S1 Power spectral slope estimate $\hat{\tilde{\mathbf{S}}}_{GFT}$ from $\hat{\mathbf{p}}$

The estimated parameter matrix

$$\hat{\mathbf{p}} = [\hat{a}_1, \hat{a}_2, ...\hat{a}_M, \hat{b}_1, ...\hat{b}_M]^T \tag{S1.1}$$

is rewritten as a complex vector

$$\hat{\mathbf{Z}} = \hat{\mathbf{a}} + \hat{\mathbf{b}}\, j. \tag{S1.2}$$

The power spectra of this vector is than

$$\hat{\tilde{\mathbf{S}}} = \frac{N_c}{2N_i}\, Real\left(\hat{\mathbf{Z}}\, \hat{\mathbf{Z}}^*\right) \tag{S1.3}$$

with $N_c = L/dx = 2500$ as the maximum possible observation and N_i as actual observations per segment.

Supplementary Tables

Track	Internal ID	ATL03 & ATL07/10
Track 1	SH_20190224_08800210	ATL03_20190224022046_08800210_005_01 ATL10-02_20190224012038_08800201_005_01
Track 2	SH_20190219_08070210	ATL03_20190219073735_08070210_005_01 ATL10-02_20190219063727_08070201_005_01
Track 3	SH_20190502_05160312	ATL03_20190502021224_05160312_005_01 ATL10-02_20190502005851_05160301_005_01
Track 4	SH_20190502_05180312	ATL03_20190502052058_05180312_005_01 ATL10-02_20190502040726_05180301_005_01

Table 1: Tracks names and conventions used in the main text.

Supplementary Figures
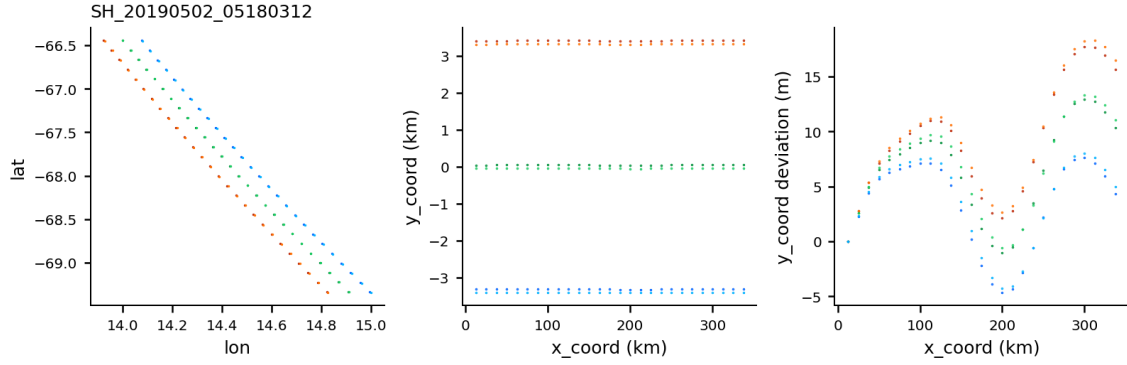
SH_20190502_05180312

Figure S1: Segment positions of the 6 beams (color coded as in Figure S2) for Track 4 in (a) latitude-longitude coordinates, (b) X-Y-coordinates defined relative to the sea-ice edge on the nominal track position (see Figure S2), and (c) same as (b) but using $Y' = Y - Y(0)$ for each beam.

