph starting in Line 187 has been revised as:

Based on diffusion phenomenology, Cui et al. (2009) presented two mathematical solutions similar to our Eqs. (3A) and (3C). In the work of Cui et al. (2009), however, one of late-time solution is missing, and error analyses are not provided. Besides, the lack of detailed analyses of τ and K_c in the constitutive equations will likely deter the practical application of Eq. (3B), which is able to cover an experimental condition of small sample mass with a greater τ (further analyzed in Section 3). Furthermore, the early-time and late-time solution criteria are not analyzed, and the pioneering work of Cui et al. (2009) does not comprehensively assess practical applications of their two solutions in real cases, which is addressed in this study.

With respect to the clarification of similarities, differences, and innovations of our work, we acknowledge your concerns about the potential overlap with the study conducted by Cui et al. (2009). Both studies focus on the diffusion decay method for permeability measurements, investigating the same phenomena and employing the same method proposed by Cui et al. (2009). The work of Cui et al. (2009) presented two notable contributions: (1) they provided two solutions, one for early-time measurements and another for late-time measurements; and (2) they established a reference value of the storage capacity (K_c) as 50 and recommended the use of the late-time solution when K_c is greater than 50.

We acknowledge the excellent work performed by Cui et al. (2009), however, we believe their studies to be incomplete. Firstly, full mathematical solutions were not provided, as actually there are three solutions rather than two. Secondly, the mathematical error associated with each solution was not discussed, as all the three solutions are an approximation rather than an exact one. Most importantly, they overlooked the early-time solution, which is truly necessary, practical, and efficient for testing tight rock media.

In our work:

- (1) We provide a comprehensive and mathematically deduction process to obtain all three solutions, rather than directly borrowing the existing solutions from the realms of heat conduction and chemical diffusion.
- (2) We explain the classification of the early-time and late-time solutions, based on the

dimensionless time τ (never discussed by Cui et al.), and determined the specific value of 0.024 as the criterion from the exact solution calculated using MATLAB.

- (3) Based on the discussion and classification of different τ values, we explain how the storage capacity (K_c) influences the solution selection.
- (4) We conduct a kinetic analysis of several gas molecules and provide detailed analyses regarding sample mass, diameter, and equipment settings.
- (5) We demonstrate the clear work-flow procedure for the application and selection of these three solutions that we've derived.
- (6) Therefore, in the revised paragraph starting in Line 187, we emphasize that Cui et al. (2009) did not provide a comprehensive analysis of τ and K_c , and they overlooked the analyses of the early-time solution (Eq. 3C). In contrast, the solution being provided in current work (Eq. 3B) is practical for most common tight geomedial.

2. Regarding the comments: In Table 2, it would be helpful to have a comparison between results of the present work and those using Cui's methods to demonstrate improvement.

The comparison of these two solutions (from Cui et al.) and the third one has been added here (Second to last column and third to last column):

Granular size (mm)	SMP-200 (nD) [§]	GPT test 1 (nD) [£]	GPT test 2 (nD) [£]	Average value (nD) [£]	Fitting duration (s)	Solution type	Dimensionless time	Comparison for second solution (nD)	Comparison for third solution (nD)	Particle density (g/cm ³)
5.18	-	1.17	1.17	1.17	50-100	ILT	0.023-0.027	239 IET	1.31 LLT	2.631
2.03	14.2	0.45	0.41	0.43	50-100	LLT	0.026-0.028	11.1 IET	0.36 ILT	2.626
1.27	-	0.10	0.10	0.10	30-60	LLT	CR*	20.5 IET	0.09 ILT	2.673
0.67	0.65	0.08	0.04	0.06	30-60	LLT	CR*	1570 IET	0.03 ILT	2.658
0.34	-	0.002	-	0.002	30-60	IET	CR*	0.00076 LLT	0.00068 BLT	2.643

[§] The results are from the SMP-200 using the GRI default method.

[£] The results are from the GPT method we proposed.

* CR means the conflict results that the verified dimensionless time does not confirm the early- or late-time solutions using the solved permeability. For example, the verified dimensionless time would be > 0.024 using the early-time solution solved result and vice versa.

According to the method from Cui et al. (2009), they prefer using the late-time solution (ILT) for all the situations, while we provide the results accordingly with ILT, LLT, and IET and demonstrated here. The conclusion from the comparison is that: there is not much value difference between LLT and ILT method, which is the same as we analyzed in the error difference. However (1) the IET solution is necessary, (2) and LLT is more eurytopic, as discussed in this paper.

Overall, except the mathematical derivation, we deem the innovation and the significant improvement from this manuscript is the methodology and criteria for the practical utilization.

Thank you very much for your valuable feedback and suggestions.