The evolution of isolated cavities and hydraulic connection at the glacier bed. Part 1: steady states and friction laws

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1 Referee #2

Referee: - In this work, once a cavity becomes hydraulically isolated, the model enforces that the cavity have constant volume until it becomes hydraulically connected once again. Under these conditions, the water pressure of the cavity is an unknown to compute. Something that came to mind is the possibility of hydraulically isolated cavities that develop an air-filled gap in between the water and the ice. I can understand the reasoning behind the constant volume condition when, e.g., the far-field ice speed decreases, since the cavity would want to close but it cannot compress the water in it. However, upon an increase in this far-field speed, I find it logical that, at a certain point, when the water pressure becomes sufficiently low (perhaps 0?), a partially air-filled cavity starts to develop. In other words, it is reasonable to assume that a hydraulically isolated cavity is still pneumatically connected, since the opposite sounds unrealistic for most glacier conditions. In the context of your model, allowing the formation of air-filled cavities might mean enforcing pneumatic connectedness when a low water pressure is reached, and therefore allowing the volume to increase beyond the water volume. I mention this point because it would mean that the unbounded increase in basal stress that you see for the single cavity in Figure 8 would perhaps no longer hold. If this comment is sensible, I think it should be commented in the discussion.
Response: Thank you for pointing this out; a good point, which I would rather not have dealt with (being lazy, as it makes the model more complicated!) — in particular, it increases the size of the parameter space by one, since N and overburden p_i are independent parameters, and it is really p_i that controls when the type of cavities described should form. I have amended the text after equation (9) to describe how the model would need to be changed if one were to deal with the complication raised: *Outside of the intervals $(b_j - \delta, b_j)$ and $(c_j, c_j + \delta)$, (9) can, and in general will, be violated somewhere as indicated in figure 1. The possibility of such underpressurized regions is the primary difference between the permeable and impermeable bed models. By not bounding compressive normal stress everywhere at the bed, the model does however not allow for vapour-filled cavity to form if the normal stress dropping to the triple-point pressure of water. In order to incorporate the latter effect, I would need to add the constraint that $p - 2\eta\partial v/\partial y > -p_i$ in C', where p_i s ice overburden, and set $p - 2\eta\partial v/\partial y = -p_i$ in any cavity that does not straddle P and in which the prescribed water volume V_j (potentially equal to zero) would lead to an effective pressure $N_j > p_i$ if the volume constraint *8) were imposed. I omit this complication here on the basis that I expect overburden p_i to be large compared with the typical normal stress variations caused by ice flow over bed undulations; suffice it to point out that the model described in part 2 can in principle describe vapour-filled cavities,*

Note that I refer to vapour-filled cavities: I don't believe that the glacier is actally likely to be pneumatically-connected in the sense that air can flow but water cannot (from personal experience, I can attest to the existence of pressurized air-filled pockets that can expel a hot water drill head when punctured; such air pockets presumably should not exist if pneumatic connection in the strict sense is maintained). That does not detract from the possibility of cavities that are not filled with liquid water as described in the exceprt from the updated paper.

As stated above, I have not tried to implement the possibility of "dry crevasses" in the steady state model. The ability to describe fluid lag does mean the model in part 2 can, in principle, describe such dry crevasses, though it would require setting the overburden to much lower values than I have tried in that paper.

With regard to preventing unbounded basal drag, I have added the following paragraph at the end of section 3.2:

As a further caveat, note that for a fixed N, unbounded τ_b as shown in figure 8 results from the ability to generate arbitrarily large compressive normal stresses on the upstream side of the smaller bed protrusion, balanced by correspondingly negative compressive normal stresses on the downstream side in the hydraulically isolated low pressure region on the downstream side of the larger protrusion (figure 7c2, where $-\sigma_{nn}^*$ is scaled with $1/u_b$, so the actual stress is the pattern shown multiplied by a coefficient proportional to u_b). As described in section 2 immediately after equation (9), arbitrarily negative normal stresses cannot be generated since a vapour-filled cavity will eventually form, and this should lead ultimately to a bounded basal drag satisfying an amended version of Iken's bound, $\tau_b \leq \max(\partial b/\partial x)p_i$, where p_i is once more overburden; the model here ignores that possibility, effectively treating p_i as infinite for the purposes of bounded basal drag.

Referee: p3, line 18: Consider starting sentence with a non-mathematical symbol.

Response: Added "Here, . . . "

Referee: p3, line 23: "the normal component of velocity vanishes" ¿ "the normal component of the velocity vanishes".

Response: Is that change strictly necessary, grammatically speaking? I'm inclined to say it is not.

Referee: p 5, line 14: In Stubblefield et al. (2021), the rate of change of the volume is enforced, not the total cavity volume. Mathematically, this is carried out by setting the integral of the normal velocity at the lower boundary equal to this prescribed rate of change. The Lagrange multiplier associated to this constraint is the water pressure.