Beams in the along-track coordinate system
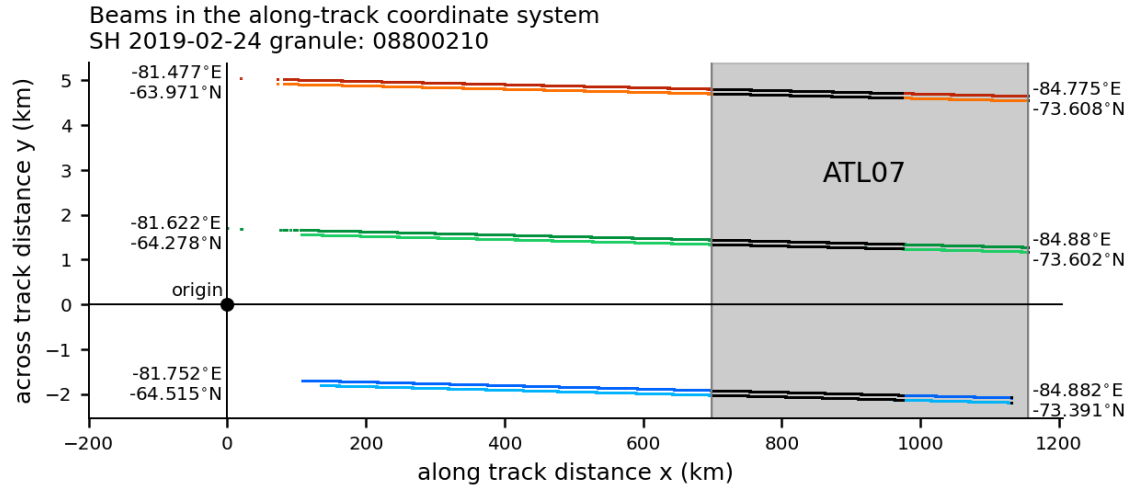SH 2019-02-24 granule: 08800210

Figure S2: ATL03 Beams with respect to the ground track in the along- and across-track coordinate system for Track 1. The reference point for this analysis is shown as black dot, and the photon position of the six beams referenced to this point are shown in red (gt1l), orange (gt1r), dark green (gt2l), light green (gt2r), dark blue (gt3l), and light blue (gt3r). The ATL07/10 data is available within the gray area and is shown as black lines. The starting and end positions of each beam pair are also shown in latitude-longitude coordinates.
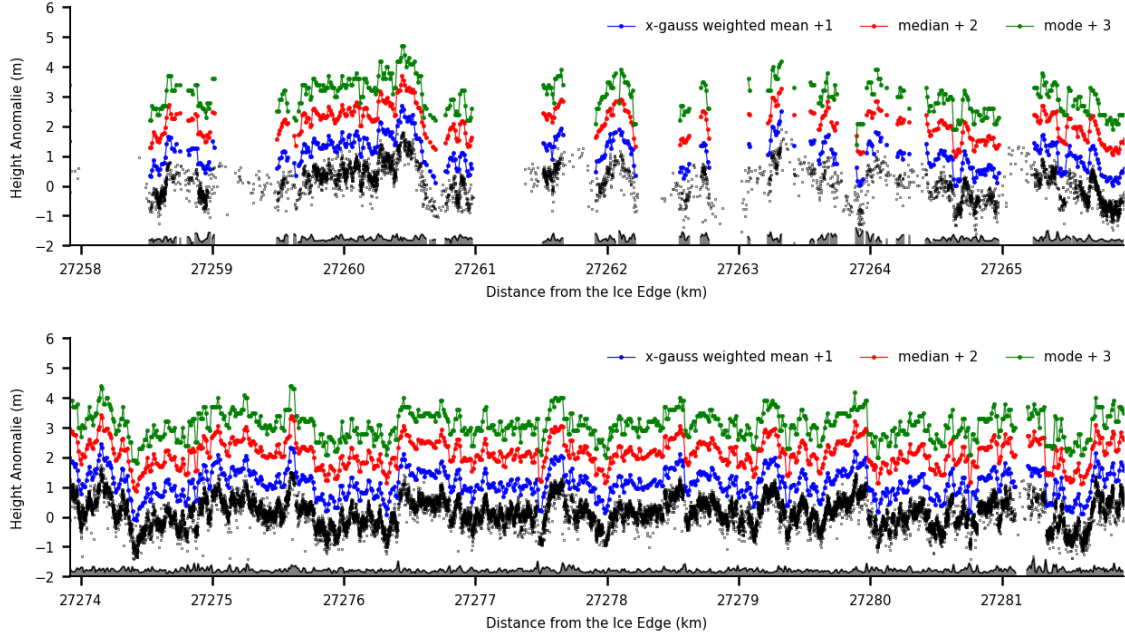
2

