Dear Editor and Referees:

Thanks for your effort to review our manuscript titled " AdaHRBF v1.0: Gradient-Adaptive Hermite-Birkhoff Radial Basis Function Interpolants for Three-dimensional Stratigraphic Implicit Modeling", and now we have just revised this manuscript according to your good suggestions. The details are as follows, and all the revisions are done using track changes in Word.

Report #1: Michalak, Michal

The authors have made an effort to more clearly explain the reasoning behind their method. This should be appreciated. However, new unexplained concepts (e.g. „well-known iterated conditioned mode method", „well known Duchon's energy") are used to support the main reasoning behind posing and solving their minimization problem. The authors write that the conditioned mode method is „well-known". I would argue that this is something very specific and needs more explanations why it is relevant to the proposed method. Especially that the conditioned mode method has something in common with Markov random fields, a concept that does not occurr in the manuscript. I don't recommend any decision (I marked major revision in the system because I have to select something) because I don't do research in functional analysis (note that the authors now cite a theoretical paper from functional analysis Duchon, 1977 for their well known Duchon's energy). So maybe colleagues doing research strictly in functional analysis could say something more decisive.

Response: We appreciate your effort to review our manuscript and grateful for that. We have addressed your main concerns in our manuscript on response to your detailed comments #1 and #3.

More detailed comments are given below.

#1 In the response file, the authors write:

We thought the Hermite-Birkhoff interpolation in pure mathematics could be referenced.

Reviewer:

Any valid source of information can be referenced. Because you deleted the Bachman&Narici book from the referenced publications, you have acknowledged that it was invalid. Thank you. Now you have Duchon, 1977 which seems to pose a new yet similar challenge – why is this publication relevant?

Response: Apologies for citing Duchon, 1977. The work of Duchon, 1977, to the best of our knowledge, is the first to define the smooth regularizer for thin-plate interpolant and give its solution. This paper could be also difficult to understand for readers of GMD due to its heavy mathematics. To better explain the smooth regularizer, we replace Duchon, 1997 with two alternative literatures (Wahba, 1990; Walder et al., 2006) that are more relevant to implicit modeling.

Wahba, G., 1990. Spline models for observational data. Society for industrial and applied mathematics.

Walder, C., Schölkopf, B. and Chapelle, O., 2006, April. Implicit surface modelling with a globally regularised basis of compact support. In Computer Graphics Forum (Vol. 25, No. 3, pp. 635-644). Amsterdam: North Holland, 1982-.

See Lines 157-158.

#2 In the response file, the authors write:

Comparing with explicit modeling, implicit modeling has the efficiency advantage of avoiding a lot of workloads of human-computer interaction. The 3D orientations are usually surveyed on the outcrops of strata.

Reviewer:

Since my comment was about comparing triangulation and interpolation methods, I will make a point about the difference in computational time.

Interpolation methods, e.g. kriging (but also in Grose et al. https://doi.org/10.5194/gmd-14-3915-2021), often require solving large systems of linear equations. For example, for kriging equations, the size of the linear systems is proportional to the number of N sampling points. This implies that the CPU time for solving this system on a computer is proportional to N^2 (Mallet, J.-L., 2002).

However, the Delaunay triangulation of n points in the plane can be computed in O(nlogn) expected time, so triangulation is faster to calculate.

References:

• Mallet, J.-L., (2002), Geomodeling, Oxford University Press, p. 510.

• De Berg M, Cheong O, Van Kreveld M, Overmars M (2008) Computational Geometry: Algorithms and Applications, 3rd Ed. Springer

Response: Thank you for pointing out this issue. We have addressed that "implicit modeling often requires a large solution system of linear equations to consume more computational time than explicit modeling, e.g., the Delaunay triangulation".

See Lines 37-39.

#3 In the response file, the authors write:

Inspired by the well-known iterated conditioned mode method, instead, we devise an iterative scheme to optimize potential function f and the gradient magnitudes $\mathbf{l}$ alternatively.

Reviewer:

The authors write that they are inspired by „conditioned mode method", however I cannot see any citation for this inspiration in the manuscript. The authors cannot assume that geological readers are just familiar with the concept of „conditioned mode method". In fact, I can see the following definition (https://en.wikipedia.org/wiki/Iterated_conditional_modes):

„In statistics, iterated conditional modes is a deterministic algorithm for obtaining a configuration of a local maximum of the joint probability of a Markov random field. It does this by iteratively maximizing the probability of each variable conditioned on the rest."