Response: If I constrain the rate of change of volume to be zero (which I believe is what Stubblefield et al do), then am I not constraining the volume to remain constant? I understand that there is a difference in terms of how this is implemented, but the result is the same. In any case, I've changed the wording to

. . . Instead, the total cavity volume is prescribed through initial conditions, the constraint itself being imposed on normal velocity so as to conserve that initial volume.

Referee: p5, line 16: As you write, I also find the specification of a permeable point x_P along the bed awkward, from a physical point of view. It seems to me that the fact that you choose it to be the location where cavities first form is effectively equivalent to choosing that a particular cavity, once it forms, is connected to the external drainage system. I think it would be helpful to the reader to clarify the intention behind choosing x_P to be this point.

Response: I think this is the same point as raised by the other referee, in slightly different form. I have reworded the section flagged above as

Below, I will typically consider either the entire bed permeable with $P = (0, a)$, or I will consider a small permeable patch around a single location, which I will denote by x_P. I will typically choose x_P to be the location of a local minimum of compressive normal stress for an uncavitated bed, since *that is where cavities first form for a permeable bed. In addition, in section 3.3 I consider larger permeable bed portions P that do not align with these normal stress minima.*

Depending on your take, there may not be a very good reason for locating x_P as described, especially if we read the model description literally, so P is a (geologically-determined) permeable part of the bed. If we suppose that P is a proxy for lateral drainage access across an unmodelled of ice-bed interface that lies to the side of the flowline that *is* modelled, then it may make more sense to consider x_P at normal stress minima.

That being said, I have introduced a new section 3.3 to investigate what happens if you locate P elsewhere. The start of that section reads

The results above were computed either for completely permeable beds, or for beds that had permeable sections located at normal stress minima prior to cavity formation. As pointed out, I view these permeable bed portions P potentiallly as proxies for lateral access from a three-dimensional ambient draiange system along an unmodelled part of the ice-bed interface, to one side of the flowline that model describes. In that case it may make sense for that laeral access to reach the modelled flowline in places where compressive normal stress has local minima. Locating the permeable where cavities form at the highest possible values of N is also convenient as it reduces the number of additional parameters that describe the bed in the absence of a more sophisticated three-dimensional model.

In order to investigate the effect of choosing different permeable bed portions P, I plot in figure 9 the dependence of cavity end point positions on effective pressure N^* for the same bed geometry (equation (10) as before, but for two alternative choices of P^*, ...

I won't reproduce the entire text of that section here, please see the revised paper for details.

Reviewer: p 6, line 29: "by plotting cavity end point position" $\mapsto$ "by plotting the cavity end point positions"

Response: corrected

Reviewer: p 8, figure 4: I think it would be helpful to the reader to include a subtle line in the plots for the normal stress that indicates the values of N^*. It would make it much easier to notice the areas of low pressure and the difference in water pressure between isolated and connected cavities. Same for figure 7.

Response: Done

Reviewer: p 9, line 3: "see also fig 3" $\mapsto$ "see also figure 3"

Response: Done

Reviewer: p 9, line 9: "decreased." $\mapsto$ "decreased"

Response: full stop replaced by comma

Reviewer: p9, line 33: "we start with an uncavitated bed an only lower N^*", this sentence does not make sense.

Response: Replaced with "We start with an uncavitated bed, and only lower N^*"

Reviewer: p10, Figure 5: Close brackets in line 3 of caption.

Response: Done.

Reviewer: p11, line 6: "cavity cavity" $\mapsto$ "cavity"

Response: oops... corrected.

Reviewer: p13, line 5: "by arrow" $\mapsto$ "by the arrow"

Response: Done

Reviewer: p13, line 12: Consider starting sentence with a non-mathematical symbol.

Response: Added "Drag"

Reviewer: 'p14, line 19: "trimple" $\mapsto$ "triple"

Response: Done

Reviewer: p16, line 7: "is" $\mapsto$ "are"

Response: Done.
Reviewer: p16, line 8: “again be unbounded” $\mapsto$ “again unbounded”
Response: Done.
Reviewer: p17, line 30: close brackets.
Response: Done.
Reviewer: p17, line 32: “it easier” $\mapsto$ “it is easier”
Response: Corrected.
Reviewer: p18, line 20: “constraint” $\mapsto$ “constraints”
Response: Done.
Reviewer: p18, line 33: “If drainage system access” $\mapsto$ “If the drainage system access”
Response: Again, I'm going to disagree on the need to use of a definite article here, on the basis that drainage system access is not countable.
Reviewer: p18, line 34: “effective pressure” $\mapsto$ “the effective pressure”
Response: Ditto.
Reviewer: p19, line 18: “bed,” $\mapsto$ “bed.”
Response: Corrected.