Figure S3: Example photon heights in the MIZ from granule SH 20190208 06440212. The ATL03 data is shown as back dots and three different binning methods offset by 1 meter.
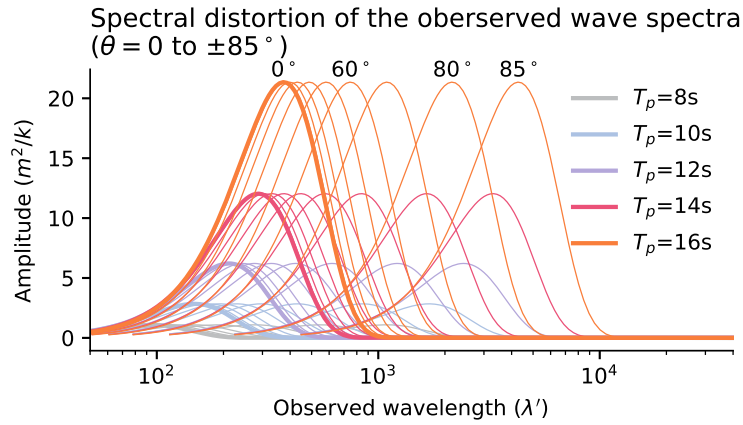

Figure S4: Illustration of the geometric distortion of open ocean spectra depending on the incident angle and peak period.

Prior
LOPS WW3-GLOB-30M
SH_2
Incident angle

9.8sec
2.3m
-122.3deg

6.3sec
1.1m 113.3d
9.2sec
1.4m -93.8d
11.3sec
0.9m 84.1d
12.4sec
0.8m 62.6d
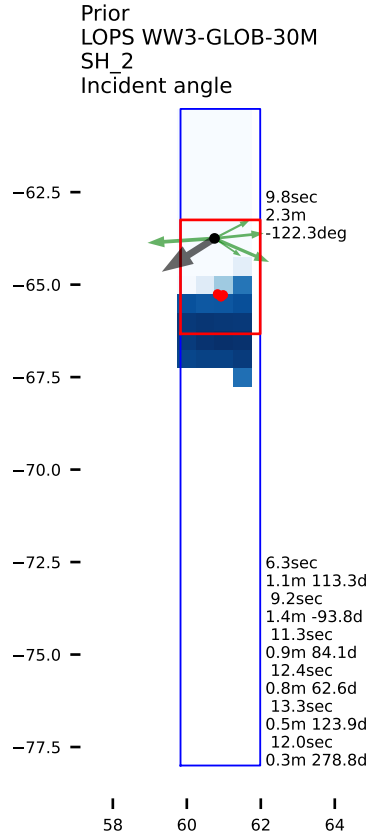13.3sec
0.5m 123.9d
12.0sec
0.3m 278.8d

