Dear Dr. Lachlan Grose:

Thanks for your effort to review our manuscript titled " AdaHRBF v1.0: Gradient-Adaptive Hermite-Birkhoff Radial Basis Function Interpolants for Three-dimensional Stratigraphic Implicit Modeling ", and now we have just revised this manuscript according to your good suggestions. The details are as follows, and all the revisions are done using track changes in Word.

RC2: **'Comment on egusphere-2022-1304', Lachlan Grose, 20 Feb 2023**

The paper presents AdaHRBF (adaptive hermite-birkhoff radial basis function), a new interpolation method for building implicit geological models. The main contribution of this the iterative process for adapting the gradient of the implicit function to prevent artefacts due to inconsistent gradient magnitude norms.

The paper is generally well written with a logical structure and I believe that it is a good contribution towards the field. In general, the authors have referenced some of the appropriate literature however I believe a deeper analysis of the different implicit modelling techniques would improve the paper – for instance when reviewing discrete smooth interpolation that papers by Mallet 1980/1992 were presented as implicit methods where these papers actually discuss the method applied to 2D surfaces.

Response: We appreciate your positive comments and grateful for that. We thought DSI interpolant could also be applied to implicit modeling with a tetrahedral mesh. We have removed these references to avoid misunderstanding.

The authors should thoroughly review the paper “Three-Dimensional Modelling of Geological Surfaces Using Generalized Interpolation with Radial BasisFunctions” as there are a lot of parallels with the presented works that seem to be missed.

If the method is presented as an approach to tackle the issues with fold geometries it would be worthwhile reviewing the relevant literature around fold modelling: Laurent et al., 2016, Grose et al., 2017,2019, 2020, Hillier et al., 2014.

Response: Thank you for providing the excellent references. We have cited these articles in the Sections Related Works and Discussions.

Don’t change between strike and dip data and attitude data, keep it consistent. Preferably something meaningful for geologists.

Response: Thank you for the suggestion. We have changed attitude data to strike and dip data.

What is wrong with the formatting of the pages with equations? All of the equations need to be carefully checked to ensure that they are readable and there are no extra symbols!

Response: We have corrected this compilation error from word file to pdf file.

The figure captions are brief and difficult to follow. They should be stand alone and provide a description and brief interpretation of the contents.

Response: Thank you for the suggestion. We have updated the captions of Figs 1, 2, new 7, new 8, new 12, new 14, and new 17.

I am not sure what the stratigraphic index adds, it is hard to interpret. For example figure 23 section 14 near D1y there is an odd geometry. What causes this? It is orthogonal to the expected orientation of stratigraphy?

Response: Thank you for the suggestion. We have removed the contents related to STI.

Major comments:

From reading the manuscript the main contribution is the ability of the interpolator to adapt to variations in the magnitude of the gradient norm. This is a problem that was discussed by Gautier Laurent in doi: 10.1007/s11004-016-9637-y but his paper does not appear to be referenced. In this paper an iterative approach for updating the gradient norm was presented in a discrete modelling approach. This paper should be discussed and compared with the work presented here, I would strongly encourage the authors to provide a comparison between the two methods.

Response: Thank you for providing this excellent reference. We have discussed this article in the Section Optimization of Gradient Magnitude.

I am not convinced that adding the constant to the diagonal component of the second derivative matrix actually changes the magnitude of the gradient norm. I believe that it will just allow for a larger misfit between the orientation observations and the implicit function – which will have the same result but means that any information in the gradient direction will not be incorporated. If the method is actually just removing outlier data then should the message of the paper be changed to this rather than for adapting the magnitude of the gradient norm?

Response: Introducing optimal term of $\boldsymbol{\lambda}$ into HRBF linear solution will simultaneously cause misfit or gradient direction and changing of gradient magnitude. However, we only update the gradient magnitude in the iterations and do not change gradient direction. Meanwhile, when $\boldsymbol{\lambda}$ is finally close to zero, the HRBF linear solution will satisfy the original gradient direction and iteratively obtained gradient magnitude.

I would be interested in seeing a comparison between this method and a discrete approach where the regularisation contribution can locally change, see LoopStructural paper in GMD for a comparison between discrete interpolation and RBF. I would also be interested in seeing the model without any orientation data and just interpolating from the contact locations and also when constraining the direction of the gradient using tangent constraints.

Response: Thank you for the suggestion. We have discussed regularization contribution in LoopStructural in the Section Optimization of Gradient Magnitude. Meanwhile, we have compared the model interpolated by RBF from the contact locations without any orientation.

