Review of

Title: The rate of information transfer as a measure of ocean-atmosphere interactions

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The authors apply the Liang (or Liang-Kleeman) information flow (LIF thereafter) in order to quantify interactions between the ocean surface and lower atmosphere over the period 1988–2017 at monthly time scale. They investigate dynamical dependencies between sea-surface temperature (SST), SST tendency and turbulent heat flux in satellite observations and find a **strong two-way influence** between SST / SST tendency and turbulent heat flux in many regions of the world.

LIF is a very interesting approach, independent of all other causality methods known in the literature. It was analytically derived for dynamical systems and its general application requires the knowledge of the underlying equations of the studied dynamical systems. The only form, available for experimental data without the knowledge of the equations, was derived **for linear systems**. In spite of this, the approach has recently been applied for data from apparently nonlinear systems, relying on a few numerical examples by the original author in which LIF was applied to nonlinear systems.

In general, there are cases when linear approaches can extract correct causal relations from nonlinear data. One example is the causality in highly nonGaussian data from space weather area [1]. In the study [1], however, the analysis started by a general nonlinear approach – conditional mutual information (CMI thereafter, a.k.a. transfer entropy). In the next step, the response to time reversal suggested [according to study 2] that although the data were nonlinear, the observed causality is a sort of linear information transfer. The latter was confirmed by the application of the linearized version of CMI as well as by LIF, both giving consistent results with the nonlinear CMI. Applying a linear approach, LIF in this MS, alone, can be dangerous.

I will explain my concern using the well-known causality benchmark of unidirectionally coupled Rössler systems, described, e.g., in [3]. Figure 1 in this review, left panel, illustrates the successful application of LIF to the unidirectionally coupled Rössler systems, presented originally by X. San Liang. It took me some time to reproduce this result. The key to obtain the correct distinction of the direction of coupling is using very high sampling frequency, which is about 6000 samples per period (or pseudoperiod in this chaotic system), see Fig. 1 right panel. With this oversampling, the nonlinear dynamics is locally linearized. In Fig. 1. left panel, we can see that LIF in the direction of coupling (“causal direction”) increases with the increase of the coupling strength ε, while the LIF in the non-causal direction (the direction with no coupling, i.e. from the effect to the cause) remains on the zero value. A slightly disturbing fact is that the ε-dependence ignores the transient to synchronization (cf Fig. 4 in [3]), however, LIF in this case correctly identifies the causal direction.

The result is different when LIF is applied to the unidirectionally coupled Rössler systems sampled with ‘’usual’’ frequency, with about 20 samples per period, which is sufficient for inferring causality using nonlinear methods [3]. We can see in Fig. 2 that LIF in the causal direction nonmonotonically increases with the increase of the coupling strength ε, however, the LIF in the non-causal direction does the same, just with the negative sign. That is, when the sampling does not allow linearization of the problem, LIF detects information flow also in the direction where there is no connection, just its value is negative. The plot of LIF as function of coupling strength ε results in a symmetric figure (Fig. 2, left panel) with the zero axis as the axis of symmetry, meaning that LIF(x->y)=−LIF(y->x).