Figure S5: Derivation of the angle prior $\mathbf{P}_\theta$ based on WW3 partition (green arrows). The red box is the region north of the sea ice (blue shading) with at least 66% open water around the ATL03 most-equatorward data point (red dots). The box mean of the partitions peak wave direction, period and significant wave height (Hs) as provided by WW3 are shown as green arrows. The total peak wave direction, period and Hs across all frequencies is shown as black arrow. The green arrows correspond to the black dots in main-text figure 7.
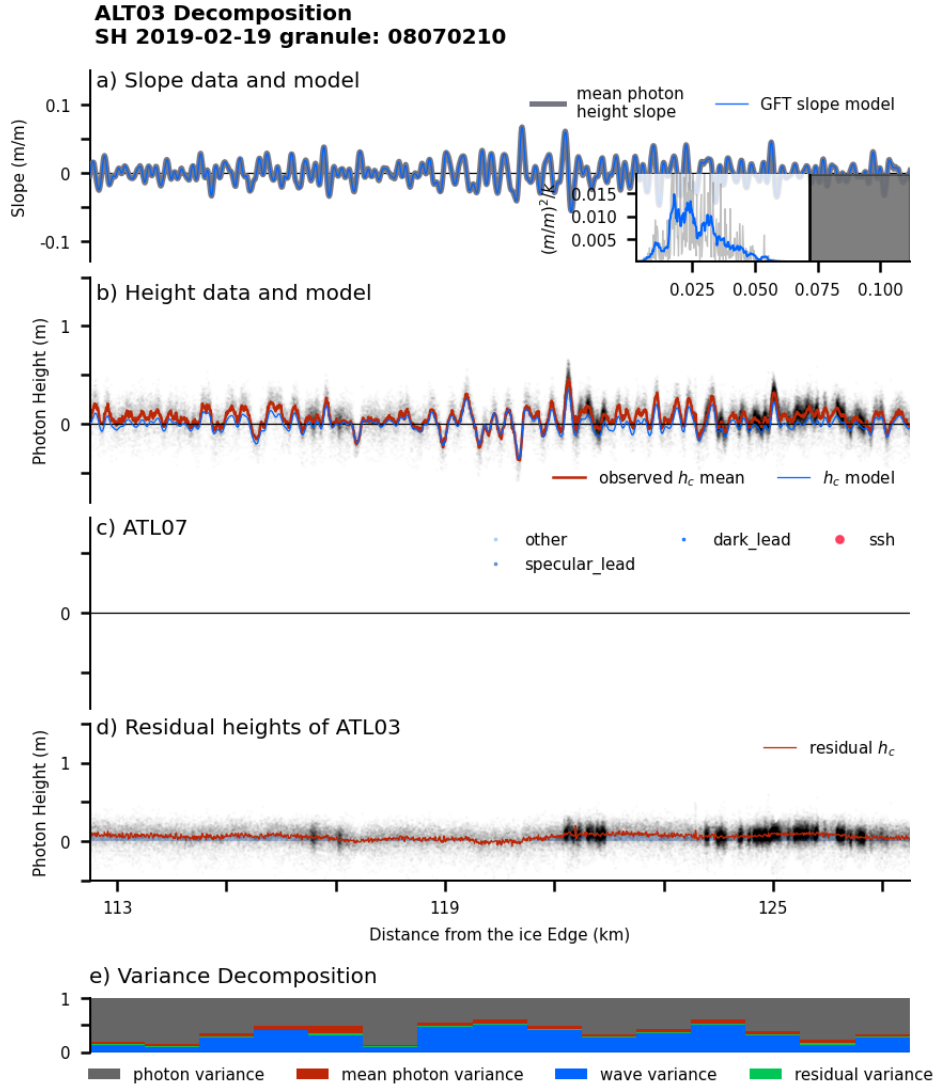
4

ALT03 Decomposition
SH 2019-02-19 granule: 08070210

a) Slope data and model

mean photon height slope   GFT slope model

b) Height data and model

observed h_c mean h_c model

c) ATL07

other   dark_lead   ssh
specular_lead

d) Residual heights of ATL03

residual h_c

Distance from the ice Edge (km)

