Origin of the NSE

This is in response to the plea by the authors for insights on the popular Nash–Sutcliffe model efficiency criterion (NSE) - Lines 79-82.

I would like to share my memory of the origin of an alternate version of NSE. In hindsight, the scientific meaning of NSE boils down to what the term "initial variance" means. This was recently reported in Ding (2018) which may have escaped the attention of the authors.

Using the original notations, the NSE scale is recast from Equation (1) as follows:

$$R^2 = (F_0 - F)/F_0,$$

$$F_0 = \sum(o_i - \mu_o)^2,$$

$$F = \sum(f_i - o_i)^2,$$

in which: R^2 is the model efficiency, i.e. NSE; F_0 is the "initial variance" of the observations, o_i; F is the "residual variance" of the forecasts, f_i; subscript i is time; and μ_o is the mean of the observations.

In the literature, there existed an alternate definition presented by Ding (1974) of the so-called "initial variance", F_0. The alternative, symbolized by, say, F_{0-d}, appeared four years after Nash and Sutcliffe (1970), not year 1974 as written in the preprint.

I defined F_{0-d} directly from F by replacing the observations by the mean, i.e.

$$F_{0-d} = \sum(f_i - \mu_o)^2,$$

F_{0-d} thus becomes the sum of squares of the error (SSE) of a model measured against the observed mean flow. This is different from F_0 which is simply the variance of the observations.

The origin of an observed-mean-flow benchmark or baseline was embedded as μ_o in the "initial variance" of both standard and alternate versions. But the alternate version, F_{0-d}, shows unambiguously the observed mean flow, a baseline.

The alternative was preserved in Ding (1974, Eqs. (40) & (47)) where the criterion or scale was called, after Nash (1968-69), the model efficiency, R^2, same in notation as the coefficient of determination (Line 39). Absent from the paper was the name of Sutcliffe.

Nash was the Journal of Hydrology editor of the 1974 paper of mine, and, as far as I remember, had kept silent on the alternate definition of the "initial variance", a technical term he first coined. In Ding (1974) paper, the raw source, Nash (1968-69), was cited, but not the subsequent one, Nash and Sutcliffe (1970). The alternate efficiency scale could be named after both Nash and Ding as NDE (1974) to differentiate it from the standard NSE (1970) after Nash and Sutcliffe.

Thanks for the opportunity to reflect on the genesis of a classical but imperfect metric in evaluating the performance of hydrologic models. So rudimentary is the observed-mean-flow baseline, and so imprecise the Nash–Sutcliffe efficiency (NSE) scale. But alternatives exist to both the baseline, e.g., Ding (2018), and the standard scale, e.g., Ding (1974). A combination of the two alternatives will lead to an invention of a new metric, "elegant" and "intuitive", both characterizations (Line 79) favoured by the authors of a metric.

References

Ding, J. (2018). "Interactive comment on "On the choice of calibration metrics for "high flow" estimation using hydrologic models" by Naoki Mizukami et al." doi: https://doi.org/10.5194/hess-2018-391-SC1.

Ding, J. Y. (1974). "Variable unit hydrograph". Journal of Hydrology, 22.1-2, pp. 53-69.