Reply to Referee

Liisa Juusola[1], Ari Viljanen[1], Andrew P. Dimmock[2], Mirjam Kellinsalmi[1], Audrey Schillings[3], and James M. Weygand[4]

[1]Finnish Meteorological Institute, Helsinki, Finland
[2]Swedish Institute of Space Physics, Uppsala, Sweden
[3]Department of Physics, Umeå University, Umeå, Sweden
[4]Department of Earth, Planetary, and Space Sciences, University of California Los Angeles, Los Angeles, CA, USA

Dear Reviewer,

thank you for a critical view-point. We have done our best to reply to each concern. The original review is written in black and our replies in blue.

I am sceptical of the authors' measures of internal and external geomagnetic field variation.

I am happy to be corrected on this, but I see no reason why the variational data could not, in principle, be fitted with a geomagnetic field model that are entirely generated by external current systems. When I say this, of course, I am not referring to the internal field generated in the Earth's core. That part of the field is essentially steady over the course of a magnetic storm. To emphasize, I'm referring to the storm-time variation in the geomagnetic field. I see no reason why that part of the field can't be entirely modelled by external sources.

The horizontal part of the geomagnetic variation field can indeed be expressed in terms of external currents only. However, the vertical (B_z) component of the field cannot, as demonstrated by Figure 1 and Figure 2, and indicates that such a reconstruction is not correct.

In this context, recall that a spherical-harmonic description of a global field has both internal and external parts (the division between the two comes with different radial functions), and this dichotomy is consistent with potential-field theory. That sort of internal-external division is rigorous, but such a division is not, to my knowledge, available for spherical elementary currents, where one starts off assuming that internal currents reside on a shell at an arbitrarily chosen depth.

Therefore, I would be extremely careful in making physical interpretations of the *internal* field while relying on an unrealistic assumption about the source of the internal field. The danger of circular reasoning, here, should be clear.

I note that the authors seem to admit most this on lines 248–250: "However, separation and interpolation of the geomagnetic field between the stations are not perfect and are affected by the density of the magnetometers as well as boundary conditions, as discussed by Juusola et al. (2020)." One wonders, then, how much the separation can be affected by the boundary conditions. This big issue is not addressed anywhere as far as I know.

My understanding is that the boundary conditions used in spherical elementary current systems are just mathematical conveniences. As such, they allow construction of specific field models, but these field models are only examples from the large set of models that can fit the data. In other words, the models are non-unique. Some of the possible models (with different chosen boundary conditions) might have lots of internal contribution, but others might have very little.

This sort of non-uniqueness is why spherical elementary currents are often described as "equivalent".

As long as such non-uniqueness exists, I don't know how the authors can come to any conclusions about the relative portions of internal and external fields.

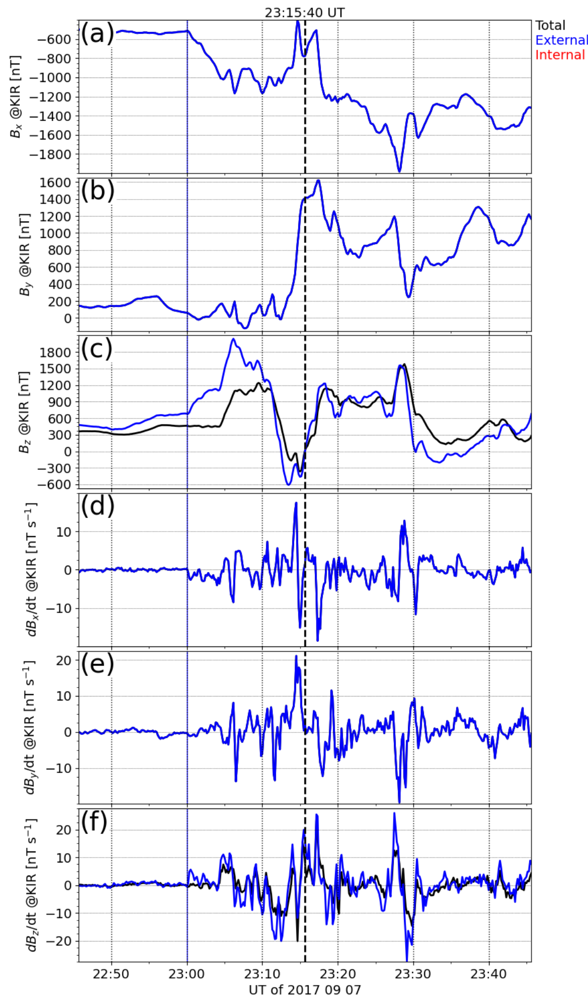

Figure 1. Observed magnetic field ("Total") in black and the 2D SECS reconstruction ("External") in blue using a single layer of SECSs at 90 km altitude. The black curve is not visible in panels a, b, d, and f, because it is perfectly covered by the blue curve. The time indicated by the dashed vertical line corresnponds to the example provided in the manuscript Figures 2a and 2d.

The total variation magnetic field at the Earth's surface is produced by currents in space (ionosphere and magnetosphere) and in the conducting ground (telluric currents). Both current systems are 3D, but they can be replaced by divergence-free currents on two spherical shells (e.g., Haines and Torta, 1994). These equivalent currents produce the same magnetic field at the Earth's surface as the true 3D currents. The location of the equivalent current layers is based on physical arguments: the upper layer is at 90 km altitude, practically below all currents in space, and the lower layer is just below the Earth's surface to represent all induced currents which can flow at any depth.