Reviewer:

You don't use Markov random fields in the manuscript, so why the „conditioned mode method" should be relevant to your method?

Response: Sorry for the lack of citation and the confusing explanation. As you pointed out, the iterated conditioned mode (ICM) method could be strange and unknown to the readers of Geoscientific Model Development. Thus, we have used the term *alternating optimization*, which would be more straightforward to present our optimization scheme, instead of original term. The alternating optimization is an optimization scheme that alternately updates just some variables at a time rather than update of all variables simultaneously. The ICM is an early and representative example of alternating optimization. In the revised manuscript, we have briefly explained the alternating optimization and cited the literature (Bezdek, J. C., & Hathaway) to show that each alternating optimization is well-suited to our optimization problem.

Bezdek, J. C., & Hathaway, R. J. (2002). Some notes on alternating optimization. In Advances in Soft Computing—AFSS 2002: 2002 AFSS International Conference on Fuzzy Systems Calcutta, India, February 3–6, 2002 Proceedings (pp. 288-300). Springer Berlin Heidelberg.

See Lines 238-246.

#4 In the response file, the authors write:

In the revised manuscript, we have discussed the limitations of our implicit modeling method in Section Discussions.

Reviewer:

However, in the Discussions section, I cannot see any new comments about limitations (e.g. use of data from outcrops discussed in the response file). There are only new comments about future work.

Response: Thank you for pointing out this issue. We have discussed the limitations of sub-domain division by fault and uncertainty introduced by using the orientations from outcrops in the Section Conclusions, and these limitations lead to the future work.

See Lines 511-513.

#5 The authors explain the three components of the energy function in the response file but not in the revised manuscript.

Response: Thank you for pointing out this issue. We have added these explanations to the newly revised manuscript.

See Lines 163-166.

#6 What is actually the „novel optimizing term" ? (Line 57)

Response: This optimizing term refers to the diagonal matrix $\mathbf{\Lambda}$ in Equation 10. We have modified this statement in Section Introduction because it is not introduced until Equation 10.

See Line 55.

#7 In the response file, the authors write:

„scattered multivariate Hermite-Birkhoff data (i.e., unstructured points and orientations)"

Reviewer:

If the definition for Hermite-Birkhoff data is „unstructured points and orientations", then why not writing only „unstructured points and orientations"? I still don't know why the term

„Hermite-Birkhoff" appears in the manuscript.
Response: Thank you for the suggestion. The difference between Hermite-Birkhoff radial basis function (HRBF) and standard radial basis function (RBF) is the presence of orientations. We use the term of Hermite-Birkhoff because of two persons' innovations in interpolating using orientations in functional analysis.
See Lines 44-46.

Report #2: Goncalves, Italo

The article seems suitable for publication in its current form.
Response: We appreciate your positive comments and grateful for that.

A final remark: it is important to mention right at the introduction that the difference between Hermite-Birkhoff and standard RBF interpolation is the presence of derivatives.
Response: Thank you for pointing out this issue. Revised.
See Lines 44-46.

Minor remarks:

Line 50: "simulate" does not seem to be the ideal word. Perhaps "models", "generates" or "interpolates".
Response: Thank you for the suggestion. Revised.
See Line 53.

Lines 70-72: Gonçalves et al. (2017) fits into the continuous category of models. If I'm not mistaken, Renaudeau et al. (2019) is a continuous model as well.
Response: Thank you for pointing out this issue. We have categorized your article into continuous models, however, Dr. Lachlan Grose suggested that Renaudeau et al. (2019) is a somewhat discrete model.
See Lines 91-93.

Report #3: Grose, Lachlan

The revisions seem to cover all of my comments. One clarification is needed on Line 243 - reference to constant gradient should also reference Frank et al 2007 and Caumon 2013. My comment about DSI was that the 1992 mallet papers do not present the 3d algorithm which is presented in frank 2007, caumon 2013.
Response: We appreciate your positive comments and grateful for that.
We also thank you for pointing out this issue. We have added these references to

methods of constant gradient.
See Lines 226-227.