e) Variance Decomposition

photon variance   mean photon variance   wave variance   residual variance

Figure S6: Same as main-text figure 10 but for another segment on Track 2. In this case no ATL07 data is not provided.
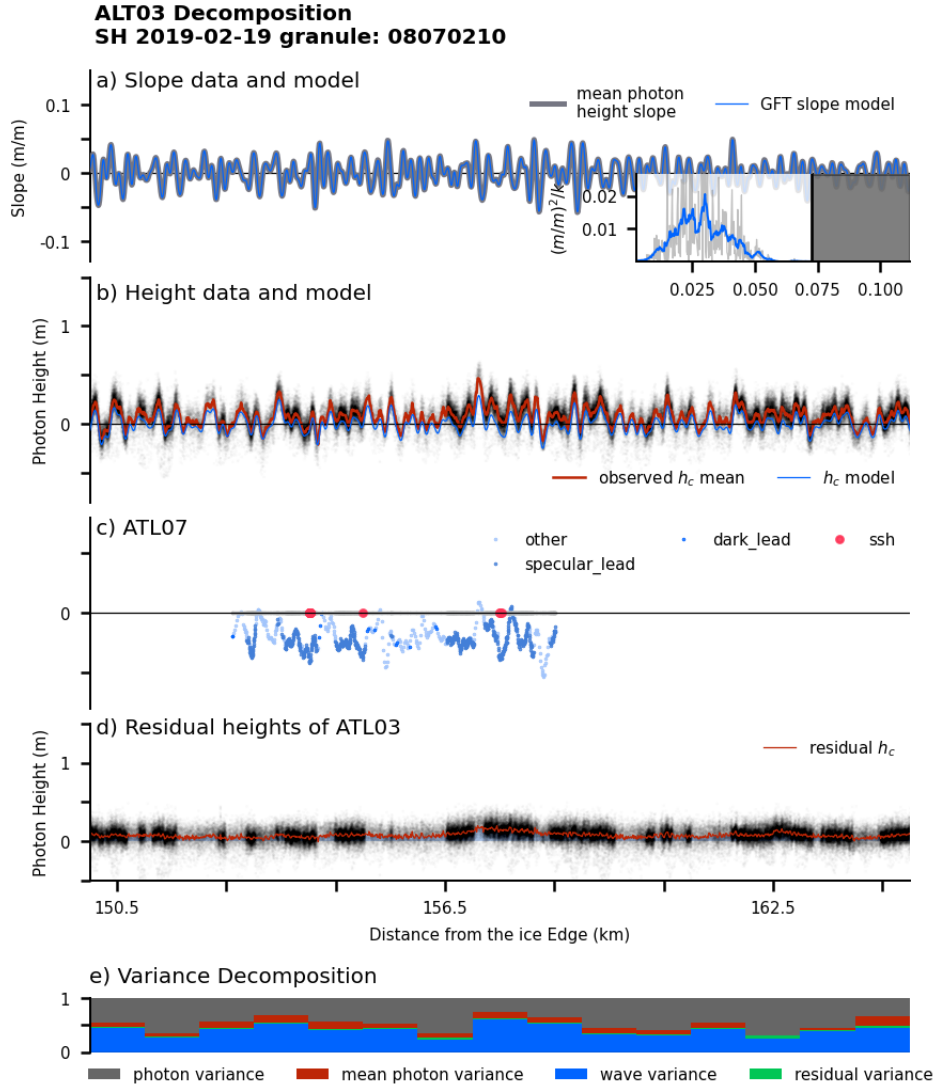
5

