Dear Dr. Italo Goncalves:

Thanks for your effort to review our manuscript titled " AdaHRBF v1.0: Gradient-Adaptive Hermite-Birkhoff Radial Basis Function Interpolants for Three-dimensional Stratigraphic Implicit Modeling ", and now we have just revised this manuscript according to your good suggestions. The details are as follows, and all the revisions are done using track changes in Word.

RC1**: 'Comment on egusphere-2022-1304', Italo Goncalves, 17 Feb 2023**

In this work, the authors introduce an iterative procedure to deal with the uncertainty in the gradient magnitudes of attitude data in implicit geological modeling, which represent local inverse thickness. The results are very solid, and I believe the manuscript is suitable for publication after the corrections pointed below.

Response: We appreciate your positive comments and grateful for that, and now we have just revised this manuscript according to your good suggestions.

Line 175: if the cubic function is the only one used, I see no reason the present other types in a table. You can point to a reference that lists them and save space. Also, in 3 dimensions the cubic function is the one that minimizes the curvature (eq. 1), the others do not necessarily do so. See Chapter 6 in Rasmussen and Williams (2006) and Wendland (2005).

Response: Thank you for providing the excellent references, we have cited them for listing other radial basis functions and removed Table 1 to save space. We also explained the reason why we use the cubic function as it satisfies Equation 1.

Line 241: isn't there a risk of obtaining a negative λ_{t+1} with eq. 8? Have you tried using something like $\lambda_{t+1} = (l_t - l_{t-1})^2$? Also, I suppose the same update is applied to the 3 directions, but it is important to emphasize this.

Response: Thank you for excellent suggestion. Yes, there is a risk of obtaining a small negative λ_{t+1} with Eq. 8. We will try $\lambda_{t+1} = (l_t - l_{t-1})^2$, therefore, Eq. 8 and Eq. 9 should converge simultaneously. We have emphasized that we apply the same λ_{t+1} to three directions.

If contact data is somehow unavailable or unreliable in part of the space, would the model be able to benefit from off-contact point data (such as indicators for stratum A, stratum B, etc.)? They could be useful to improve the classification accuracy close to the DTM where this information is available. See Hillier et al. (2014).

Response: We do not integrate the off-contact point indicating a lithologic marker in our linear AdaHRBF system. However, the data source of our method mainly includes geological map and cross-section. In geological mapping, geologist will consider this kind of point when connecting the boundary of a stratum.

This might already be published elsewhere, but I could not see the difference between HRBF and basic RBF. The equations presented seem the same as the basic RBF equations seen in the books. Please clarify the difference and point to the reference that introduced HRBF.

Response: Thank you for the suggestion. Revised.
Generally, the basic RBF reconstructs an implicit function with constraint $f(\boldsymbol{p}_i) = f_i$, however, the HRBF reconstruct an implicit function which interpolates scattered multivariate Hermite-Birkhoff data (i.e., unstructured points and orientations). See Macedo et al. (2011).

A few points to improve the discussion. Please elaborate on how the present work compares to these (which have already been cited):

- This approach is very similar to von Harten et al. (2021), with the difference that here the diagonal matrix is applied to the gradients instead of the contacts.
- Gonçalves et al. (2017) use the strike and dip vectors as zero-gradient directions in order to avoid assigning an arbitrary magnitude to the normals. Have you tried this approach?
- By extending the inequality constraints by Hillier et al. (2014) to the gradients, it is possible to obtain the same results presented here, in principle.

Response: Thank you for the suggestion. Revised.
(1) Because the diagonal matrix directly relates to the input data, von Harten et al. (2021) add variation σ^2 to the contacts in the contact diagonal matrix to realize the local smoothing as well as we add optimal item λ on the gradient diagonal matrix to iteratively get the optimized gradient magnitude.
(2) Your method used a novel zero-magnitude gradient to avoid assigning a magnitude or modulus, which is the tangent constraint. However, we emphasize on iteratively getting the optimized gradient to represent changing trend of stratigraphic potential field.
(3) Besides constraints of scattered multivariate Hermite-Birkhoff data, the Generalized RBF, proposed by Hiller et al. (2014), reconstructs an implicit function with more constraints of lithologic markers (inequality) and lineations (tangent). A goal for future work is to integrate these constraints in our solution to utilize more kinds of modeling data. In our method, the gradients are transformed from off-contact or on-contact structural, and we could not connect the gradient with the inequality, see Fig.1 of Hiller et al. (2014).

I did not find the STI to be very informative of the stratigraphic characteristics of the strata. It seems to be a little erratic and can vary from the minimum to maximum value within the same stratum, which seems to defeat its very purpose. Perhaps trying to assign a geological meaning to the gradients is not a good idea, as they can be very dependent of the specific data points that were used and are a result of the minimum-curvature characteristic of RBF. I think the manuscript would not suffer with the removal of this section.

Response: Thank you for the suggestion. We have removed the contents related to STI.

Minor points:

The manuscript seems to suffer from a compilation error. See pages 6 and 7. All the R symbols are displaced.

Response: We have corrected the compilation error of pdf file.

Lines 59, 348: “true” gradient magnitudes seem a rather strong term. I would call it an optimized gradient.

Response: Thank you for the suggestion. Revised.

Line 184: a line break after the semicolon would improve readability.

Response: Thank you for the suggestion. Revised.

Line 200: it might be worth mentioning that $\theta_2 = \theta_1 + 90^{\circ}$.

Response: Thank you for the suggestion. Revised.

Figure 2: if the vectors g and n have the same direction but not necessarily the same magnitude, I think the figure could be improved by adding a vector n with a different length.

Response: Thank you for your good suggestion. We have updated Figure 2.

Line 212: please rephrase to avoid starting a new section with “however”.

Response: Thank you for the suggestion. Revised.

Line 218: suppress “the”.

Response: Revised.

Figure 3: was it hand-drawn or computed? A computed example might make the point clearer.

Response: Thank you for your good suggestion. We have removed the hand-drawn Figure 3 and provided two computed examples (new Figures 4 & 5) to show the inconsistent of SPF caused by forcing equal gradient magnitude for each attitude point.

Line 231: it is worth emphasizing that n is a unit vector.

Response: Thank you for the suggestion. Revised.

Figure 7: I think this example is unnecessary given the previous two, but I leave it at the authors’ discretion.

Response: Thank you for the suggestion. We have removed the Figure 7.

Figure 12: is this field value the burial depth that was mentioned before? How was it measured? Is it constant within a given contact?

Response: Yes, the relative burial depth was mentioned in new Figure 8 (Comprehensive stratigraphic column of the study area) from regional geological studies. We assigned an approximate constant for each contact in the whole study area. We have also updated the legend of new Figure 10 using contact points with different colors.

Line 349: “The changes of gradient magnitude **are** shown…”

Response: Revised.

Line 475: “attitude”

Response: Revised.

References:

Rasmussen, C. E., & Williams, C. K. I. (2006). Gaussian processes for machine learning. MIT Press. https://doi.org/10.1142/S0129065704001899

Wendland, H. (2005). Scattered Data Approximation. Cambridge University Press.

Response: Thank you for providing the excellent references, we have cited them for listing other radial basis functions.