Dear Dr. Michal Michalak:

Thanks for your effort to review our manuscript titled " AdaHRBF v1.0: Gradient-Adaptive Hermite-Birkhoff Radial Basis Function Interpolants for Three-dimensional Stratigraphic Implicit Modeling ", and now we have just revised this manuscript according to your good suggestions. The details are as follows, and all the revisions are done using track changes in Word.

RC3: 'Comment on egusphere-2022-1304', Michal Michalak, 03 Mar 2023

I decided to review this paper from a more optimization perspective since authors use "optimization" terms very often throughout the manuscript. And my field is rather data science in geology where optimization problems are common. But I'm glad to see proper implicit interpolation guys among reviewers who can better evaluate the contribution to their field.

My general opinion is that in the present form it is difficult to evaluate the contribution because it seems that the key methods are not referenced properly. For example, a textbook about Functional Analysis (pure mathematics) is referenced to support applications of interpolation concepts in geology - an unlikely source of information, where in fact there already exist very specific papers about Hermite-Birkhoff interpolation. Moreover, the authors use standard terms (optimization) in a non-standard context which makes the paper difficult to read: I am looking for an optimization criterion but I can't find it.

Response: We appreciate your review efforts. We thought the Hermite-Birkhoff interpolation in pure mathematics could be referenced. We have replaced a new paper about HRBF applied in geoscience. We have also explained why we regard our solution as an optimization.

A positive note: the paper has a logical structure and a neglible overlap with previous work of the author.

Response: We appreciate your positive comments and grateful for that.

Line 20: What do you mean by optimization? Optimization is usually considered either as minimizing something bad (e.g. misfit function) or maximizing something good (e.g. profit). The Wikipedia definition says: "mathematical optimization is the selection of a best element, with regard to some criterion, from some set of available alternatives". Despite many occurrences of "optimization" throughout the manuscript, I cannot find a criterion that is optimized. Instead, I can hypothesize that by "optimizing" the authors mean learning the true value of something. So I would argue that this research is not about optimization. If the authors do not agree, I would like to see an explanation of:

1) why should we consider the results obtained by authors as optimal, i.e. why any other candidate solution is worse than the results proposed by authors in relation to some criterion

2) a thorough description of methods assumed to give optimal results

As a side note, I was once requested by a reviewer regarding why calculating eigenvectors from an orientation matrix should give optimal results. You can see how it was done in the 4.4.2 section of the below paper (the content rather irrelevant for your paper):

Michalak, M. P., Kuzak, R., Gładki, P., Kulawik, A., & Ge, Y. (2021). Constraining uncertainty of fault orientation using a combinatorial algorithm. Computers and Geosciences, 154, 104777.

(https://doi.org/10.1016/j.cageo.2021.104777) , or here (free access): https://github.com/michalmichalak997/3GeoCombine/blob/master/Michalak_2021_combinatorial_accepted.pdf

Response: Thank you for pointing out this issue. We are sorry for the lack of definition of objective function of the optimization. In our setting, both the potential function f and the gradient magnitudes $\mathbf{l}$ are known. Thus, they are optimized by minimizing an objective function, which leads to a minimization problem as:

$$\min_{f,\mathbf{l}} \sum_{i=1}^{N} (f(\boldsymbol{p}_i) - f_i)^2 + \sum_{j=1}^{M} \left(\frac{\partial f(\boldsymbol{p}_j)}{\partial x} - l_j g_j^x\right)^2 + \left(\frac{\partial f(\boldsymbol{p}_j)}{\partial y} - l_j g_j^y\right)^2 + \left(\frac{\partial f(\boldsymbol{p}_j)}{\partial z} - l_j g_j^z\right)^2$$
$$+ \int_{\mathbf{R}^3} \frac{\partial^2 f(\boldsymbol{p})}{\partial^2 x} + \frac{\partial^2 f(\boldsymbol{p})}{\partial^2 y} + \frac{\partial^2 f(\boldsymbol{p})}{\partial^2 z} + 2\frac{\partial^2 f(\boldsymbol{p})}{\partial x \partial y} + 2\frac{\partial^2 f(\boldsymbol{p})}{\partial y \partial z}$$
$$+ 2\frac{\partial^2 f(\boldsymbol{p})}{\partial z \partial x} dxdydz$$

Given such a challenging optimization problem, it is intractable to solve it directly using common optimization techniques such as variational approach. Inspired by the well-known iterated conditioned mode method, instead, we devise an iterative scheme to optimize potential function f and the gradient magnitudes $\mathbf{l}$ alternatively. In the revised manuscript, we have presented a thorough description of method to show that the iterative update of gradient magnitudes $\mathbf{l}$ in the current form is equivalent to optimize the objective function. Thus, we consider the resulting gradient magnitudes are optimal.