The SECS method is one option for deriving the divergence-free equivalent currents and separating the variation magnetic field into its external and internal parts. It is based on explicit current distributions from which the magnetic field is calculated according to the Maxwell equations. So the SECS method is as rigorous and unique as techniques based on spherical harmonics or Fourier analysis, for example, and it has certain practical advantages (Amm and Viljanen, 1999). In real-life applications,

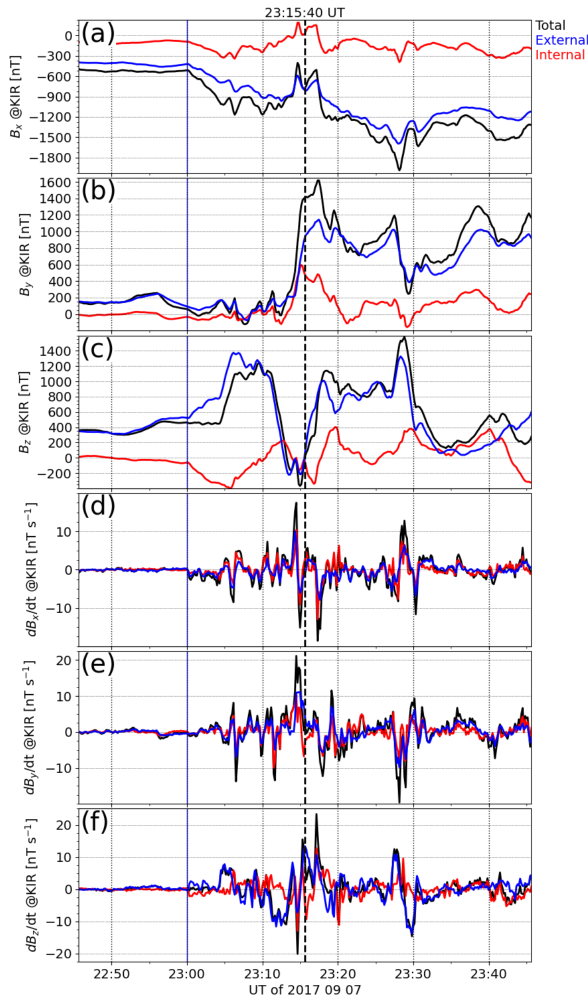

Figure 2. Observed magnetic field ("Total") in black and the 2D SECS reconstruction in blue ("External") and red ("Internal") using one layer of SECSs at 90 km altitude and another at 1 m depth. Summing up the blue and red curve produces a perfect match to the black curve in all panels. The time indicated by the dashed vertical line corresponds to the example provided in the manuscript Figures 2b, 2c, 2e, and 2f.

availability of the measured magnetic field from a finite set of points on a limited area instead of the whole globe causes some uncertainty as discussed by Vanhamäki and Juusola (2020). Similar issues naturally concern other methods too. We have shown in a previous study (Juusola et al., 2016) that the ionospheric divergence-free currents produced using the separation correspond very well to currents derived independently from low-orbit satellite-based magnetic field measurements. If the separation is not carried out, the correspondence is clearly worse.

It is true that we should be very careful about making physical interpretations about the 3D currents based on the reconstructed equivalent currents, but the magnetic field as decribed by the equivalent currents between the current sheets should be fairly reliable. An analysis concerning the uncertainties of the reconstruction would be very useful indeed. However, such an analysis is out of the scope of the present study, because it is far from straightforward and would require careful attention, preferably in a dedicated study.

Please consider these issues.

In order to clarify these issues, we suggest to add at line 25:

“Both current systems are 3D, but they can be replaced by divergence-free currents on two spherical shells (e.g., Haines and Torta, 1994). These equivalent currents produce the same magnetic field at the Earth’s surface as the true 3D currents. The locations of the equivalent current layers are based on physical arguments: the upper layer is at 90 km altitude, practically below all currents in space, and the lower layer is just below the Earth’s surface to represent all induced currents which can flow at any depth. The two-dimensional Spherical Elementary Current Systems (2D SECS) (Vanhamäki and Juusola, 2020) (SECS) method is one option for deriving the divergence-free equivalent currents and separating the variation magnetic field into its external and internal parts. It is based on explicit current distributions from which the magnetic field is calculated according to the Maxwell equations. In real-life applications, availability of the measured magnetic field from a finite set of points in a limited area instead of the whole globe causes some uncertainty, as discussed by Vanhamäki and Juusola (2020). Similar issues naturally concern other methods as well, such as those based on spherical harmonics or Fourier analysis.”

References

Amm, O. and Viljanen, A.: Ionospheric disturbance magnetic field continuation from the ground to ionosphere using spherical elementary current systems, Earth Planets Space, 51, 431–440, https://doi.org/https://dx.doi.org/10.1186/BF03352247, 1999.

Haines, G. V. and Torta, J. M.: Determination of equivalent current sources from spherical cap harmonic models of geomagnetic field variations, Geophysical Journal International, 118, 3, 499–514, https://doi.org/https://doi.org/10.1111/j.1365-246X.1994.tb03981.x, 1994.

Juusola, L., Kauristie, K., Vanhamäki, H., and Aikio, A.: Comparison of auroral ionospheric and field-aligned currents derived from Swarm and ground magnetic field measurements, J. Geophys. Res. Space Physics, 121, 9256–9283, https://doi.org/https://doi.org/10.1002/2016JA022961, 2016.

Vanhamäki, H. and Juusola, L.: Introduction to Spherical Elementary Current Systems, in: Ionospheric Multi-Spacecraft Analysis Tools, pp. 5–33, ISSI Scientific Report Series 17, https://doi.org/https://doi.org/10.1007/978-3-030-26732-2, 2020.