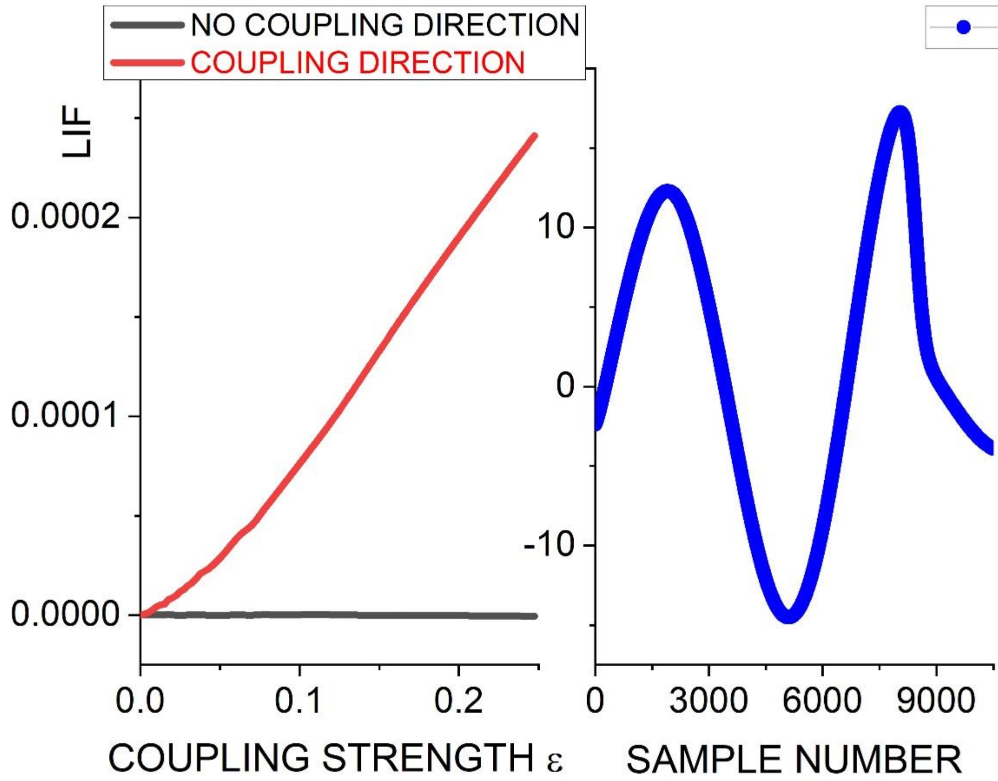

Fig. 1.: Left panel: LIF applied to coupled Rössler systems with very high sampling frequency, illustrated in the right panel.

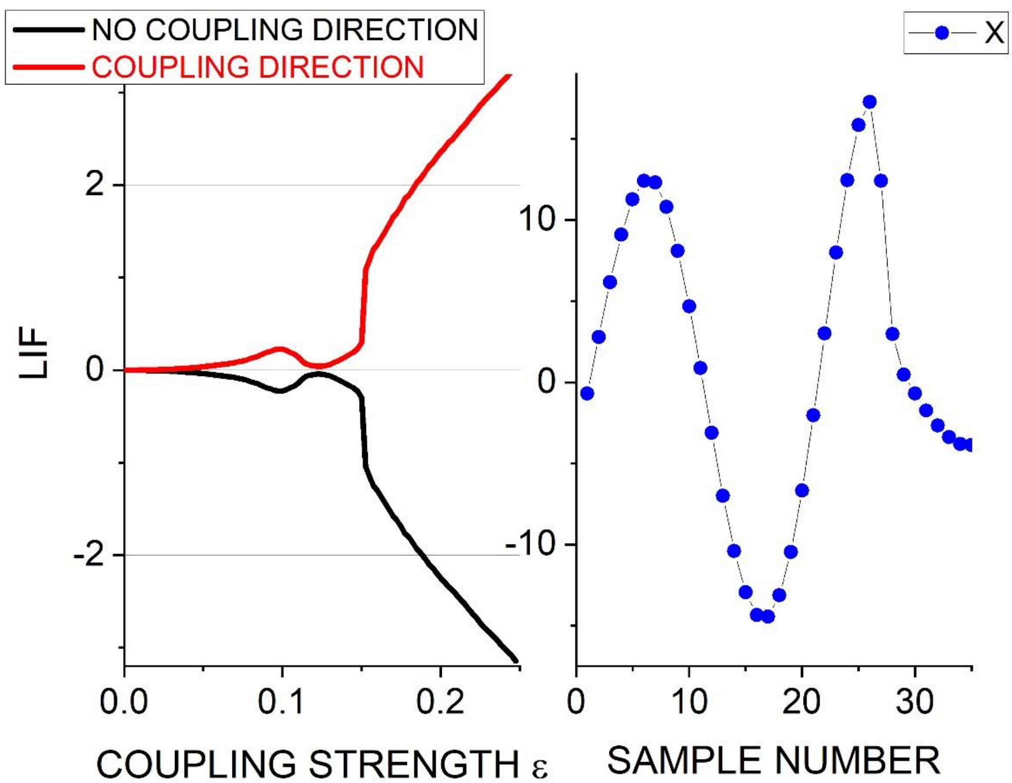

Fig. 2.: Left panel: LIF applied to coupled Rössler systems with usual sampling frequency, illustrated in the right panel.

The results presented in the current manuscript, e.g. MS Fig. 2 for the relation between SST and THF reflect the same symmetric pattern: LIF(SST->THF) is positive, marked by red color in the used color scale, while LIF(THF->SST) is negative, marked by blue color in the used color scale, and the red and blue patterns in parts (a) and (b) are the same. This is the same results as in the case of the unidirectionally coupled Rössler systems with usual sampling frequency, presented in our Fig. 2 above, i.e. the authors obtained that LIF(SST->THF)=−LIF(THF->SST). For any further discussion of the results presented in this MS, the authors should provide an evidence, based on an independent, nonlinear method, that the "symmetric information flow" (interpreted as a **strong two-way influence)** between SST or tSST and THF is indeed a physical phenomenon and not just a failure of the linear LIF applied to nonlinear data, as observed in the case of the unidirectionally coupled Rössler systems above.

[1] Manshour, Pouya, et al. "Causality and information transfer between the solar wind and the magnetosphere–ionosphere system." Entropy 23.4 (2021): 390.

[2] Paluš, Milan, et al. "Causality, dynamical systems and the arrow of time." Chaos: An Interdisciplinary Journal of Nonlinear Science 28.7 (2018): 075307.

[3] Paluš, M., & Vejmelka, M. (2007). Directionality of coupling from bivariate time series: How to avoid false causalities and missed connections. Physical Review E, 75(5), 056211.

M. Paluš, December 10, 2022