Line 36: "Speed" - when I build a triangulated surface using 800 points, it doesn't take more than two seconds. But when I try to do a similar thing using interpolation methods in GemPy, it takes really long - so I would argue that speed may not be the best marketing candidate for implicit interpolation methods. Moreover, in cokriging methods it is not enough to add surface points - you need to add 3D orientations. But if it is a subsurface terrain, then how do you get an independent orientation measurement? To sum up, I would like to see a discussion about limitations of implicit methods.

Response: Thank you for the suggestion. Comparing with explicit modeling, implicit modeling has the efficiency advantage of avoiding a lot of workloads of human-computer interaction. The 3D orientations are usually surveyed on the outcrops of strata. However, if it is a totally subsurface terrain, we could model it according to other orientations of its conformable strata with outcrops. In the revised manuscript, we have discussed the limitations of our implicit modeling method in Section Discussions.

Line 48: This is the first occurrence of HRBF in the manuscript so it should be preceded with full name. However, here you point to some weaknesses of HRBF and in line 57 you propose HRBF as your main contribution. I'm confused with this presentation.

Response: Revised. We have given the full name in first occurrence of HRBF. What we propose is AdaHRBF, a gradient-adaptive HRBF framework for SPF modeling. We have highlighted it in the revised manuscript.

Line 57: what is actually Hermite-Birkhoff interpolation? The concept should be explained. In the paper, I can see only one reference (except rather inadequate one about functional analysis) about using Hermite interpolation theory in geology (Wang et al. 2018). I would say that the referenced paper better presents the foundational aspect of the method.

Response: Thank you for the suggestion. Revised.

"Generally, the basic RBF reconstructs an implicit function with constraint $f(\boldsymbol{p}_i) = f_i$, however, the HRBF reconstruct an implicit function which interpolates scattered multivariate Hermite-Birkhoff data (i.e., unstructured points and orientations)"

Lines 155-157: I can see three components of the energy function E (two sums and one integral).

1) What do these components represent and how they can be interpreted?

2) I can see that a textbook about functional analysis is referenced to support the equation (Eq.1). Where exactly in this book did you find information about "minimizing smoothness and unevenness of the energy function"? It seems that it is a general mathematical textbook so I would be surprised to see there notions such as "energy function of stratigraphic potential field" or "degree of unevenness". In fact, I have checked the 1972 edition of the Bachman&Narici book and I could not find such concepts. If you found them in 2000 edition, please provide a scan.

3) can you reference other works where Eq. 1 is used?

Response: Thank you for the suggestion. The first and second components represent the misfit between the estimated values and observed contact points and orientation points, respectively. The third component is the second-order derivative of implicit function to represent smoothness of SPF implicit function. What we do is to minimize the energy function. We have removed this reference to avoid misunderstanding.

Line 511: does your work address the problem of subjectivity in implicit methods mentioned by Grose et al. 2021 (text below)? Please discuss.

"The fundamental reasoning behind our approach is that the subjective constraints that are required to capture the geological features with standard implicit algorithms will be one of the greatest sources of uncertainty in the model." (https://doi.org/10.5194/gmd-14-3915-2021)

Response: Thank you for pointing out this issue. We are suffering the same uncertainty problem from the subjectivity in implicit methods so that additional geophysical exploration data and geological interpretation should be incorporated into the modeling constraints in the future.

Title: I would suggest to change the title so that it presents the main value of the research. As of now, the first part of the title contains some technical terms but in my opinion it should point to the added value for three-dimensional stratigraphic implicit modelling. So if it is optimization, then I would like to see the reflection of optimization in the title.

Response: Thank you for the suggestion. We have changed the title as "AdaHRBF v1.0: Gradient-Adaptive Optimized Hermite-Birkhoff Radial Basis Function Interpolants for Three-dimensional Stratigraphic Implicit Modeling"