Figure S7: Same as main-text figure 10 but for another segment on Track 2. In this case most of the ATL07 data is not provided.
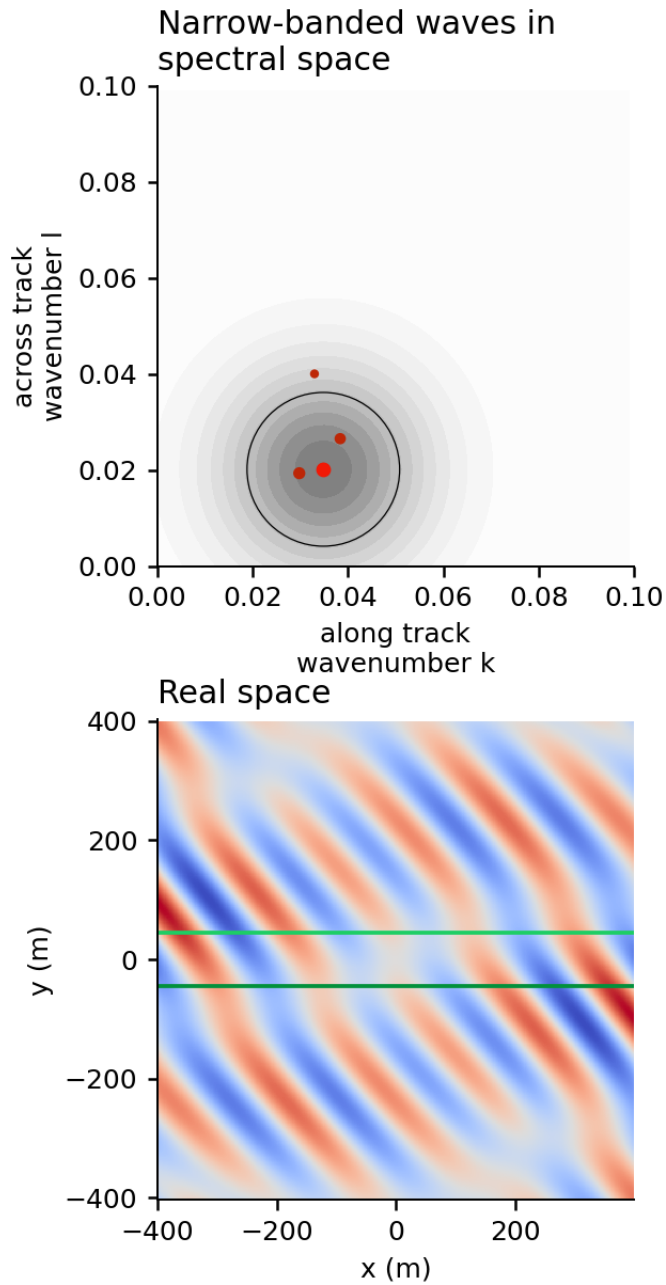
Figure S8: Synthetic narrow-banded wave spectrum. (a) for a given peak direction (thick red dot) and standard deviation (black circle) three additional random wavenumbers where chosen (darker red dots). (b) Realization of the wave field defined by the 4 wavenumbers in (a) with random phase. The green lines illustrate two neighboring ICESat-2 tracks.

Weighted marginal PDFs for
X_i=62 km for track 07190212
on 20190213

a) gt1 pair weighted mean

b) gt2 pair weighted mean

c) gt3 pair weighted mean

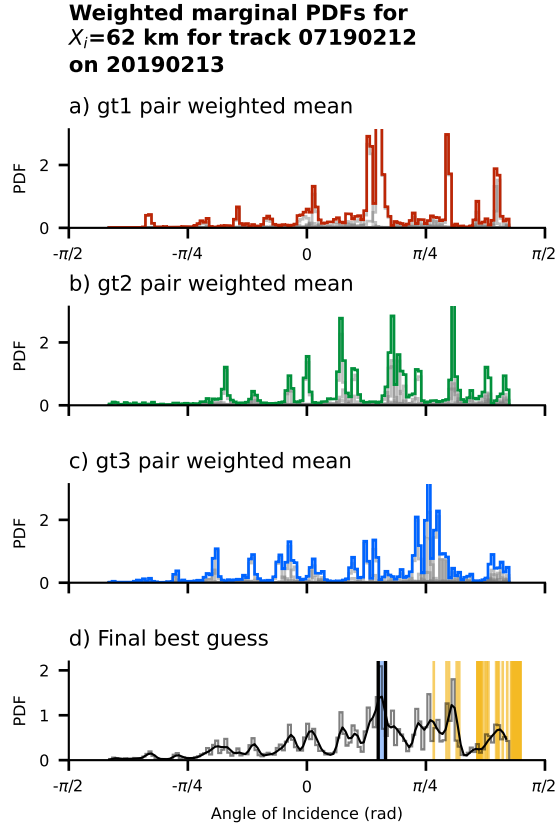d) Final best guess

Angle of Incidence (rad)

Figure S9: Weighted marginal PDF at $X_i = 62$ km for the gt1 pair in red (a), gt2 pair in green (b) and gt3 pair in blue (c). Each marginal distribution is weighted by the power of the corresponding wavenumber and the number of data points of the beam segment. The marginal distributions for each wavenumber are shown in gray. d) The weighted mean across the pairs with the most likely incident angle as thick blue line and the WW3 priors for all used wave numbers as orange lines.