Details comments:

Line 26: replace significance with importance

Response: Thank you for the suggestion. Revised.

Line 29: delete “and has garnered extensive attention from geologists”

Response: Thank you for the suggestion. Revised.

Line 30: Implicit/explicit definition should refer to the approaches as ways of representing surfaces not as methods for building models

Response: Thank you for the suggestion. Revised.

Line 39-46: A lot of the mentioned studies are not reliant on implicit modelling, you could do the same thing with explicit models. E.g. implicit function is not combined directly with geophysics the model is discretized first which means it could be replaced

with a model defined by explicit surfaces. Same point for uncertainty analysis. I would replace this section with relevant references to implicit modelling not just a list of all studies that use implicit modelling

Response: Thank you for the suggestion. We have removed those irrelevant references, and references of implicit modelling are listed in the Section 2 Related Works.

Line 48: delete “in HRBF method”, you can do the same in discrete as well

Response: Thank you for the suggestion. Revised.

Line 60: What was adaHRBF method compared with? It is presented as better than an alternative

Response: Thank you for the suggestion. We have revised that AdaHRBF is compared with HRBF interpolant using constant unit normal gradients and RBF interpolant only using contact locations without orientations.

Line 63: “Distribution of attribute” what do you refer to here it is unclear

Response: Revised. We refer to the attribute of scalar field, i.e., the relative buried depth in our manuscript.

Line 70- 73: Mallet reference is for DSI applied to nodes of triangular surface, do you mean to reference mallet 2004 or frank 2007/ caumon 2013?

Response: Thank you for the suggestion. We thought improved DSI interpolant could then be applied to implicit modeling with a tetrahedral mesh. We have removed Mallet (1989,1992) to avoid misunderstanding. These three references better represent implicit interpolation in GoCAD.

Line 76-80: This paragraph started as discrete modelling and then jumps to rbf methods, perhaps keeping them separate will make it easier for the reader

Response: Thank you for the good suggestion. We have separated Section Related Works into two parts, i.e., discrete interpolants and continuous interpolants.

Line 92-94: Renaudeau and Irakarma are discrete or somewhat discrete methods

Response: Thank you for the suggestion. We have reclassified them into discrete interpolants.

Line 118-121: “Moreover, RBF/HRBF-based methods construct implicit field functions separately for each geological interface and extract the zero value equipotential surfaces to locate the geological interface. Therefore, it is difficult to maintain topological consistency between geological bodies, let alone to represent their internal attributes and structural attitudes.”

This is not true, the surfe library by Hillier et al 2014 can do all of these points…

Response: Thank you for the suggestion. We have removed this statement.

Line 126: Reference first sentence of paragraph

Line 126: What do you mean by geological maps? Do you mean the outcrop pattern of contacts?

Response: Yes, we mean the outcrop pattern of contacts on the planar geological maps.

Line 130: Change annotation of f1/f2 to something that can’t be confused with faults

Response: Revised.

Line 153: optional? Should be optimal?

Response: Yes, it is “optimal”.

Line 162: Can you not change the order of the polynomial trend? Hillier et al can?

Response: As same as Hillier et al (2014) mentioned, the degree of the polynomial is restricted to be at most (m − 1) for CPD functions of order m, whereas for SPD functions there is no restriction.

Line 165: define the meaning of f*

Response: f* is the estimation function of f.

Line 174-178: delete table and reference to other basis functions. If you only include r3 then why introduce the others. You could refer the reader to Hillier et al

Response: Thank you for providing this excellent reference, we have cited Hillier et al (2014) for referring other radial basis functions and removed Table 1 to save space.

Line 179-180: The explanation of the construction of the matrices is not clear, it is not obvious what each component represents. Either leave this information for supplementary material if its not necessary for understanding or add more explanation about the different terms.

Response: Thank you for the suggestion. We have added more explanations about the different terms of matrices.

Line 196: “added into modelling process” add references to all of the work that already does this e.g. hillier et al 2014, caumon 2013 etc

Response: Revised. We have added these references.

Line 212: “it is difficult to obtain the gradient magnitude through any geological observation.” Delete

Response: Thank you for the suggestion. Revised.

Line 222: “the same gradient magnitudes” what do the red circles indicate

Response: We have removed the hand-drawn Figure 3 and provided two computed examples (new Figures 4 & 5) to show the inconsistent of SPF caused by forcing equal gradient magnitude for each attitude point.

