Reply to Dr. Ding's comment

We would like to thank Dr. Ding for the sharing on the interesting history of developing the Nash-Sutcliffe efficiency (NSE) and a related variants which was proposed but has never been considered in practice. Here are our explanations for this variant:

The following efficiency was introduced in Ding (1974) four year after the introduction of NSE in Nash and Sutcliffe (1970)

$NDE = 1 - \frac{\sum(f_i - o_i)^2}{\sum(f_i - \mu_o)^2} = 1 - \frac{\overline{(f-o)^2}}{\overline{(f-\mu_o)^2}}$, (1)

where we call this efficiency NDE as in your comment. This means that instead of regressing observations on forecasts as in the case of NSE $o = f + \varepsilon$, NDE regresses forecasts on observations $f = o + \varepsilon$ and takes the resulting coefficient of determination as a similarity measure.

Under the additive error model

$f = o + b + \varepsilon$, (2)

NDE becomes

$NDE = 1 - \frac{b^2 + \sigma_e^2}{\sigma_o^2 + b^2 + \sigma_e^2} = \frac{\sigma_o^2}{\sigma_o^2 + b^2 + \sigma_e^2}$. (3)

This score is surprisingly the square of the correlation efficiency CE that we show in Equation 25 in the manuscript. We derived CE to illustrate that any score can be constructed from a monotonic function of the correlation coefficient ρ, and did not know that CE was indeed proposed in the literature. In the revised manuscript we will add a citation to your study for this efficiency.

Thus, NDE is equivalent to NSE, and both measure the noise-to-signal ratio in forecasts. However, there is a potential problem here if NDE was chosen instead of NSE in the history. From (1) like NSE we may wrongly conclude that NDE=0 marks the boundary between skillful and unskillful forecasts. But from (3) this threshold of NDE should be NDE=1/2 when the power of noise starts dominating the power of variation of observations $\sigma_o^2 = b^2 + \sigma_e^2$. Note that in the case of NSE, both equations similar to (1) and (3) yield the same threshold NSE=0.

Reference

Ding, J. Y.: Variable unit hydrograph. *Journal of Hydrology*, **22.1-2**, 53-69, 1970.

Nash, J. E., and Sutcliffe, J. V.: River flow forecasting through conceptual models. Part 1: A discussion of principles. *Journal of Hydrology*, **10(3)**, 282–290, doi:10.1016/0022-1694(70)90255-6, 1970.