Replies to D. Ringeisen:

Thanks for your detailed and helpful comments and suggestions. Please find below point-by-point replies (in blue) to your comments and questions (which are reprinted in black).

This manuscript presents a new model for the fracture of sea ice at the intermediate scale, from 100 m to 10 km, using the phase-field method. The manuscript describes the model and then presents some tests where the orientation of the inserted fracture set is studied as a function of the critical failure stress. The authors then discuss the challenges of using this model inside a sea ice DEM and the outlook for ice field campaigns.

The manuscript is well-written, concise, clear, and well-presented. It is a valuable contribution to the field of sea ice modeling, especially to the current effort of high resolution sea ice DEMs. However, I have some comments: I think there is some missing key literature related to sea ice modeling, especially to non-DEM sea ice models and fractures, the relationship between the orientation and the critical stress is a bit unclear, and compressive and shear tests are missing. Finally, I wonder if this manuscript should be better suited for Copernicus' Geoscientific Model Development (GMD) journal instead of The Cryosphere.

Therefore, I recommend this manuscript for publication with major revisions after my comments have been addressed. Below you will find my general comments, specific comments, and technical corrections. References are listed at the end of this document.

General Comments

Research paper or model development paper

I really like this manuscript; it is a good step for the fracture of floes in DEM models. However, I wonder if it would not be better suited for the Geoscientific Model Development (GMD) journal instead of The Cryosphere (TC).
The main result is that the orientation of the lines of reduced thickness is an important factor for fracturing the floe and determines the critical stress. This result is not surprising as embedded lines of reduced thickness reduce the ice strength, so the results look more like a proof-of-concept for the phase-field model for floe fracture than results about sea ice physics. Alternatively, some simulations could be added to describe more the physics of such a model and the behavior of the modeled ice floe and strengthen the manuscript, see my specific comments below.

Specific comments

The notation u-displacement or velocity?

There is a confusing notation that needs to be addressed. The authors use the notation on L66 for the displacement vector. Then, they define as the velocity vector on L140. Per convention and common use, is better suited for velocity. I suggest using something like for the displacement field and keeping for the velocity field. This way, any confusion with the sea ice VP models, where the strain-rate with the velocity, would be avoided.
We will clarify the notation when we revise the manuscript. Thanks for pointing this out.

Only tension tests

This manuscript only shows tensile tests, although observations of sea ice show more compressive or shear deformations. Is there a reason for this choice? I would like to see how the model behaves with shear and

compressive tests, with one example of each. It would strengthen the manuscript to have more examples of how sea ice behaves. I would guess these are forcing situations that are very likely to happen in the ICEx 2018 datasets.
We will study shearing and compression experiments when we update the manuscript, and add some of these tests to the paper. If one of the experiments (we believe the shearing experiment is more likely to be promising) proves to be physically interesting, we will perform a statistical study with random ice impurities similar to what we did for the tension test and add it to the manuscript. Thanks for the suggestion—we agree that only tension tests is somewhat restrictive.

Missing literature

I think this manuscript is missing some key literature about sea ice models and observations.
Page 1, ca. L20:: I think the author should mention the Elastic Anisotropic Plastic model at this point, which takes into account anisotropy in the ice.
Yes, that reference will make a nice addition to the other intermediate-scale models.

Page 2, L38 to L40: There are many studies studying the self-similarity of sea ice in observations and models. I think those should be cited here. See, for example, Hutter et al. (2019), Bouchat et al. (2022), Rampal (2019)
We will review and cite these papers. They are more recent than the currently cited papers on self-similarity in sea ice.

Page 10 and introduction: The literature linking fracture angles of sea ice to stress in other types of models is missing (e.g., Hibler & Schulson (2000), Dansereau et al. (2019), Plante and Tremblay (2021), Ringeisen et al. (2021), Wilchinsky et al. (2011))
Those papers will be added. Thank you for pointing them out.

Relationship between critical stress and orientation of thickness lines

Page 10, L210 to 220: The correlation between and critical force appears inexistent to me, maybe due to the choice of figure. If the correlation is weak, a correlation coefficient and significance number should be given.
We will add the correlation coefficient and some measure of correlation strength. We do not claim that a strong relation exists and we will emphasize this point in our next update.

To improve the figure, you could use two panels, one for critical force and and one for average thickness and α.
The figure will be improved. We agree that it looks confusing.

The orientation of fractures in sea ice is being investigated in many sea ice models with different physics. It is usually done at larger scales than the ones presented here, but I think it is nevertheless important to mention them. Sea ice rheological models like the VP or the brittle models (MEB/BBM) set a preferred angle in their physics. I would like to know if this model sets preferred angles (Dansereau et al. (2019), Plante and Tremblay (2021), Ringeisen et al. (2021))).
This is an interesting question. However, as you point out, these studies consider much larger spatial scales than the individual ice floe scale we consider here. For floe-size scale (i.e., hundreds of meters or a few kilometers), the nucleation and direction of fracture is likely be governed by impurities, by geometric singularities that lead to stress localization, and by concentrated forces due to collisions. Preferred fracture direction can be engineered into the phase field model using anisotropic elastic parameters, but this does

not seem desirable for (rotating and moving) ice floes.