Line 224: “to the Eq. 4” change to “to Eq. 4”

Response: Revised.

Line 224: “used” replace with “and used”

Response: Revised.

Line 227: There are similarities to this diagonal block to the smoothing parameter in Surfe – this was used in Grose et al., 2020 to show a comparison between smoothing regularisation in rbf to the regularisation used by discrete interpolation

Response: We have discussed regularization contribution in LoopStructural in the Section Optimization of Gradient Magnitude.

Line 238-239: My understanding of adding a constraint to the diagonal of the matrix is it allows for the interpolant to have a larger misfit to the constraint. So this means that by iteratively adjusting the diagonal for specific gradient constraint you are actually changing how well those constraints are honoured by the interpolant which includes not just the gradient magnitude but also the orientation constraint.

Response: Introducing optimal term of $\boldsymbol{\lambda}$ into HRBF linear solution will simultaneously cause misfit or gradient direction and changing of gradient magnitude. However, we only update the gradient magnitude in the iterations and do not change gradient direction. Meanwhile, when $\boldsymbol{\lambda}$ is finally close to zero, the HRBF linear solution will honor the original gradient direction and iteratively obtained gradient magnitude.

Line 247: replace convergency with “ when convergence is reached”

Response: Revised.

Line 268: “distribution of attribute and attitude points;”

Are the attribute points constraining the value of the implicit field or are they "interface" points as per calcagno where they set the implicit field to be constant along all points related to a single contact? I don't understand how if the points aren't constraining the value of the implicit field this results in a scalar field with the same range as the original dataset when the gradient norms are unit vectors.

Response: Yes, indeed. The attribute point includes an implicit field value.

Line 330: explain the cross section more

Response: Thank you for the suggestion. We have added more explains of cross-sections.

Line 336: “attitude points”

Personally I don't think attitude points speaks to me as a geologist, could you refer to orientation observations or structural observations. At least make it consistent with the geological map, you have angle of strike and dip vector as the legend

Response: Thank you for the suggestion. We have changed attitude data to strike and dip data.

Line 358: “he” replace with “the”

Response: Revised.

Line 470: “However, existing RBF and HRBF interpolants implicitly reconstruct a single geological interface and extract it as the zero-value equipotential surface.”

Not true, read and reference Hillier et al., 2014

Response: Revised. We have removed this paragraph.

Line 472: “Moreover, existing RBF and HRBF interpolants need several independent scalar fields to simulate geological interfaces”

This is also not true, Hillier et al use a single scalar field. You also use several scalar field because you represent each fault block independently. If you see the fault modelling method in Grose et al., 2021, this can be used with Surfe and would allow for a single scalar field for stratigraphy.

Response: Revised. We have removed this paragraph.

Line 484: “they are incapable of interpolating or extrapolating a fold series within a continuous structural style” this point by Jessell 2014 was addressed by a few publications Laurent et al., 2016, and Grose et al., 2017,2018,2019,2020 as well as Hillier et al., 2014

Response: Thank you for providing the excellent references. We have cited these articles in the Sections Related Works and Discussions.

Line 486-489: How have you addressed point 1)? You don't extrapolate a fold series in this manuscript, you interpolate a fold shape from gradient constraints but that is not the same. I would remove this section as it is not consistent with the literature.

Response: Revised. We have removed “extrapolate” from point 1).

Line 490-496: This section makes no sense, needs revisiting.

I don't see using the burial depth as being a new contribution from this paper, it is the same method used by various authors Caumon 2013, Hillier 2014, Grose et al., 2020

Response: Revised. We are explaining the burial depth is more suitable in our solution than other type of potential field value.

Line 505: “Because 3D stratigraphic potential fields can be coupled with various geoscience numerical simulation methods, they have a broad prospect for application in related fields such as metallogenic prediction.” Delete or move to discussion, don’t introduce a new idea in the conclusion

Response: Thank you for the suggestion. We have removed this sentence from Section Conclusion.

Line 510: “A goal for future work is to introduce a drift function in the model to accommodate discontinuity of fault planes. In addition, the uncertainty of the model should be considered in the modeling process, and additional geophysical exploration data and geological interpretation should be incorporated into the modeling constraints.”

Move to discussion, but also please ensure that you reference the limitations of a drift function.

Response: Thank you for providing your excellent reference. Revised as “A goal for future work is to introduce a fault integrating way into the implicit model to accommodate discontinuity of fault planes.”