It would be interesting to see the critical stress dependence with the orientation of a single all-through line of the same reduced thickness, a bit like the study done in Fig. 5., but when the goal is to find the lowest bound of critical stress instead of the highest.
We will include a study of the floe strength depending of the angle of a single all-through line of reduced thickness. We agree that it's not clear that a vertical line results in the lowest strength for tension experiments.

Numerical cost - advantages compared to other models

I could imagine doing similar tests with a VP model. However, it would take an enormous amount of computer time because the numerical convergence of the solver is very slow, especially at a 5m resolution. The cost is discussed on page 12, but this model seems much more efficient than the VP, with which I cannot fathom doing 1000s of simulations. I would be interested to know how fast the model presented here can predict a fracture, e.g., how much time it takes per processor per 1000 random samples.
We appreciate your comments, but believe that such a comparison in favor of the approach presented here is not fair towards VP models. The phase field approach we present is instantaneous, so it does not include time evolution (like VP), and it's also only useful for initially undamaged ice floes. As discussed in the manuscript, if such a phase field model is part of a discrete element model, then in absence of a faster approximate model, many phase field computations for floes in each (or at least many) time steps are necessary, and the time evolution still needs to be taken care of by another model (e.g., the DEM model).

Technical corrections

- Page 1, L10-12: I find the sentence unclear
  We will rephrase the sentence. We agree that sentence sounds vague.
- Page 3, L58 to L63: This paragraph is unnecessary. The titles of the subsections are sufficient, and these section introductions are unnecessary.
  We will consider removing it.
- Page 3, L70: The name of the operator should be given for clarity.
  We will name the operator.
- Page 6, L146 to L150: I think this paragraph is unnecessary (same as above)
  We will consider removing both.
- Page 7, L175: is it the ice floe domain?
  Yes. We will consider reword the sentence for clarity.
- Page 7, L180: So the resolution of the experiment is $dx = 5m$. Can you say the number for completeness?
  We added this number to the manuscript.
- Page 11, L232 to L235: I think this paragraph is unnecessary (same as above)
  We will consider removing this paragraph.

Bibliography

1. Bouchat, A., Hutter, N., Chanut, J., Dupont, F., Dukhovskoy, D., Garric, G., Lee, Y. J., Lemieux, J.-F., Lique, C., Losch, M., Maslowski, W., Myers, P. G., Olason, E., Rampal, P., Rasmussen, T.,

Talandier, C., Tremblay, B., & Wang, Q. (2022). Sea Ice Rheology Experiment (SIREx): 1. Scaling and Statistical Properties of Sea-Ice Deformation Fields. Journal of Geophysical Research: Oceans, 127(4), e2021JC017667. https://doi.org/10.1029/2021JC017667

2. Dansereau, V., Demery, V., Berthier, E., Weiss, J., & Ponson, L. (2019). Collective Damage Growth Controls Fault Orientation in Quasibrittle Compressive Failure. Physical Review Letters, 122(8), 085501. https://doi.org/10.1103/PhysRevLett.122.085501

3. Hibler, W. D., & Schulson, E. M. (2000). On modeling the anisotropic failure and flow of flawed sea ice. Journal of Geophysical Research: Oceans, 105(C7), 17105-17120. https://doi.org/10.1029/2000JC900045

4. Hutter, N., Zampieri, L., & Losch, M. (2019). Leads and ridges in Arctic sea ice from RGPS data and a new tracking algorithm. The Cryosphere, 13(2), 627-645. https://doi.org/10.5194/tc-13-627-2019

5. Plante, M., & Tremblay, L. B. (2021). A generalized stress correction scheme for the Maxwell elasto-brittle rheology: Impact on the fracture angles and deformations. The Cryosphere, 15(12), 5623-5638. https://doi.org/10.5194/tc-15-5623-2021 Rampal, P., Dansereau, V., Olason, E., Bouillon, S., Williams, T., Korosov, A., & Samake, A. (2019). On the multi-fractal scaling properties of sea ice deformation. The Cryosphere, 13(9), 2457-2474. https://doi.org/10.5194/tc-13-2457-2019

6. Ringeisen, D., Tremblay, L. B., & Losch, M. (2021). Non-normal flow rules affect fracture angles in sea ice viscous-plastic rheologies. The Cryosphere, 15(6), 2873-2888. https://doi.org/10.5194/tc-15-2873-2021

7. Wilchinsky, A. V., Feltham, D. L., & Hopkins, M. A. (2011). Modelling the reorientation of sea-ice faults as the wind changes direction. Annals of Glaciology, 52(57), 83-90. https://doi.org/10.3189/1727564117959